The Cost of Non-Decreasing Pay: Tenured Academics and Civil Servants

Stanimir Morfov*

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Abstract

This paper formally analyzes the optimal implementation of compensation schemes where the wage cannot decrease in time. Such schemes are commonly applied to tenured academics and civil servants. We use a finite-horizon model of moral hazard to analyze the disincentive effects and the cost of the scheme. Myopia is shown to hurt the principal two-fold. If the principal is myopic, she is slow to adjust the contract to binding wages in the future. As a result, any adverse effect on incentives shows up late in the relationship. If the agent is myopic, the principal cannot decrease current wages sufficiently to offset the expected slackness in agent’s future individual rationality. We also build an infinite horizon model where current wage binds future wages from below. The dynamic problem is reduced to a series of single-period problems stationary on a properly chosen state space, which allows for computing the model iteratively.

*State University - Higher School of Economics, International College of Economics and Finance, Pokrovski bulvar 11, Moscow 109028, Russian Federation, smorfov@hse.ru
1 Introduction

Recently, there has been some heated debate over the pay of civil servants. In the US, their compensation is often a result of collective bargaining and may not be so easy to cut as illustrated by the recent events in Wisconsin. In Europe, civil servants are protected from firing and their wages are categorized in such a way that an increasing profile is guaranteed. Moreover, indexation is common practice. Tenured academics enjoy similar privileges worldwide. They are difficult to fire and in practice face wages that do not decrease in time.\footnote{For example, in Russia where academic tenure is not legally sanctioned, it is still practiced in the New Economic School and the Higher School of Economics in Moscow.} Whether the non-decreasing pay arrangement is based on legislation or usual practice, it does apply to a substantial number of employees worldwide.

Consider academic tenure. Why provide it at all? Carmichael (1988) uses an overlapping generations framework to demonstrate that academic tenure guarantees that the university hires the best candidates by creating the proper incentives for the incumbent faculty to reveal truthfully their signals about the quality of junior candidates. McPherson and Schapiro (1999) and Siow (1998) highlight the role of academic tenure in alleviating the problems of obsolescence and underspecialization. Freeman (1977) shows that academic tenure is inherent to environments where employers and employees have symmetric information about research productivity. Glaeser (2002) relates academic tenure to the non-profit orientation of the research industry and the growth in university resources leading to an increased influence of faculty over administration. Khovanskaya, Sonin, and Yudkevich (2007) use a dynamic model to analyze the relationship between tenure and university budgets first identified by Alchian (1959). Li and Ou-Yang (2003) consider 249 economists and 68 financial economists and find that their impact measured by total citations does not decline significantly after tenure. They explain their result by the existence of implicit incentives for tenured researchers: becoming a field leader, winning a prestigious prize, etc. Interestingly, Oyer (2007) finds that there is an insider advantage in getting tenure at least as far as academic economists are concerned.


Academic wages have usually been analyzed in models of hidden information. However, it would be interesting to see whether a tenure-like non-decreasing compensation scheme can be supported in dynamic models of hidden action. Wang (1997) generates optimal time paths of CEO compensation in a repeated moral hazard setting where managerial effort is unobservable. The average wage exhibits a slightly positive trend but is marked by negative autocorrelation. Morfov (2010) analyzes a similar framework where both the principal and the agent cannot commit to long-term contracts and face history-dependent outside options. Following a parameterization by Aseff and Santos (2005) and Aseff (2004), he computes the dynamically optimal executive pay and obtains smoother time paths which tend to grow significantly at the lower end of the wealth distribution. Wages are still negatively autocorrelated for wealthier CEOs.

The current paper models the optimal tenure grant in a dynamic interaction between a principal and an agent. The principal (she) wants the agent (him) to operate some stochastic technology mapping agent’s effort to outcomes. Effort, however, is unobservable and the principal has to specify outcome-contingent wages instead. Moreover, in order to induce a certain level of effort,
the contract should be incentive-compatible. The principal can commit to long term contracts, but the agent has a constant outside option, which requires that the contract should be individually rational. The non-decreasing wage constraint is exogenous and requires that the compensation scheme in the current period is not below the wage received by the agent in the previous period. In other words, by designing an outcome contingent pay scheme today, the principal effectively sets tomorrow’s minimum wages.

We first consider a two-period model and analyze the optimal contracts observed under non-decreasing wage constraint. We consider three subcases: when both the principal and the agent are myopic, when only the agent is myopic, and when nobody is myopic. Myopia is taken as equivalent to short-termism in the sense that a myopic party to the contract will only care about current period utility ignoring the implications of his/her decisions on future utility. We identify parameter conditions that determine the observed optimal contracts. In general, the non-decreasing wage constraint does hurt the principal and may provoke her to switch her effort recommendation from high to low. In case the principal is not myopic, such a switch may also occur in the initial period. Interestingly, when nobody is myopic, the principal can adjust initial wages more efficiently to partially offset the expected wage increase.

Next, we consider the infinite horizon and reduce the dynamic problem to a series of single-period problems stationary to a properly chosen state space. The state space contains the agent’s continuation utilities and wage lower bounds that can be supported by single-period, feasible and consistent contracts that are temporary incentive-compatible in the sense of Green (1987) and guarantee the participation of the agent. Note that the state space is two-dimensional. In the spirit of Spear and Srivastava (1987), it includes the agent’s expected discounted utility, but we also add the endogenous “minimum” wage. We construct a set operator that satisfies Abreu, Pearce and Stacchetti (1990)’s self-generation and factorization with respect to the state space. Iterating the operator on a proper initial guess leads to a convergence to its largest fixed point; namely, the state space. We characterize the maximum utility the principal can obtain on each point of the state space by a generalized Bellman equation on bounded upper semi-continuous functions. The value function can be derived by dynamic programming techniques. The dynamic contract can then be recovered by successively applying the optimal policies and laws of motion for the state variables.

We also discuss extensions of the dynamic model where the lower bound on tomorrow’s wage depends on today’s contingent wages in a more general way. For example, assume that there are only two possible outcomes: boom and bust. Suppose that the principal offers the agent a compensation scheme that promises \( w_{\text{boom}} \) if a boom has been observed today and \( w_{\text{bust}} \) if a bust has been observed instead. In our original model, the feasible compensation schemes tomorrow will be the ones that pay the agent at least \( w_{\text{boom}} \) if tomorrow’s outcome is a boom and at least \( w_{\text{bust}} \) if it is a bust. We can extend the model so that tomorrow’s contingent minimum wages should be some continuous function of both \( w_{\text{boom}} \) and \( w_{\text{bust}} \). For example, it may be \( \min \{ w_{\text{boom}}, w_{\text{bust}} \} \).

Section 2 analyzes the effect of restricting the principal to non-decreasing wage trajectories in a simple two-period model. Section 3 discusses the recursive structure of a related infinite horizon model. Section 4 concludes. Appendix A contains the proofs. Appendix B extends the infinite horizon analysis to a more general set-up where wages depend on all the components of the previous period compensation scheme.
2 A two-period model

The model considers a moral hazard problem in a principal-agent framework with one-sided commitment. Each period, the principal (she) needs the agent (him) to implement some action that stochastically affects a variable of principal’s interest, but suffers from the fact that the action is observable only by the agent. Given that the variable of interest to the principal is publicly observable, the principal may want to condition the wage of the agent on the realization of this variable instead. However, the issue of inducing the proper incentives is further complicated by the lack of commitment on part of the agent. He faces a constant outside option and may always choose to leave the contract and pursue it instead.

The innovation to the optimal contract literature is that the current-period wage scheme is bounded from below by the wage received by the agent in the previous period. The restriction is exogenous and reflects existing legislation or usual practice in rewarding tenured academics and civil servants.

Consider academic tenure. It is a contractual arrangement which is common to research centers and universities. A tenure grant provides a faculty member with a high degree of job security. It usually amounts to a “lifetime” employment or, at least, severely restricts the ability of the academic institution to fire senior faculty. Tenure is often accompanied by some form of salary guarantee which may or may not be made explicit in the contract. An interesting case is when the tenure decision forces the employer to honor a long-term contract providing the employee with compensation that cannot decrease in time. Contractual arrangements of this type are not restricted to academia. Indeed, civil servants are usually employed under “lifetime” compensation-guaranteed schemes.

In such a framework, by setting today’s compensation, the principal effectively determines the wage bound tomorrow. Therefore, if the constraint binds, the model will exhibit history dependence. Moreover, providing the necessary incentives for the agent to work hard will be much more complicated. Under standard assumptions, implementing high effort requires a wage scheme that increases in the outcome. Since future wages should be at least the wage corresponding to today’s outcome realization, higher realizations would make it much more difficult to punish the agent for shirking tomorrow. The higher cost of implementing high effort will hurt the principal and may provoke her to allow the agent to shirk. Such a decision is more likely to follow good outcomes since then the lower bound to the wage will be higher. Nevertheless, if the discounted cost is high enough, shirking may become optimal even after low outcomes.

To analyze the effect of the non-decreasing wage constraint, we will start by a two-period model that can trivially be extended to any finite horizon.

For simplicity, we assume two possible outcomes of the variable of interest to principal and two possible action/effort levels. Let the outcome and action spaces are respectively $Y = \{y, \overline{y}\}$, where $y < \overline{y}$, and $A = \{a, \overline{a}\}$, where $a < \overline{a}$. The distribution of the outcome conditional on agent’s effort is described by a probability mass function $\pi : Y \times A \rightarrow (0,1)$. Let $\underline{\pi} := \pi(y, a)$ be the probability of low outcome conditional on low effort and $\overline{\pi} := \pi(y, \overline{a})$ be the probability of low outcome conditional on high effort. We assume that outcome distribution conditional on high effort stochastically dominates the distribution conditional on low effort, i.e., $\overline{\pi} > \underline{\pi}$, which also guarantees that strong monotonicity of the likelihood ratio (SMLR) holds. Given an outcome $y$, effort $a$, and wage $w$, the agent’s period utility is $v(w) - a$, where $v$ is twice continuously differentiable, strictly increasing and strictly concave. The respective utility of the principal is given by $y - w$. Both the agent and the principal discount the future by a factor $\beta \in (0,1)$. Each period the agent can
walk out of the contract and receive a pay of $v^{-1}(V)$ without exerting any effort. $V$ is known by both the principal and the agent and is the same each period. The timing is as follows. In the beginning of each period a contract is signed that recommends an effort level and specifies an outcome-contingent compensation scheme for the agent. The agent exerts some effort. Then, nature picks up an outcome according to the respective conditional distribution. The outcome is observed by both parties. The principal pays the agent the wage corresponding to the observed outcome. The agent consumes all his pay. Given the unobservability of effort, the principal offers an incentive compatible contract guaranteeing that the effort recommended by the contract is actually implemented. The contract also needs to be individually rational since otherwise the agent will revert to his outside option.

We will first analyze the model where no restriction is imposed on second period wages.

2.1 Optimal contract with no restrictions on pay

Without the history dependence stemming from the constraint on future wages, the principal’s two-period problem is equivalent to two single-period problems. Let us consider one of them:

\[
\begin{align*}
\text{[PP]} & \quad \max_{a, w} \sum_{y \in Y} (y - w) \pi(y, a) \quad \text{s.t.:} \\
& \quad a \in A \\
& \quad \sum_{y \in Y} (v(w) - a) \pi(y, a) \geq V \\
& \quad a \in \arg\max_{a' \in A} \sum_{y \in Y} (v(w) - a') \pi(y, a')
\end{align*}
\]

(1) is a feasibility constraint for the exerted effort. It simply says that the recommended effort is an element of $A$. (2) is the individual rationality constraint for the agent which requires that his expected utility under the contract is not less than his reservation utility. (3) is the incentive compatibility constraint which guarantees that the agent chooses to exert the recommended effort. Since we have two feasible levels of effort, (3) is in fact a single inequality constraint.

It is straightforward to show\(^2\) that low effort is implemented by a fixed wage $v^{-1}(v)$, while high effort is implemented by a compensation scheme that offers $v^{-1}(v + (1 - \pi)k)$ after a bad outcome $y$ and $v^{-1}(v + \pi k)$ after a good outcome $\overline{y}$, where $v := V + q$ and $k := \frac{\pi - a}{\pi - \pi} > 0$. Since SMLR holds, the wage implementing high effort is strictly increasing in the outcome. The wage implementing low effort is strictly between the two components of the compensation scheme implementing high effort. The term $k$ measures the direct utility loss of an agent switching from low to high effort over the respective gain in the probability of success. Since success means a higher wage, an increase in

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\(^2\)Following, for example, Mas-Colell, Whinston and Green (1995), low effort is implemented by a fixed wage such that individual rationality binds, while high effort is implemented by a compensation scheme for which both individual rationality and incentive compatibility bind.

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the probability of success increases the agent’s expected utility of consumption. Therefore, we can regard $k$ as a measure of the difficulty of implementing high over low effort. Another way to view $k$ is as the utility premium that the agent requires in order to exert high and not low effort. It is his realized utility-of-consumption gain in good versus bad-outcome states.

Which effort level will actually be implemented depends on the respective utility of the principal. She compares the gain in expected output associated with higher effort with the rise in the expected cost of its implementation.\(^3\) High effort will be optimal if $C := E(y|\overline{a}) - \pi v^{-1}(v + (1 - \pi)k) - (1 - \pi) v^{-1}(v + \pi k) - E(y|\underline{a}) + v^{-1}(\underline{v}) \geq 0$, low effort will be optimal otherwise.\(^4\)

The previous results are summarized in Proposition 1. In the proposition and hereafter, the compensation scheme is represented by a vector of two components where the first denotes the pay after a bad outcome, while the second denotes the pay after a good outcome. In case both components are the same, the compensation scheme is represented by a single element equal to the fixed pay.

**Proposition 1** In a two-period model with no restrictions on pay, the following holds:
(a) if $C < 0$, then $\underline{a}$ is optimal in both periods and is implemented by a fixed wage $v^{-1}(\underline{v})$;
(b) if $C \geq 0$, then $\overline{a}$ is optimal in both periods and is implemented by $(v^{-1}(v + (1 - \pi)k), v^{-1}(v + \pi k))$.

We are now ready to analyze the effect of restricting the principal to wages that do not decrease in time.

### 2.2 Optimal contract with non-decreasing pay

The constraint of non-decreasing pay introduces history dependence in the problem. In the second period, the contingent wage scheme should offer the agent at least as much as the wage he received at the end of the first period. Therefore, the second-period compensation depends on the first-period wage.

We will begin with a case where both the principal and the agent care only about current-period utility.

To facilitate the exposition, we denote the initial contracting node by 0, and the second period contracting node by 0B if the first-period outcome was bad and 0G if the first-period outcome was good.

#### 2.2.1 Myopic principal and myopic agent

Assume that both the principal and the agent are myopic in the sense that they do not account for the effect of their current period choices on their utility next period. While the inter-period linkage exists since in the second period the compensation is bound from below by the wage paid at the end of the first period, both parties ignore it when agreeing on first-period wages. Contracts are single-period and the second-period contracts face an effective minimum wage which is the wage paid to the agent in the previous period. Such a framework can be valid if both parties are short-termist. For example, the current administration may expect to be in office only for the current period and

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\(^3\) Since agent’s utility is concave in consumption and individual rationality binds for either effort level, it is more costly in expected terms to implement high versus low effort. Formally, $E(v^{-1}(.|\overline{a}) > v^{-1}(E(.|\overline{a})) = v^{-1}(\overline{v} + \overline{\pi}) > v^{-1}(\underline{v} + \underline{\pi}) = v^{-1}(\underline{v})$.

\(^4\) Here and hereafter, we assume that the principal will choose to implement high effort if indifferent.
the civil servant may expect to stay on this job for one period only or to see the legislation or common practice of non-decreasing wages overturned or respectively changed next period.

The principal’s first-period problem is the same as [PP]. In the second period, however, it matters whether a good or a bad outcome was observed in the first period.

Let us assume that low effort was optimal in period 1, i.e., \( C < 0 \). This means that the agent has received \( v^{-1}(y) \) at the end of the period no matter the outcome. Therefore, in the second period, the effective minimum wage is \( v^{-1}(y) \). Since low effort was optimal in the first period, it should also be optimal in the second period if there was not a minimum wage constraint. However, since low effort is implemented by a fixed wage \( v^{-1}(y) \), the constraint will not be violated in the second period and the contract will be the same as the one without any constraint on wages. Therefore, if \( C < 0 \), low effort will be optimal at any node and implemented by \( v^{-1}(y) \).

Now, let us assume that high effort was optimal in period 1, i.e., \( C \geq 0 \). Then, the effective minimum wage is \( v^{-1}(y + (1 - \pi) k) \) at 0B and \( v^{-1}(y + \pi k) \) at 0G. Since high effort was optimal in period 1, it should also be optimal in period 2 if there were no minimum wage constraints. At node 0B, the compensation scheme implementing high effort at 0 will not violate the minimum wage constraint and will therefore implement high effort at 0B. Since high effort was optimal at 0, it should also be optimal at 0B. What about 0G? The principal can implement low effort by offering a fixed wage equal to the minimum one, i.e., \( v^{-1}(y + \pi k) \). Indeed, a fixed wage satisfies the incentive compatibility for low effort. Since individual rationality binds at 0 and the fixed wage contracted at 0G is equal to the higher component of the compensation scheme offered at 0, the agent’s expected utility at 0G is strictly above his reservation utility. Any compensation scheme that satisfies the minimum reservation constraint at 0G and has some component greater than \( v^{-1}(y + \pi k) \) is more expensive than the proposed one. How can the principal implement high effort at 0G? By SMLR, the compensation should increase in the future outcome. Moreover, any compensation scheme satisfying the minimum wage constraint should bring the agent an expected utility which is strictly higher than the reservation one. Lemma 1 in Appendix A shows that two constraints in the principal’s problem at 0G are binding: the incentive compatibility constraint and the minimum wage constraint for the lower component of the pay, i.e., for the wage received following a bad outcome. The corresponding compensation scheme offers \( v^{-1}(y + \pi k) \) following a bad outcome and \( v^{-1}(y + (1 - \pi) k) \) following a good outcome. Which effort level will be chosen at 0G depends on the respective utility of the principal. High effort will be optimal if \( D := E(y_1|a) - E(y_1|a) + (1 - \pi) (v^{-1}(y + \pi k) - v^{-1}(y + (1 + \pi) k)) \geq 0 \), low effort will be optimal otherwise.

The previous results are summarized in Proposition 2. To ease up the exposition, we define the following three cases.

\[ G_0 \]

\[ G_1 \]

\[ G_2 \]

\( G_0 \) is the case where \( a \) is optimal at 0, 0B and 0G and is implemented by a compensation
scheme \( v^{-1}(u) \) at any node. \( G_1 \) is the case where \( \pi \) is optimal at 0 and 0B and is implemented by \((v^{-1}(u + (1 - \pi) k), v^{-1}(u + \pi k))\), while \( \alpha \) is optimal at 0G and is implemented by \( v^{-1}(u + \pi k) \). Finally, \( G_2 \) is the case where \( \pi \) is optimal at 0, 0B and 0G and is implemented by \((v^{-1}(u + (1 - \pi) k), v^{-1}(u + k))\) at 0 and 0B, and by \((v^{-1}(u + \pi k), v^{-1}(u + (1 + \pi) k))\) at 0G. Note that each of these cases fully describes the optimal contracts signed at each node 0, 0B, and 0G of the decision tree.

**Proposition 2** In a two-period model where both the principal and the agent are myopic, the following holds:

(a) \( C < 0 \) implies \( G_0 \);
(b) \( C \geq 0 \) and \( D < 0 \) imply \( G_1 \);
(c) \( C \geq 0 \) and \( D \geq 0 \) imply \( G_2 \).

In order to understand how the restriction to non-decreasing wages affects the optimal contracts, we compare these results with the results of Proposition 1. Since both the principal and the agent are myopic, the restriction on pay will not affect the contract signed at the initial node 0. There will be no change in the second period contract signed at 0B either. However, there might be changes in the second period contract signed at 0G. The restriction on wages increases the cost of implementation of high effort at 0G. If low effort is optimal with unconstrained wages, the restriction on pay will not affect the contract at 0G since the cost of implementation of low effort remains the same while the cost of implementation of high effort increases. If high effort is optimal with unconstrained wages, it may still be chosen by the principal at 0G if the restriction on pay has not raised the cost of its implementation too much relative to low effort. However, if \( D < 0 \), implementing high effort becomes too costly and the principal will choose low effort at 0G. The fixed wage that would implement this low effort would in fact equal the higher component of the compensation scheme implementing high effort with no restriction to pay.

To summarize, unless low effort was optimal with unconstrained wages, the restriction hurts the principal after a good first-period outcome. Moreover, she may find it too costly to incentivize the agent at this point and may leave him shirk instead.

### 2.2.2 Myopic agent

Here, we keep the agent myopic, but allow the principal to see the big picture. She now cares about her expected discounted utility from both periods when signing the contract in period 1. Therefore, the first-period contract is affected by the existence of minimum wage restrictions in the second period. The agent, however, is short-termist and ignores the restrictions when signing the contract in period 1. This means that while the principal maximizes expected discounted utility in period 1, her choices are subject to single-period individual rationality and incentive compatibility constraints of the form (2) and (3) respectively. The following proposition demonstrates how the principal implements a particular effort level in the second period.

**Proposition 3** Given a minimum wage \( w_1 \) in the second period of a two-period model where only the agent is myopic, the principal can implement low effort by \( \max \{ w_1, v^{-1}(u) \} \) and high effort by \( (w_2, v^{-1}(u + (w_2 + k))) \), where \( w_2 := \max \{ w_1, v^{-1}(u - (1 - \pi) k) \} \).

The proposition says that a principal facing a minimum wage constraint will try to implement a particular effort level as she would have without the constraint. If the respective compensation
scheme violates the constraint, she sets the bad-outcome wage equal to the minimum level. If implementing low effort, the good-outcome wage is also set equal to the minimum level, i.e., the principal offers a fixed compensation equal to the minimum wage. If implementing high effort, the good-outcome wage is chosen so that the agent is indifferent between working hard and shirking. In particular, consuming the good-outcome wage brings the agent utility that exceed the utility of consuming the minimum age by $k$.

Note that setting the minimum wage equal to $v^{-1}(v + (1 - \bar{\pi})k)$, $v^{-1}(v)$, or $v^{-1}(v + \bar{\pi}k)$ and combining it with the effort choices from Proposition 2 would result in the respective compensation schemes under each of the cases listed in Proposition 2. However, the principal may choose different levels of effort depending on whether she is myopic or not. Indeed, the increase in the second-period compensation above the one implied by the optimal contract with no restrictions on pay can make her switch her recommendation for the first period from high to low effort. Proposition 4 identifies the conditions under which each of the three cases will dominate the other two.

Let $K := E(y|\bar{\pi}) - \bar{\pi}v^{-1}(v + \bar{\pi}k) - (1 - \bar{\pi})v^{-1}(v + (1 + \pi)k) - E(y|\pi) + v^{-1}(v)$, and $L := v^{-1}(v) - v^{-1}(v + \pi k)$.

Proposition 4 In a two-period model where only the agent is myopic and the principal chooses among $G_0, G_1$ and $G_2$

(a) $K < 0$ and $(1 + \beta \bar{\pi})C + (1 - \beta)\bar{\pi} \max\{K, L\} < 0$ imply $G_0$;

(b) $(1 + \beta \bar{\pi})C + (1 - \beta)\bar{\pi}L \geq 0$ and $K < L$ imply $G_1$;

(c) $(1 + \beta \bar{\pi})C + (1 - \beta)\bar{\pi}K \geq 0$ and $L \leq K < 0$; or $K \geq 0$ imply $G_2$.

We should note that the proposition does not necessarily indicate principal’s optimal choices. The reason is that while disjoint, events $G_0, G_1$ and $G_2$ are no longer all inclusive. Now, there are three other possible cases which we call $G_3$, $G_4$ and $G_5$. In $G_3$, the principal implements low effort at 0 by $(w_{B,3}, w_{G,3})$ and at $0G$ by $w_{G,3}$ and she implements high effort in $0B$ by $(w_{B,3}, v^{-1}(v(w_{B,3} + k)))$. In $G_4$, the principal implements low effort at 0 by $(w_{B,4}, w_{G,4})$ and at $0B$ by $w_{B,4}$ and she implements high effort in $0G$ by $(w_{G,4}, v^{-1}(v(w_{G,4} + k)))$. The respective wages $(w_{B,3}, w_{G,3})$ and $(w_{B,4}, w_{G,4})$ are characterized in Lemma 2 in Appendix A. In $G_5$, the principal implements low effort at 0 by $v^{-1}(v)$ and high effort at $0B$ and $0G$ by $(v^{-1}(v), v^{-1}(v + k))$.

In the following proposition, we show that the principal will indeed choose one of the six cases identified above.

Proposition 5 In a two-period model where only the agent is myopic, the events $G_0, G_1, G_2, G_3, G_4$, and $G_5$ are mutually exclusive and collectively exhaustive.
Therefore, in order to determine the optimal contract, we need to find which case maximizes the expected discounted utility of the principal. Unfortunately, since we are not able to obtain a close-formed solution for \((w_{B,3}, w_{G,3})\) and \((w_{B,4}, w_{G,4})\) without an explicit formula for the first derivative of the agent’s utility of consumption, we cannot establish nice parameter conditions as to which case is indeed optimal as in Proposition 2. What we can say is that if low effort was optimal without restrictions on pay, i.e., \(C \geq 0\), it would also be optimal with non-decreasing wages. However, if high effort was initially optimal, i.e., \(C < 0\), a long-term utility maximizing principal may find it optimal to change her recommendation from high to low at some or all contracting nodes.

2.2.3 No myopia

Here, both the principal and the agent consider their expected discounted utility from both periods when contracting in period 1. Interestingly, the principal would do better than in the previous case where the agent was myopic. She can use the fact that the agent receives more than his reservation utility in the second period which is now also understood by the agent to decrease his first-period compensation such that individual rationality in a single-period contract would be in fact violated.

The agent will not walk away from the two-period contract since the decrease in his current utility due to the decrease in first-period pay will be compensated by higher wages in the second period. The agent’s utility decrease in the first period should of course be lower than his utility increase in the second period due to discounting.

3 An infinite-horizon model

Here, we extend the analysis to the infinite horizon. Time is discrete and indexed by \(t\). There is an initial period of contracting which is normalized to 0. The possible set of outcomes \(Y\) is stationary and consists of \(N\) distinct elements. The sets of possible actions and wages are both stationary and compact and are denoted by \(A\) and \(W\) respectively. The distribution of the current outcome conditional on current action is stationary and has full support. The principal’s period utility is given by \(u : W \times Y \rightarrow \mathbb{R}\) which is continuous, decreasing in the wage, and increasing in the outcome. She discounts the future by a factor \(\beta_p \in (0, 1)\) and commits to long-term contracts.

The agent’s period utility is given by \(\nu : W \times A \rightarrow \mathbb{R}\) which is continuous, increasing in the wage, and decreasing in the action. He discounts the future by a factor \(\beta_A \in (0, 1)\) and has a reservation utility \(V\) each period. Let \(c := (a, w)\) be a supercontract signed at the beginning of period 0. Denote by \(U_t(c, y^{t-1})\) and \(V_t(c, y^{t-1})\) the expected discounted utility of the principal and respectively the agent at node \(y^{t-1}\). Then, the optimal dynamic contract with non-decreasing pay is the solution of the following problem.

\[
\sup_{c} U_0(c) \quad \text{s.t.:} \\
\quad a_t(.) \in A \quad (4) \\
\quad w_t(y^{t-1}, .) \in W \cap [w_{t-1}(y^{t-1}), \infty) \quad (5) \\
\quad V_t(a, .) \geq V_t(a', .), \forall \text{ feasible } a' \\
\quad V_t(.) \geq V \quad (7)
\]

where \((4)\) is action feasibility, \((5)\) guarantees wage feasibility and a non-decreasing wage trajectory, \((6)\) is incentive compatibility and \((7)\) is individual rationality. The dynamic contract can be
represented recursively by a series of single-period contracts stationary on the space of wage lower bounds $W$ and agent’s corresponding continuation utilities $\{V^C(w) : w \in W\}$. In particular, let $c_s := \{(a_s, w_s(y), V_s(y)) : y \in Y\}$ be such a stationary contract specifying an action $a_s$, a contingent wage, or alternatively a continuation wage lower bound, $w_s(.)$ and a continuation utility $V_s(.)$. Take $w \in W$ and $V \in V^C(w)$. Then, we have the following generalized Bellman equation:

$$[\text{BE-1}]$$

$$U(V, w) = \max_{c_s} E_{a_s} \{u(w_s, \cdot) + \beta_p U(V_s)\} \quad \text{s.t.:}$$

$$a_s \in A \quad (8)$$

$$w_s(.) \in W \cap [w, +\infty) \quad (9)$$

$$V = E_{a_s} \{\nu(w_s, a_s) + \beta_A V_s\} \geq E_{a'} \{\nu(w_s, a') + \beta_A V_s\}, \forall a' \in A \quad (10)$$

$$V_s(.) \in V^C(w_s(.) \}) \quad (11)$$

where $E_{a_s} \{\cdot\}$ denotes the conditional expectation $E\{\cdot|a_s\}$. Constraint (8) is action feasibility, (9) guarantees that wage is feasible and above the relevant minimum wage, (10) imposes promise keeping and Green (1987)'s temporary incentive compatibility, while (11) guarantees promise consistency.

If there were no dynamic minimum-wage constraints, the state space would only consist of the agent’s continuation utilities. The algorithm of obtaining it is as follows. We start with a large initial guess, iterate on an APS operator and converge to the largest fixed point of the operator. This fixed point is the endogenous state space. A natural initial guess is $V_0 := \left[\nu(\min W, \max A), \nu(\max W, \min A)\right]$. With dynamic minimum wage constraints which guarantee non-decreasing wage trajectories, the state space needs to be enlarged by including the lower bounds on wages: $\{\{V, w\} : V \in V^C(w), w \in W\}$. To obtain it, we should start with a large initial guess, iterate on a modified APS operator and converge to the largest fixed point which would be the endogenous state space. A natural initial guess is $\{\{V, w\} : V \in V_0, w \in W\}$.

The numerical computation requires iteration on operators defined on two-dimensional sets. Initial experiments suggest that the state space shrinks pretty quickly requiring a frequent update of the two-dimensional grid.

Before concluding, we note that the infinite horizon framework presented above can be extended to handle more general cases of wage history dependence. The Bellman equation [BE-2] presented in Appendix B requires that the current wage is above the lowest component of the previous-period compensation scheme. This gives the principal more freedom in setting contingent compensation. The agent’s worst wage (under SMLR, the one following the worst outcome) is what sets the minimum wage tomorrow, so while keeping it low, the principal can reward good performance (by offering a bonus above the base wage) without affecting future pay.

An even more general version is [BE-3] in Appendix B which specifies today’s minimum wage as a continuous function of all the components of the compensation scheme yesterday. Both cases presented above can be encompassed in such a framework.

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5 See, for example, Spear and Srivastava (1987).
6 See, for example, Wang (1997).
7 The operator is based on the results of Abreu, Pearce and Stacchetti (1990).
4 Conclusion

The paper considers a dynamic model of moral hazard where the admissible wage schemes are restricted to the ones exhibiting non-decreasing time trajectories. We analyze the effect of the restriction in a setting of optimal contracting. Both finite and infinite horizon models are considered. In the infinite horizon, we manage to characterize the dynamic contract recursively, which allows the application of numerical techniques in computing the optimal contract. Numerical experiments could further be used to investigate the cost of the restriction, and the incentive structure of the resulting contracts. This would provide us with some important intuition of the role of tenure-like arrangements as insurance vehicles, the scope of compatible incentives, and their cost of implementation. By simulating time paths according to the optimal policy and the related law of motion under the restriction, we should be able to analyze the dynamics and the long-term behavior of the scheme. We can also investigate how the model responds to shocks: distribution of returns, degree of risk aversion, disutility of effort, reservation utility. A step further would be parameterizing the model and quantifying the results.

APPENDIX A

Lemma 1 In a two-period model where both the principal and the agent are myopic and high effort is optimal at 0, high effort can be implemented at 0G by a compensation scheme offering \( v^{-1}(\bar{y} + \bar{\pi}k) \) after a bad outcome and \( v^{-1}(\bar{y} + (1 + \bar{\pi})k) \) after a good outcome.

Proof. Consider the principal’s problem at 0G. It is the same as [PP], but with the additional constraint that \( w(y) \geq v^{-1}(\bar{y} + \bar{\pi}k) \). Let \( \gamma : Y \rightarrow \mathbb{R}_+ \) be the Lagrange multipliers associated with the minimum wage constraint. Denote the Lagrange multiplier of the incentive compatibility constraint by \( \mu \geq 0 \). Then, since individual rationality is trivially satisfied, the first order condition with respect to \( w(y) \) is as follows:

\[
-\pi(y, \bar{\pi}) + \mu (\pi(y, \bar{\pi}) - \pi(y, \bar{\pi})) v'(w(y)) + \gamma(y) = 0, \forall y \in Y
\]  

(A1)

If \( \mu = 0 \), (A1) implies \( \gamma(y) > 0, \forall y \in Y \) which would result in \( w(y) = w(\bar{y}) \) violating incentive compatibility. Therefore, it must be that \( \mu > 0 \), i.e. incentive compatibility binds. From (A1), we have \( \gamma(y) = \bar{\pi} - \mu(\bar{\pi} - \bar{\pi}) > 0 \) given \( \bar{\pi} < \bar{\pi} \) and \( \mu > 0 \). Therefore, \( w(y) = v^{-1}(\bar{y} + \bar{\pi}k) \). Plugging this in incentive compatibility which holds with equality results in \( w(\bar{y}) = v^{-1}(\bar{y} + (1 + \bar{\pi})k) \).

Proof of Proposition 3. Without the minimum wage constraint, low effort will be implemented by \( v^{-1}(\bar{y}) \). If \( v^{-1}(\bar{y}) < w_1 \), consider a compensation scheme with both components equal to \( w_1 \). The scheme satisfies individual rationality and incentive compatibility for low effort. A decrease in either component of the scheme violates the minimum wage constraint, while an increase raises the cost borne by the principal.

Now, we consider the implementation of high effort. If \( v^{-1}(\bar{y} - (1 - \bar{\pi})k) \geq w_1 \), the principal will use \( (v^{-1}(\bar{y} - (1 - \bar{\pi})k), v^{-1}(\bar{y} + \bar{\pi}k)) \) which is the compensation scheme that implements high effort with no restrictions to pay. Assume that \( v^{-1}(\bar{y} - (1 - \bar{\pi})k) < w_1 \) instead. Consider the first order condition with respect to \( w(y) \) is:

\[
-\pi(y, \bar{\pi}) + \lambda \pi(y, \bar{\pi}) v' \left( w(y) \right) + \mu (\pi(y, \bar{\pi}) - \pi(y, \bar{\pi})) v' \left( w(y) \right) + \gamma(y) = 0, \forall y \in Y,
\]  

(A2)
where \( \lambda \geq 0 \) is the Lagrange multiplier of the individual rationality constraints, while \( \mu \) and \( \gamma (y) \) are as defined in Lemma 1.

Let \( \lambda > 0 \) which implies that individual rationality binds. A binding incentive compatibility would then result in the previously described compensation scheme which given our assumptions violates at least one of the minimum wage constraints. Therefore, incentive compatibility for high effort holds as a strict inequality, from where the wage is strictly increasing in the output. Then, if the lower wage component is greater than \( w_1 \), (A2) implies that \( v'(w(y)) = \frac{1}{x} \), from where both wage components should be equal since \( v \) is strictly concave. This is a contradiction, so it must be that the lower wage component equals \( w_1 \) which implies that \( \gamma (y) = 0 \). Then, from the first order conditions, we obtain \( \gamma (y) = \pi \left( 1 - \frac{v'(w(y))}{v'(w(y) + k)} \right) < 0 \), which is not possible since \( \gamma (y) \) is a Lagrange multiplier.

Therefore, it must be the case that \( \lambda = 0 \). Then, (A2) reduces to (A1). Following the analysis of Lemma 1, we obtain the scheme \((w_1, v^{-1}(v(w_1) + k))\).

**Proof of Proposition 4.** The result is obtained by comparing the principal’s utility under each of the cases \( G_1, G_2 \) and \( G_3 \).

**Lemma 2** Let \( f(x, y) := \pi v(x) + (1 - \pi) v(y) - y \) and \( g(x, y) := \beta \left( \pi + (1 - \pi) \frac{v'(x)}{v'(w(x) + k)} \right) + 1 - (1 + \beta) \frac{v(x)}{v'(y)} \).

(a) The cheapest implementation of the effort configuration in \( G_3 \) (low at 0 and 0G, high at 0B) is by \((w_{B,3}, w_{G,3})\) at 0, \((w_{B,3}, v^{-1}(v(w_{B,3}) + k))\) at 0B and \( w_{G,3} \) at 0G, where wages satisfy \( v - (1 - \pi) k \leq \bar{v} < v(w_{B,3}) \leq \bar{v} + \pi k \) and:

\[
\begin{align*}
    f(w_{B,3}, w_{G,3}) &= 0 \\
    g(w_{B,3}, w_{G,3}) &= 0
\end{align*}
\]

(b) The cheapest implementation of the effort configuration in \( G_4 \) (low at 0 and 0B, high at 0G) is by \((w_{B,4}, w_{G,4})\) at 0, \( w_{B,4} \) at 0B and \((w_{G,4}, v^{-1}(v(w_{G,4}) + k))\) at 0G, where wages satisfy \( v - (1 - \pi) k \leq \bar{v} < v(w_{G,4}) \leq \bar{v} + \frac{(1 - \pi)^2 k}{2} \) and:

\[
\begin{align*}
    f(w_{B,4}, w_{G,4}) &= 0 \\
    g(w_{G,4}, w_{B,4}) &= 0
\end{align*}
\]

(c) The cheapest implementation of the effort configuration in \( G_5 \) (low at 0, high at 0B and 0G) is by \( v^{-1}(\bar{v}) \) at 0 and \((v^{-1}(\bar{v}) + 1, v^{-1}(\bar{v} + k))\) at 0B and 0G.

**Proof.** We present only the proof of (a). The proofs of (b) and (c) are analogous. Consider the principal’s two-period optimization problem. Note that the effort configuration in \( G_3 \) is only possible if high effort was optimal when pay was not restricted; otherwise, low effort would be optimal at every node. Then, since high effort is optimal at 0B and not at 0G, it must be that the minimum wage is higher at 0G than at 0B. Therefore, \( w_G > w_B \). Then, from first-period individual rationality, we have \( v(w_G) > \bar{v} \) and so from Proposition 3 the principal will implement low effort.
at 0G by setting a fixed pay equal to \( w_G \). Let \( w'_B \) denote the bad-outcome wage contracted at node 0B. Then, the compensation scheme implementing high effort at this node will be given by \( (w'_B, v^{-1}(w'_B + k)) \), where \( w'_B \geq v^{-1}(\bar{y} - (1 - \bar{\pi}) k) \) \( w'_B \geq v_G \). Let \( \varphi \) and \( \gamma \) be the respective Lagrange multipliers of these constraints, while \( \lambda \) and \( \mu \) are the Lagrange multipliers corresponding to first-period individual rationality and first-period incentive compatibility. Then, the first-order conditions with respect to \( w_B, w_G \), and \( w'_B \) are as follows:

\[
-\bar{\pi} + \lambda \pi v'(w_B) + \mu (\pi - \bar{\pi}) - \gamma = 0 \tag{A3}
\]
\[
-(1 - \bar{\pi})(1 + \beta) + \lambda (1 - \bar{\pi}) v'(w_G) - \mu (\pi - \bar{\pi}) = 0 \tag{A4}
\]
\[
-\beta \pi \left( \pi + (1 - \bar{\pi}) \frac{v'(w'_B)}{v'(v^{-1}(v(w'_B) + k))} \right) + \varphi + \gamma = 0 \tag{A5}
\]

A4 implies that \( \lambda > 0 \), so individual rationality binds, i.e., (12) holds. From there, \( v(w_{B,3}) < \bar{y} \). Assume \( \mu = 0 \). If \( \gamma = 0 \), (A3) and (A4) imply \( 1 + \beta = \frac{v'(w_B)}{v'(w_G)} \) which contradicts \( w_G > w_B \). Therefore, we should have \( \gamma > 0 \). If \( \varphi > 0 \) as well, it must be that \( w'_B = w_B = v^{-1}(\bar{y} - (1 - \bar{\pi}) k) \). From (12), we obtain \( w_G = v^{-1}(\bar{y} + \pi k) \). Let \( z := \frac{v^{-1}(\bar{y} - (1 - \bar{\pi}) k)}{v'(v^{-1}(\bar{y} + \pi k))} > 1 \) and note that (A3), (A4) and (A5) imply \( \varphi = \pi \pi v^{-1}(\bar{y} - (1 - \pi) k), v^{-1}(\bar{y} + \pi k) = \pi (1 + \beta \pi)(1 - z) < 0 \). Therefore, it must be that \( \varphi = 0 \) from where \( w'_B = w_B \geq v^{-1}(\bar{y} - (1 - \bar{\pi}) k) \), which implies \( w_{G,3} \leq v^{-1}(\bar{y} + \pi k) \). Then, (13) follows from (A3), (A4), and (A5). Note that we still need to prove that \( \mu = 0 \). Assume not. Then, first period incentive compatibility binds which together with (12) implies \( w'_B = w_B = v^{-1}(\bar{y} - (1 - \pi) k) \) and \( w_G = v^{-1}(\bar{y} + \pi k) \). From (A3), (A4) and (A5), we obtain \( \varphi = \pi (1 + \beta \pi)(1 - z) - \mu(\pi - \bar{\pi}) \frac{v'(w'_B)}{1 + \pi} < 0 \), which is impossible since \( \varphi \) is a Lagrange multiplier.

Note that the principal should find it optimal to implement the effort configuration specified in \( G_3 \). The optimality of her second period effort choices requires \( (1 - \pi) \left( v^{-1}(v(w)_G + k) - w_G \right) \geq E(y|\pi) \geq \bar{\pi}v^{-1}(w_B) + (1 - \bar{\pi}) v^{-1}(v(w_B) + k) - v^{-1}(\bar{y}) \). The optimality of the contract requires comparing it with all the other five cases. ■

**Proof of Proposition 5.** It is easy to demonstrate that each case implements its respective effort configuration in the cheapest possible way. We have explicitly done that for cases \( G_3, G_4 \) and \( G_5 \) in Lemma 2. We just need to show that there are no other implementable effort configurations. Assume low effort is optimal without restrictions on pay. With non-decreasing wages, it can still be implemented at any node in exactly the same way as before. Therefore, implementing high effort at any node will be suboptimal. Now, assume high effort is optimal with no restrictions on pay. Can implementing high effort at 0 and 0G and low effort at 0B be optimal with non-decreasing wage profiles? No, because implementing high effort at 0 requires that wage increases with outcome (SMLR) and since high effort is optimal at 0G, it should be also optimal at 0B where the minimum wage is lower. ■
Let $c_s := \{(a_s, w_s, w_s(y), V_s(y)) : y \in Y\}$. Take $w \in W$ and $V \in V^{C'}(w)$.

[BE-2]

$$U(V, w) = \max_{c_s} E_{a_s} \{u(w_s, \cdot) + \beta_p U(V_s)\} \quad \text{s.t.}:
\begin{align*}
& a_s \in A \\
& w_s(\cdot) \in W \cap [w, +\infty) \\
& w_s = \min_{y \in Y} w_s(y) \\
& V = E_{a_s} \{\nu(w_s, a_s) + \beta_A V_s\} \geq E_{a'} \{\nu(w_s, a') + \beta_A V_s\}, \forall a' \in A \\
& V_s(\cdot) \in V^{C'}(w_s)
\end{align*}
$$

Let $c_s := \{(a_s, w_s(y), w_s(y), V_s(y)) : y \in Y\}$. Let $f : W^N \to W^N$ continuous. Take $w \in W$ and $V \in V^{C''}(w)$.

[BE-3]

$$U(V, w) = \max_{c_s} E_{a_s} \{u(w_s, \cdot) + \beta_p U(V_s)\} \quad \text{s.t.}:
\begin{align*}
& a_s \in A \\
& w_s(\cdot) \in W \cap [w, +\infty) \\
& w_s(y_n) = f_n(\{w_s\}), \forall y_n \in Y \\
& V = E_{a_s} \{\nu(w_s, a_s) + \beta_A V_s\} \geq E_{a'} \{\nu(w_s, a') + \beta_A V_s\}, \forall a' \in A \\
& V_s(\cdot) \in V^{C''}(w_s(\cdot))
\end{align*}
$$

Note that [BE-3] is reduced to [BE-2] if $f_n(\{w_s\}) = \min_s \{w_s\}$ and to [BE-1] if $f_n(\{w_s\}) = w_n$.
References


