Market Size, Productivity, Entrepreneurship and Income Inequality in a Model a'la Melitz

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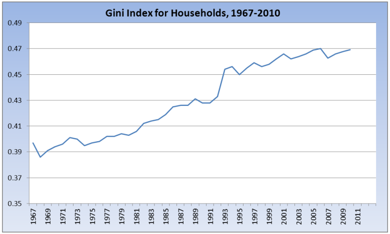
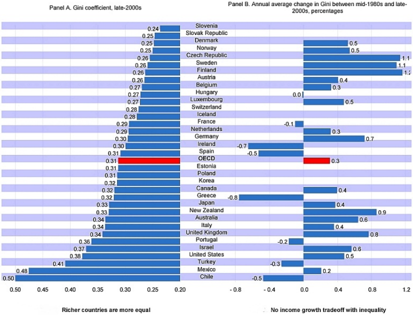
3rd NES Summer School of HESP Project, Nizhny Novgorod, July 16th - July 28th

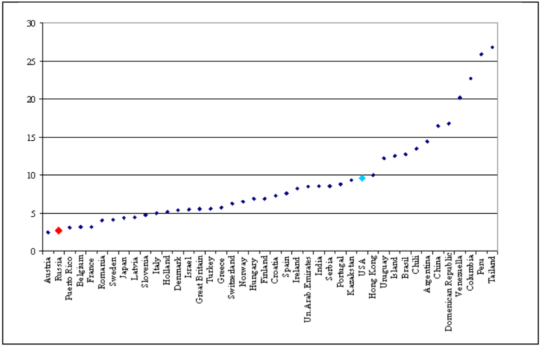
Gini Index


Gini Index for Households, 1967-2010
0.49
0.47
0.45
0.43
0.41
0.39
0.37
0.35
1967
1969
1971
1973
1975
1977
1979
1981
1983
1985
1987
1989
1991
1993
1995
1997
1999
2001
2003
2005
2007
2009
2011


Panel A. Gini coefficient, late-2000s
Panel B. Annual average change in Gini between mid-1980s and late-2000s, percentages
Slovenia
Slovak Republic
Denmark
Norway
Czech Republic
Sweden
Finland
Austria
Belgium
Hungary
Luxembourg
Switzerland
Iceland
France
Netherlands
Germany
Ireland
Spain
OECD
Estonia
Poland
Korea
Canada
Greece
Japan
New Zealand
Australia
Italy
United Kingdom
Portugal
Israel
United States
Turkey
Mexico
Chile
Richer countries are more equal
No income growth tradeoff with inequality

Entrepreneurship Index

Index of Total Entrepreneurial Activity



Research questions

1. Does expanding market influence employment structure?
2. Does market size matter for income inequality?

Our purpose - to explain impact of market size on employment structure and income inequality in a heterogeneous population

Literature Review

Key Ideas

- **Lucas (1978)** - endogenous formation of entrepreneurship

Homogeneous agents compare their reservation wage and potential profit.

- **Melitz (2003)** - heterogeneous firms productivity and endogeneous trashold

Firms become an active one if only they have a productivity level bigger than treshold.

Settings simmilar to ours

- **Oyama D., Sato Y., Tabuchi T., Thisse J.-F. (2011)**
- **Kukharsky B. (2011)**

Assumptions

- **2-sector economy** (diversified sector with increasing return in scale and traditional one with constant return in scale)
- **one production factor** - labor
- L **agents** (potential workers/entrepreneurs), **mobile between sectors**
- Agents are **homogenous** in their **preferences**

Economy, ctd.

- Agents are **homogenous** in their **worker's abilities**
- Agents are **heterogeneous** in their **entrepreneurship abilities**, describing by the parameter c - marginal cost of production if organizing a firm (the smaller c the higher entrepreneurial ability)
- Each agent knows his type c and chooses between being an entrepreneur or a worker
- The parameter c is distributed on $[0; +\infty)$ with d.d.f. $\gamma_c \equiv \gamma(c)$
- Each c-type is represented by $L\gamma_c$ agents in the population.

Consumer problem

Each agent consumes all varieties and homogenous good to maximize

$$\ln\left(\int_0^{\bar{c}} u(x_c)\, L\gamma_c\, dc\right) + A \to \max_{x,A}$$

$$\text{s.t.} \int_0^{\bar{c}} p_c x_c L\gamma_c\, dc + A = I$$

I is agent's income (that equals wage for workers but profit for entrepreneurs)

$\bar{c}$ is the (endogenous) "cutoff" type of entreprenerial abilities

$\lambda \equiv \int_0^{\bar{c}} u(x_c)\, L\gamma_c dc$ - marginal utility of spending on varieties

First order condition for subprogram of purchasing varieties is

$$p_c \equiv p(x_c;\lambda) = \frac{u'(x_c)}{\lambda},\ \forall c \in [0;\bar{c}]$$

Producer problem

A firm has only Variable Costs, proportional to its output Lx_c and maximizes profit:

$$\pi_c = (p(x_c;\lambda) - c)\, Lx_c = \left(\frac{u'(x_c)}{\lambda} - c\right) Lx_c \to \max_{x_c} \forall c \leq \bar{c}$$

$$\textbf{F.O.C.} \quad \frac{u'(x_c) + x_c u''(x_c)}{\lambda} = c \implies M_c \equiv \frac{p_c - c}{p_c} = r_u(x_c)$$

$$\textbf{S.O.C.} \quad r_{u'}(x_c) < 2$$

here:

$\lambda \equiv \int_0^{\bar{c}} u(x_c)\, L\gamma_c dc$ - market statistic

$r_u(x) \equiv -\frac{u''(x)x}{u'(x)} \in (0;1)$ - measure of concavity of utility function

Cutoff Condition

An agent with **cutoff** entrepreneurial **abilities** $\overline{c}$ is indiferent between being a worker or an entrepreneur:

$$\pi_{\overline{c}} = w \Leftrightarrow \frac{r_u\left(x_{\overline{c}}\right) x_{\overline{c}}}{1 - r_u\left(x_{\overline{c}}\right)} = \frac{w}{L\overline{c}}$$

Equilibrium

Definition

The equilibrium is a bundle $\left(\overline{c},\ \lambda,\ \{p_c;\ x_c\}_{c\in[0;\overline{c}]}\right)$ such that consumption x maximizes each consumer's utility under price vector $\{p_c\}_{c\in[0;\overline{c}]}$ and solves each producer's problem under $\overline{c},\ \lambda,\ \mathbf{p}(\cdot)$, the cut-off condition holds and $\lambda = L\int_0^{\bar{c}} u(x_c)\gamma_c dc$. For a given equilibrium, consumption of the numerarie A_c for each type follows from the budget constraint, that entails also the labor balance under our normalization $w = 1$

Equilibrium Equations

Proposition 1

The equlibrium bundle $\left(\overline{c},\ x_{\overline{c}},\ \{x_c\}_{c\in[0;\overline{c})}\right)$ is defined by the system of equations:

$$\begin{cases} \frac{u'(x_c)(1-r_u(x_c))}{c} = \frac{u'(x_{\overline{c}})(1-r_u(x_{\overline{c}}))}{\overline{c}}, \quad \forall c\in[0;\overline{c}) \\ \frac{u'(x_{\overline{c}})(1-r_u(x_{\overline{c}}))}{\overline{c}} = \int_0^{\overline{c}} u(x_c)\, L\gamma_c dc \\ \frac{r_u(x_{\overline{c}})x_{\overline{c}}}{1-r_u(x_{\overline{c}})} = \frac{1}{L\overline{c}} \end{cases}$$

Other equlibrium variables follow from **F.O.C.** equations

Elasticities and RLV

Proposition 2

The elasticity of the cutoff (and others endogeneous variables) to the market size as well as bounding interval(s) are determined with love to variety.

$$\mathcal{E}_{L\overline{c}} = \frac{\overline{r}_u}{1-\overline{r}_u} \frac{\frac{1}{r_V} - \frac{1}{\overline{r}_u} + \frac{J}{\overline{\Gamma}}}{\frac{1}{r_V} + \frac{J}{\overline{\Gamma}} + \frac{1}{1-\overline{r}_u}\frac{\overline{u}}{\tilde{u}}\frac{\overline{\gamma c}}{\overline{\Gamma}}} \in \left(-1; \frac{\overline{r}_u}{1-\overline{r}_u}\right)$$

here: bar signs the value expressed at the point $x_{\overline{c}}$, tilde singes an average under condition $c < \overline{c}$,

$$\overline{\Gamma} = \int_0^{\overline{c}} \gamma_c dc, \quad \mathscr{U} = \int_0^{\overline{c}} u(x_c)\gamma_c dc, \quad \tilde{u} = \frac{\mathscr{U}}{\overline{\Gamma}}$$

$$J = \int_0^{\overline{c}} \frac{\mathcal{E}_{x_c} u}{r_{u_c}} \cdot \frac{1 - r_{u_c}}{2 - r_{u'_c}} \frac{u(x_c)}{\tilde{u}} \gamma_c dc$$

Direct and Indirect Effetcs

Proposition 3

Influence of the market expands on the competitive intensity (and on others endogeneous variables) might be decomposed into direct (unrelated with change of cut-off) and indirect effects (related with change of cut-off)

the direct effect is positive: $\mathscr{E}_L^d \lambda = \bar{r}_u > 0$
the indirect effectis opposite to changing of share of entrepreneurs: $\mathscr{E}_L^i \lambda = -(1 - \bar{r}_u)\mathscr{E}_L \bar{c}$

Total effect is positive but no too strong:

$$\mathscr{E}_L \lambda = \mathscr{E}_L^d \lambda + \mathscr{E}_L^i \lambda = \frac{1 + \frac{\bar{r}_u}{1 - \bar{r}_u} \frac{\bar{u}}{\tilde{u}} \frac{\overline{\gamma c}}{\bar{\Gamma}}}{\frac{1}{r_V} + \frac{J}{\bar{\bar{\Gamma}}} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u}}{\tilde{u}} \frac{\overline{\gamma c}}{\bar{\Gamma}}} \in (0;1)$$

$$\mathscr{E}_L \lambda \leq \max\{\bar{r}_u; r_V\}$$

Comparative statics: Employment structure

Prpoposition 4

Increasing market size ($L \uparrow$) pushes each purchase down ($x_c \downarrow$), and impacts the equilibrium cutoff $\bar{c}$ (employment structure), all prices (p_c) according to several patterns:

(*pointwise* at x)	DED: $r_u' < 0$	CED: $r_u' = 0$	IED: $r_u' > 0$
increasing $\mathscr{E}_x u$	$\bar{c}$?, $p_c \uparrow$	$\bar{c} \downarrow$, $p_c' = 0$	$\bar{c} \downarrow$, $p_c \downarrow$
constant $\mathscr{E}_x u$	$\bar{c} \uparrow$, $p_c \uparrow$	$\bar{c}' = 0$, $p_c' = 0$	
decreasing $\mathscr{E}_x u$		$\bar{c} \uparrow 0$, $p_c' = 0$	$\bar{c}$?, $p_c \downarrow$

If the share of entrepreneurs decreases then total amount might decrease despite market expanding

$$N = L \int_0^{\bar{c}} \gamma_c dc$$

Comparative Statics: income inequality

Proposition 5

Higher entrepreneurial ability implies bigger profit and vice verse:

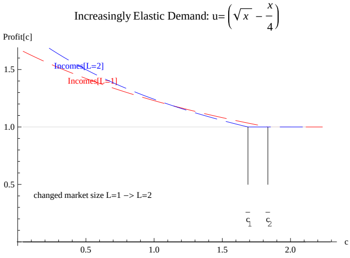
$$\pi_{c_1} \gtreqless \pi_{c_2} \Leftrightarrow c_1 \lesseqgtr c_2$$

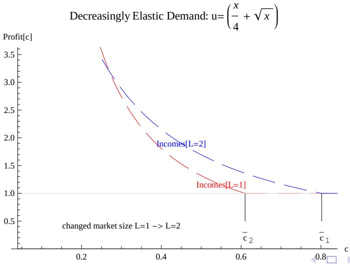
Proposition 6

Expanding market reduces income inequality between entrepreneurs if and only if concavity measure r_u is decreasing:

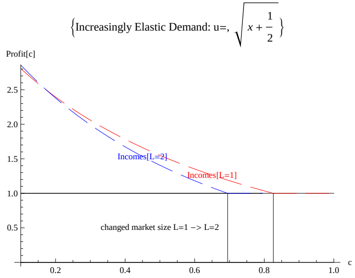
$$c_1 < c_2 \;\Rightarrow\; \left[\mathscr{E}_L \pi_{c_2} \gtreqless \mathscr{E}_L \pi_{c_1} \;\Leftrightarrow\; r_u(x_{c_1}) \lesseqgtr r_u(x_{c_2})\right]$$

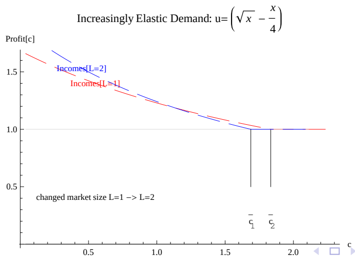
$$1.\ \text{IED}\ (r_u' > 0) \implies \frac{\pi_{c_1}}{\pi_{c_2}} \uparrow; \quad 2.\ \text{DED}\ (r_u' < 0) \implies \frac{\pi_{c_1}}{\pi_{c_2}} \downarrow$$





1. IED $(r'_u > 0)$, $\mathcal{E}_x u \uparrow \implies \bar{c} \downarrow$; 2. IED $(r'_u > 0)$, $\mathcal{E}_x u \downarrow \implies \bar{c}$?





Additional results

Also, we have results about influence of market size on other endogeneous variables:

- individual consumptions
- firms size
- prices
- market statistics

We have a few preliminary results for more generalized specification than log-linear.

Further and possible directions

- One sector model in general case (or specific cases)
- Developing two-parametric model (ability to entrepreneurship and risk aversing/quality of instituts etc.)
- Trade and labor market outcomes

THANK YOU FOR ATTENTION!