

**State University –
Higher school of economics**

International college of economics and finance

Syllabus

Calculus

for Bachelors degree in Economics 080100.62

Alexei Akhmetshin

based on the program by

J. Lockshin

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1. Teachers

Lecturer: Alexei Akhmetshin

Class teachers: Maria Derkach, Larissa Panina, Antonina Sorokina, Alexander Galybin, Anatoly Patric, Alexei Akhmetshin.

2. Course description

This course is designed to introduce students to the basic ideas and methods of mathematical analysis and their application to mathematical modeling. This course helps lay the foundation for the entire block of quantitative disciplines that are studied at ICEF, and it also provides some of the analytical tools that are required by advanced courses in economics. This course provides students with experience in the methods and applications of calculus to a wide range of theoretical and practical situations. The course is taught in English.

Prerequisites

Students are expected to have a firm grounding in elementary mathematics, algebra, trigonometry, and geometry on the coordinate plane, the properties and graphs of elementary functions at the level of Russian high school.

Course objectives:

By the end of this course the students should:

- be able to analyse functions represented in a variety of ways: graphical, numerical, analytical, or verbal, and understand the relationships between these various representations.
- understand the meaning of the derivative in terms of a rate of change and local linear approximation, and be able to use derivatives to solve a variety of problems.
- understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change, and be able to use integrals to solve a variety of problems.
- understand the relationship between the derivative and the definite integral, as expressed by the Fundamental Theorem of Calculus.
- be able to communicate mathematics in well-written sentences and to explain the solutions to problems.
- be able to model a written description of a simple economic or physical situation with a function, differential equation, or an integral.
- be able to use mathematical analysis to solve problems, interpret results, and verify conclusions.
- be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.

3. Topical Outline

No.	Topic titles	TOTAL (hours)	Contact hours		Self-study
			Lectures	Classes	
1	Sequences. Limit of a sequence	8	2	2	4
2	Limit of a function	8	2	2	4
3	Continuity	8	2	2	4
4	The derivative	16	4	4	8
5	Applications of the derivative	16	4	4	8
6	Series and Taylor expansions.	24	6	6	12
7	Antiderivatives and the indefinite integral	24	6	6	12
8	Differential equations	16	4	4	8

9	The definite integral	24	6	6	12
10	Applications of the definite integral	24	6	6	12
11	Improper integrals	8	2	2	4
12	The double integral	8	2	2	4
13	Course review	16	4	4	8
14	AP exam preparation	32	8	8	16
	Total:	232	58	58	116

4. Assessment

The academic year 2012-2013 at ICEF is divided into 4 modules as follows:

- Module 1: from 1st September 2012 till 22nd October 2012;
- Module 2: from 31st October 2012 till 23rd December 2012;
- Module 3: from 16th January 2013 till 24th March 2013;
- Module 4: from 2nd April 2013 till 5th May 2013.

At the end of each module the students will sit an exam.

In the Fall term there will be a so-called mock exam during the week of 22nd October and a mid-year during the last week of December. Both exams are to be set in the APT format by the local teachers and graded in accordance with APT rules. The final mark for the Fall term is given on a 100-point scale, which is then converted to the HSE standard 10-point scale and the usual 5-point scale.. The term mark will be mostly based on the results of the October mock (roughly 40%), the December exam (about 55%) and the bonus points for the special home assignments and quizzes.

In the Spring term students will sit a mock exam in the full APT format during the week of 25th March. This exam is set and graded by local teachers. The last module of the course is fully devoted to the preparation for the external AP Calculus AB exam, which is set and graded by the AP Board. The exact exam date is available at the College Board official web page (see a link below). The AP marks are on a scale from 1 to 5. This grade is used as enter grade to the University of London external program. It also gives 35% of the final ICEF Calculus course grade. The other constituents of the final course grade are the Fall term grade (35%) and the March mock exam grade (30%) and the bonus points.

Students who have a valid excuse for missing one of the locally set exams may be allowed to retake it on a later date or may receive an automated grade according to the ICEF rules.

During the year students must also complete weekly home assignments, which are marked by class teachers. Quizzes are held regularly in classes. For some of these activities students may receive bonus points which influence the Fall term mark and the final course mark. This influence is expected not to exceed 10% of the grade.

5. Main Reading

1. Dowling E.T. Introduction to Mathematical Economics. McGraw-Hill, 1980. (D)
2. King K.J. Calculus AB Preparation Guide. Cliffs, 1998.
3. Красс М. С., Высшая математика для экономиста. М., 1998 (Кр).
4. Кремер Н.Ш., Путко И.М., Фридман М.Н. Высшая математика для экономистов. М, 2000 (КПФ)
5. Фихтенгольц Г.М. Курс дифференциального и интегрального исчисления (в трех томах). М., 1998 (Ф).
6. Stewart J. Calculus. Early Transcendentals. 6 edition. Thomson Brooks/Cole, 2008 (S)

6. Supplementary reading:

1. Simon C.P., Blume L. Mathematics for Economists. W.W.Norton&Company, 1994.
2. Chiang A.C. Fundamental Methods of Mathematical Economics. McGraw-Hill, 1984
3. Anthony M., Biggs N. Mathematics for Economics and Finance. CUP, 1996.
4. Демидович Б.М. Сборник задач и упражнений по математическому анализу. М., Наука, 1996.
5. Зорич В.П. Математический анализ (в 2-х) томах. Фазис, 1997-1998.
6. The Student Guide to the AP Calculus Courses and Examinations. The College Board, 1998.
7. Advanced Placement Course Description. The College Board, 1998.
8. Free-Response questions. The College Board, 1995.

7. Internet resources:

1. The College Board, creators of the APT exams: <http://apcentral.collegeboard.com>
2. A very good comprehensive math resource that completely covers the program: <http://www.sosmath.com>
3. Java applets that demonstrate various concepts and methods in calculus:
<http://cs.jsu.edu/mcis/faculty/leathrum/Mathlets/topframe.html>
<http://www.ies.co.jp/math/java/calc/index.html>
<http://www.plu.edu/~heathdj/java/>
<http://www.slu.edu/classes/maymk/MathApplets-SLU.html>
4. Many practice multiple-choice problems can be found at <http://homepages.seresc.net/~sray/alvirne.html>

8. Course outline:

1. Introduction

The application of mathematics to describing phenomena. The role of mathematics and mathematical modeling in economics. Different forms of representation of functions. Elementary concepts: domain and range of a function, even and odd functions, periodic functions. Graphs of elementary functions. Shifts and distortions of graphs. Implicit functions. Examples of functions in economics: utility function, production function, cost function, demand and supply functions. (D. Ch. 7; K. – pp. 23-51; Кр. - pp. 11-14, 46-58, 88-91, 155-161; КПФ – pp. 125-137; Ф. - V.1, pp. 93-114; S. – pp. 10-72)

2. Sequences. Limit of a sequence

Sequences: bounded and unbounded, infinitely small and infinitely large. Limit of a sequence. Limit theorems for sequences: arithmetic operations, sandwich theorem. Monotone sequences. Convergence of a monotone increasing sequence. The number e . (Кр. - pp. 24-45; КПФ – pp. 141-142; Ф. - V.1, pp. 43-92; S. – pp. 674-686)

3. Limit of a function

The limit of a function at infinity. Asymptotes of a function at infinity. The limit of a function at a point. Limit theorems for functions. Functions that tend to zero, functions that tend to infinity. First and Second Special Limits. Types of indeterminate forms. Finding limits. Left and right limits.

(D. - Ch. 3.1; K. - pp. 71-91; Kp. - pp. 58-73; KΠΦ – pp. 143-160; Φ. - V.1, pp. 115-145; S. – pp. 88-118)

4. Continuity

Definition of continuity of a function at point and on an interval. Continuity of elementary functions. Properties of continuous functions. Points of discontinuity. Classification of points of discontinuity. Vertical asymptotes.

(D. – Ch. 3.2; K. – pp. 92-95; Kp. - pp. 74-87; KΠΦ – pp. 161-164; Φ. - V.1, pp. 146-185; S. – pp. 119-143)

5. The derivative

Definition of the derivative. Tangent lines and normal lines. Geometric, physical and economic interpretations of the derivative. Right and left derivatives. Differentiability at a point. Differentiability and continuity. Differentiation. Rules of differentiation. Derivatives of elementary functions. Differentiation of inverse functions. Logarithmic differentiation. Differentiation of implicit functions. Existence of a differentiable implicit function. Definition and geometric interpretation of differentials. Approximate calculations using differentials. The second derivative. The economic meaning of the second derivative. Higher-order derivatives and differentials. Properties of differentiable functions: Rolle's theorem, the Mean Value theorem, Cauchy's theorem, and their geometric interpretation.

(D. - pp. 41-47; K. pp. 109-144, 163-166, 211-214, 216-218; Kp. - pp. 98-123; KΠΦ – pp. 176-198, 209-211; Φ. V.1, pp. 186-222, 231-245, S. – pp. 143-166, 172-261)

6. Applications of the derivative

L'Hospital's rule. Necessary and sufficient conditions for increasing/decreasing functions. Related rates. Concave and convex functions. Different ways of expressing concavity. Economic interpretation of concave and convex functions. Points of inflection. Local extrema. First-order necessary and sufficient conditions for a local extremum. Second-order necessary and sufficient conditions for a local extremum. Maximum and minimum values of a function on an interval. Geometric and economic applications of optimisation. Curve sketching.

(D. – Ch. 4; K. – pp. 167-210; Kp. - pp. 124-132, 140-161; KΠΦ – pp.212-234, 240-241; Φ. – V.1, pp. 268-336, S. – pp. 270-333)

7. Number series, power series, and Taylor expansions

Necessary condition for convergence of a series. Harmonic series and power series. The ratio test. Comparing series to test for convergence. Alternating series. Sufficient condition for convergence of an alternating series. Absolute convergence. Radius and interval of convergence of a power series. Abel's theorems. Taylor's formula. Taylor and Maclaurin series. Taylor and Maclaurin expansions for elementary functions. Application of Taylor series for analyzing the behavior of a function at a point and for conducting approximate calculations.

(Kp. - pp. 133-139; KΠΦ – pp. 356-372, 379-390; Φ. - V.1, pp. 246-262; S. – Ch. 30.2, S. – pp. 687-747)

8. Antiderivatives and the indefinite integral

Antiderivatives. The indefinite integral and its properties. Table of indefinite integrals. Basic methods of integration: direct integration, substitution and integration by parts. Integration of rational functions.

(*D.* –pp. 357-362; *K.* – pp. 234-257; *Kp.* - pp. 162-186; *KПФ* – pp. 251-270; *Ф.* – V.2, pp. 11-93, *S.* – pp. 340-345, 452-489)

9. Differential equations and slope fields

Definition of first order differential equations. General and particular solutions. Existence and uniqueness theorem. Isoclines and direction fields. Solution of separable differential equations. Solution of homogeneous differential equations and first-order linear equations. Application of differential equations to physics and economics.

(*D.* –pp. 392, 395-396; *K.* – pp. 316-323; *Kp.* - pp. 477-544; *KПФ* – pp. 325-336; *Ф.* - V.2, pp. 244-257, *S.* – pp. 566-607)

10. The definite integral

Problems that require the definite integral. Definition of the definite integral using Riemann sums. Sufficient condition for the existence of the definite integral. Approximate calculation of definite integrals using rectangles and trapezoids. Simpson's rule. Properties of the definite integral. Differentiation of a definite integral with variable upper bound. The fundamental theorem of calculus. Substitution and integration by parts.

(*D.* –pp. 373-375; *K.* – pp. 273-285; *Kp.* - pp. 187 - 210, 233-236; *KПФ* – pp. 283-296, 312-317; *Ф.* - V.2, pp. 94-168, *S.* – pp. 354 - 390)

11. Applications of the definite integral

Applications of the definite integral in geometry, economics and physics. Area of a flat region, volume of a solid of revolution, volume of a solid with known cross-sections. Use of definite integrals to solve separable differential equations.

(*D.* – p. 376; *K.* – pp. 286-315; *Kp.* - pp. 211-232; *KПФ* – pp. 298-306; *Ф.* - V.2, pp. 169-243, *S.* – pp. 414-447, 524-561)

12. Improper Integrals

Integrals with infinite bounds. Improper integrals of the first kind. Integration of unbounded functions. Improper integrals of the second kind. Principle value. Convergence tests for improper integrals. Absolute and relative convergence of improper integrals.

(*Kp.* - pp. 237-248; *KПФ* – pp. 307-311; *Ф.* - V.2, pp. 552-653, *S.* – pp.508-517)

13. The double integral

Definition of double integrals. Reduction of double integrals to iterated integrals. Changing the order of integration in iterated integrals. The geometric interpretation and main properties of double integrals. (*Kp.* – c. 390-405; *KПФ*: pp. 425---427; *Ф.*: V. 3, pp. 136---141, 154---167, *S.* – pp. 950- 973)

Author of the original course program:
Updated by

J. Lockshin, 2010.
A. Akhmetshin, 2012.