Further Mathematics

Lecturer: Grigoriy G. Kantorovich
Class teacher: Grigoriy G. Kantorovich

Course description

The course is an extension of course Further Mathematics for students of specialization 'Economics'.

The structure of the course includes advanced elements of linear algebra, calculus of functions of several variables, a general optimization problem of function of several variables without restrictions and with restrictions such as equalities and inequalities. The course material should teach students to understand and prove the basic formulas of linear algebra and calculus, and also to investigate the economic problems of comparative statics and optimization within the framework of a advanced tools of mathematical models.

The course program provides lecturing and teaching classes, and also regular self-study of students. Self-study includes deepening of theoretical material offered at lectures, and solutions of the offered home assignments. During each semester 1 intermediate examination is set.

Teaching objectives

The purpose of the course is not so much acquisition of new skills in a solution of mathematical problems with an economic applications, but study of methods of proofs and more strict reviewing of some sections of mathematics.

As a result of study of material of Fall semester the student should master and be able to prove the basic facts of strict abstract construction of linear algebra.

As a result of study of material of Spring semester the student should know the basic facts of calculus of functions of several variables, including calculation of partial derivatives of explicit and implicit functions, solutions of problems of unconditional and conditional optimization. The student should be able to investigate economic problems of comparative statics with the methods of a calculus, to discover points of maximum and minimum of functions of several variables, the method of Lagrange multiplier, to find extreme points of functions subject to constraints. He should master the basic facts of nonlinear and linear programming, be able to investigate economic problems of optimization, to solve problems of linear programming with application of concepts of the duality theory, to discover Von Neumann and Nash equilibrium in matrix games of two persons.

The student should have skills of application of the indicated mathematical tools and methods to solution of problems in Micro- and Macroeconomics.

Teaching methods

The following methods and forms of study are used in the course:

- lectures;
- classes;
- homework;
- teachers' consultations;
- self study.

Grade determination

Monitoring of knowledge of students provides an evaluation of carried out home assignments, of activity of students at classes, of intermediate examinations. Final monitoring is carried out by results of examination written paper which makes 70% of final mark. 30% of final mark are determined by home assignments done and activity of students during classes.

Main reading

1. Carl P. Simon and Lawrence Blume. Mathematics for Economists, W. W. Norton and Company, 1994.

2. A. C. Chiang. Fundemental Methods of Mathematical Economics, 3-rd edition, McGrow-Hill, 1984.

3. Б. П. Демидович. Сборник задач и упражнений по математическому анализу, М., "Наука", 1966.

4. И. М. Гельфанд. Лекции по линейной алгебре. М., "Наука", 1999.

Additional reading

5. Anthony M. and Biggs N., Mathematics for Economics and Finance, Cambridge University Press, Cambridge, UK, 1996.

6. Anthony M., Reader in Mathematics, LSE, University of London; Mathematics for Economists, Study Guide, University of London.

7. Robert Gibbons. A Primer in Game Theory. Harvester Wheatsheaf, 1992.

8. M. Anthony. Further mathematics for economists. University of London, 1999.

9. Leon, S. j., Linear Algebra with Applications (5th edition). Prentice Hall, New Jersey, 1998.

10. Ильин, Ким. Линейная алгебра и аналитическая геометрия. М., Издательство Московского университета, 1998.

11. Проскуряков Сборник задач по линейной алгебре. М. “Наука”. 1985.

12. Фаддеев и Соминский. Сборник задач по алгебре. М. “Наука”. 1998.

Course outline

First semester

1. Linear (affine) n-dimensional space

Definition of a vector space. Linear independence of a system of vectors. Dimension of a linear space. Bases and coordinates in n-dimensional space. Isomorphism of n-dimensional spaces.

1, 27.1 – 27.2, p. 750 – 756; 4, page 7 – 20

2. Subspaces of a vector space

A linear span of a system of vectors. The subspaces connected with matrices. Direct lines and planes in a vector space. Expansion of a space in the direct sum of sub-spaces. The union and the intersection of subspaces. A transformation of coordinates under a change of a basis.

1, 27.3 – 27.5, p. 757–770; 4, page 21 – 30

3. Euclidean spaces

An inner (dot) product. Distance and an angle in Euclidean spaces. Cauchy-Bunyakovsky-Schwarz inequality. A triangle inequality. Orthogonal basis. Gram Matrix. Gram-Schmidt orthogonalization process. Isomorphism of Euclidean spaces.

1, 10.1 – 10.7, p. 199 – 236; 4, page 30 – 54

4. Linear transformations

Matrices and linear transformations. Addition and multiplication of linear transformations. Inverse transformation. A Null-space and a Range of a transformation.

8, p. 30 – 36; 4, page 95 – 110

5. Eigenvalues and eigenvectors of a matrix

A characteristic equation. Complex eigenvalues and eigenvectors. Diagonalization of a square matrix. Orthogonality of eigenvectors of a symmetric matrix. The matrices which are not diagonalizable.

1, 23.1 – 23.9, p. 579 – 632; 4, page 111 – 122

6. Application of diagonalisation of a matrix

Powers of matrices. Solution of homogeneous systems of difference equations. Solution of homogeneous systems of linear differential equations. Quadratic forms. Definiteness of quadratic forms and eigenvalues.

1, 25.2, p. 678 – 681; 8, p. 56 – 74

Second semester

7. The basic concepts of Set Theory

Properties of real numbers: a supremum and an infimum. A limit of a sequence. A limit and continuity of real-valued functions of one variable. Neighborhoods and open sets in $\mathbb{R}^n$. Sequences in $\mathbb{R}^n$ and their limits. Closed sets in $\mathbb{R}^n$. Closure and boundary of sets. Compact sets.

1, 2.1 – 2.2, p. 10 – 20; 10.1 – 10.4, p. 199 – 221; 12.1 – 12.6, p. 253 – 274; 2, 1.1 – 2.7, p. 3 – 31

8. Number series

Converging series. Criteria of convergence of series. Functional sequences and series. Convergence of functional sequences and series. Ascending power series. A radius of convergence. The formula of Cauchy-Hadamard. Expansion of functions in ascending power series. Taylor series.

1, 30.2, p. 827 – 831; 2, 9.5, p. 254 – 262

9. Functions of several variables

Functions from $\mathbb{R}^n$ to $\mathbb{R}^1$. Functions from $\mathbb{R}^n$ to $\mathbb{R}^k$ (vector functions of several variables). Level sets of functions of several variables. Continuity of a function of several variables. Partial derivatives of functions of several variables. Geometrical interpretation of partial derivatives. Chain rule for functions of several variables. A total differential. Geometrical interpretation of partial derivatives and a total differential. Linear approximation. A differentiability of functions of several variables. C^1 functions. Directional derivatives and a gradient of function of several variables. Meaning of a gradient.

1, 14.1 – 14.6, p. 300 – 322; 2, 7.4, p. 174 – 177, 8.1 – 8.7, p. 187 – 230

10. Optimization of functions of several variables

Stationary points and first order conditions. The second differential of functions of several variables. Second order conditions for a maximum and a minimum of functions of several variables.

1, 16.1 – 16.2, p. 375 – 385; 17.1 – 17.4, p. 396 – 410; 2, 11.1 – 11.7, p. 307 – 368

11. A constrained optimization

Lagrangean function and Lagrange multipliers. First order conditions. Regularity conditions of systems of restrictions. The second differential in case of dependant variables. Definiteness of quadratic form at linear constraints. Second order conditions for a problem of a constrained extremum. The bordered Hessian. Type of an extremum and signs of minors of the bordered Hessian.

1, 16.3 – 16.4, p. 386 – 395; 18.1 – 18.2, p. 411 – 423; 19.3, p. 457 – 465; 2, 12.1 – 12.3, p. 369 – 386

12. Economic meaning of Lagrange multipliers

Economic examples of application of method of Lagrange. A maximization of Utility function and a consumer demand. Slutsky Equation. Smooth dependence on parameter of a solution of a problem of constrained optimization. The Envelope Theorems.

1, 18.7 – 19.2, p. 442 – 456; 19.4, p. 469 – 471; 2, 12.5, p. 400 – 409

13. A maximization of a function of several variables with inequality constraints

Complementary slackness condition. A problem of constrained minimization. Kunn-Tacker formulation of the first order conditions under non-negativity restrictions for all variables. The mixed constrained: inequalities and equalities.

1, 18.3 – 18.6, p. 424 – 442; 2, Ch. 21: 21.1 – 21.4, p. 716 – 744) (1, 18.3, p. 430 – 434; 2, 21.3, 21.4, p. 731 – 738; 6, p. 144 – 150

14. Economic applications of nonlinear programming

Economic meaning of Lagrange multiplier. The Envelope Theorem. Smooth dependence of an extreme value on parameters.

1, 18.4 – 18.7, p. 442 – 447; 19.1 – 19.2, 19.4, p. 448 – 457; 2, 21.6, p. 747 – 754

15. Homogeneous functions

Properties of homogeneous functions. Homogenizing of functions. Homothetic functions.

1, 20.1 – 20.4, p. 483 – 504; 2, 12.6 – 12.8, p. 410 – 434

16. Convex and concave functions

Properties of convex functions. Quasiconvex and quasiconcave functions. Pseudoconvex functions. Convex programming.

1, 21.1 – 21.6, p. 505 – 543; 2, 12.6 – 12.8, p. 410 – 434

17. Linear programming

The standard form of a general linear program. The first order conditions for a linear program, and properties of a solution. Dividing and supporting hyperplanes.

2, 19.1 – 19.6, p. 651 – 687; 6, p. 146 – 150

18. A dual problem for linear program

Theorems of a linear programming. An existence theorem. A duality theorem. The complementary slackness theorem.

2, 20.2, p. 696 – 700; 6, p. 146 – 150

19. Games

Players and strategies. Representation of static game in a normal form. Elimination of strictly dominated strategy. Solution of a game. Zero-sum games. Von Neumann equilibrium. Optimal strategies in zero-sum games and dual problems of linear programming. Nash equilibrium. Cournaut Model. Bertrand Model. Nash theorem. Existence and finding of equilibria in pure and mixed strategies.

6, p. 167 – 171; 7, 1.1. A – 1.1. C, p. 1 – 48

Distribution of hours

#	Topic	Total hours	Contact hours		Self study
			Lectures	Seminars	
1.	Linear (affine) n-dimensional space	10	2	2	6
2.	Subspaces of a vector space	10	2	2	6
3.	Euclidean spaces. A scalar product	10	2	2	6
4.	Linear transformation	10	2	2	6
5.	Eigenvalues and eigenvectors of a matrix	10	2	2	6
6.	Applications of diagonalization of a matrix	20	4	4	12
7.	A limit of sequence. A limit and continuity of real-valued functions of one variable	20	4	4	12
8.	Numerical series. Converging series. Criteria of convergence of series	20	4	4	12

#	Topic	Total hours	Contact hours		Self study
			Lectures	Seminars	
9.	Function of several variables. A total differential. Directional Derivatives and a gradient of function of several variables	20	4	4	12
10.	Optimization of functions of many variables	20	4	4	12
11.	A conditional extremum. Lagrangean function and Lagrange multipliers	20	4	4	12
12.	Economic sense of Lagrange multiplier. The Envelope theorem	20	4	4	12
13.	Maximization of a function of several variables subject to inequality constraints	14	2	2	10
14.	Homogeneous functions. Homothetic functions	14	2	2	10
15.	Convex and concave functions. Quasiconvex and quasicave functions. Pseudoconvex functions. Convex programming	14	2	2	10
16.	The standard form of a general linear program	14	2	2	10
17.	A dual program in linear programming. Theorems of liner programming	14	2	2	10
18.	Games. Zero-sum games. Optimum strategy in zero-sum games and dual problems of linear programming. Nash equilibrium	10	2	2	6
	Total:	270	50	50	170