STATE UNIVERSITY —
HIGHER SCHOOL OF ECONOMICS

INTERNATIONAL COLLEGE OF ECONOMICS AND FINANCE

SYLLABUS

LINEAR ALGEBRA

for Bachelor degree in Economics 080100.62

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Introduction

Course title: Linear algebra

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Discussion section teachers: Dmitri D. Pervouchine, Ivan O. Kachkovsky, Daniel M. Esaulov, Artyom O. Kal'chenko, Stephan A. Provornikov

Course description

Linear Algebra is a half-semester (12 weeks) class that is obligatory for the curriculum of the second-year MIEF students. The course was originally designed as an instrumental supplement to the principal quantitative block subjects such as "Methods of optimization", "Time series analysis", and "Econometrics". Linear Algebra shares many exam topics with the program of London University, for instance in "Mathematics 1", "Mathematics 2" и "Further mathematics for economists". At the same time, the class of Linear Algebra in MIEF is taught on its own to deliver basic principles of matrix calculus. From a broader prospective, the aim of the course is to deliver one of the most general mathematical concepts - the idea of linearity.

The course splits naturally into the following three parts:

1. Problems that are immediately related to linear equations and, consequently, to the expansion 2D- and 3D- intuition onto linear spaces of higher dimensions. This part includes the concepts of basis, rank, dimension, linear subspace, etc.
2. Problems that involve antisymmetric polylinear forms (determinants) and their applications (eigenvectors and eigenvalues, matrix diagonalization, etc). I suggest this section also include complex numbers.
3. Problems that arise from the calculus of bilinear forms: quadratic forms, orthogonalization, and other geometric issues in higher-dimensional Eucledian spaces.

In Linear Algebra, it is critically important to teach not only the technique of manipulations with matrices and vectors, but also general algebraic concepts that are used, for instance, in problems that involve linear differential equations or linear differential difference equations

Improvements

Compared to the class of the previous year, this syllabus contains the following changes.

1. The technique of determinants is not discussed until all other topics which don't involve determinants (such as “fundamental system of solutions”) have been studied. This is natural from both algebraic point of view and from the consequent course delivery.
2. In order to solve characteristic equations that arise in eigenvector problems, one needs complex numbers. It appears to be quite natural to introduce complex numbers in the Linear Algebra, not in any other discipline. This will require one extra lecture.

Teaching methods

- Lectures (2 hours per week)
- Discussion sections (2 hours per week)
- Homeworks (weekly)
- Self-study

Control methods

- Homeworks (weekly)
- Online home teat
- Mock exam
- Final exam

The mock exam that is held after the 5th lecture and the final exam that is held when the course is completed both consist of two parts, Multiple choice and Free response. The mock exam takes 90 minutes; the final exam takes 150 minutes. The final exam is cumulative, i.e., it covers the entire course, not only topics since the mock exam. The online home test, which is available through internet, consists of multiple choice questions only and doesn't contribute to the final grade.

Grades

The weekly homeworks constitute 10% of the final grade. The mock exam contributes 40% to the final grade. The final exam is 50% of the final grade.

Basic reading

1. Chernyak V. Lecture Notes on Linear Algebra. Introductory course. Dialog, MSU, 1998, 2000 (Chernyak)
2. Pervouchine DD Lecture Notes in Linear Algebra. ICEF MOSCOW 2011

3. Carl P. Simon and Lawrence Blume. Mathematics for Economists, W.W. Norton & Company, 1994 (Simon, Blume)

Advanced reading

1. Chiang, Fundamental Methods of Mathematical Economics, McGraw-Hill, 3rd ed., 1984
2. R.O.Hill, Elementary Linear Algebra, Academic Press, 1986
3. Гельфанд И.М. Лекции по линейной алгебре Москва, Наука, 1999.
4. Кострикин А.И., Манин Ю.И., Линейная алгебра и геометрия, Москва, Наука 1986.
5. Проскуряков И.В. Сборник задач по линейной алгебре, Москва, Наука, 1985.

Course description

1. Systems of linear equations in matrix form. Basic concepts and geometric interpretation. Consistency. Elementary transformations of equations. Gauss and Gauss-Jordan methods. (Chernyak, ch. 1 - 5; Simon & Blume, ch. 7)

2. Linear space. Linear independence. Rank. Linear span. Bases and dimension of a linear space. Ordered bases and coordinates. Transition from one basis to another. Properties of linearly dependent and linearly independent vectors. Examples. (Chernyak, ch. 9-11; Simon & Blume, ch. 7,11)

3. Linear subspace. The set of solutions as a linear subspace. General and particular solutions. Fundamental set of solutions. (Chernyak, ch. 11; Simon & Blume, ch. 11)

4. Matrix as a set of columns and as a set of rows. Linear operations on matrices. Transpose matrix and matrix algebra. Special types of matrices. Matrices of elementary transformations. (Chernyak, ch. 2-3; Simon & Blume, ch. 8)

5. Determinant of a set of vectors. Geometric interpretation. Determinant of a matrix. Computation and basic properties of determinants. Cramer's rule. Applications to rank computation. (Chernyak, ch. 6-8; Simon & Blume, ch. 9)

6. Inverse matrix. Degenerate matrices. Computation of the inverse matrix by the extended Gauss algorithm and by using algebraic complements. (Chernyak, ch. 12; Simon & Blume, ch. 8)

7. Linear operator as a geometric object. Matrix of a linear operator. Examples, including linear operators in functional spaces. Transformations of vectors and matrices of linear operators induced by a change of coordinates. Conjugate matrices. (Chernyak, ch. 15)

8. Complex numbers. Motivation. The fundamental theorem of algebra. Complex plane. Absolute value and argument of a complex number. Power of a complex number. Examples. (Simon & Blume, appendix A3)

9. Eigenvalues, eigenvectors and their properties. Characteristic equation. Basis and dimension of eigenspaces. Diagonalization and its applications. (Chernyak, ch. 13-14; Simon & Blume, ch. 23)

10. Bilinear and quadratic forms. Canonical representation. Full squares method. Symmetric matrices and quadratic forms. Definite, indefinite, and semidefinite forms. Silvester's criterion. (Simon & Blume, ch. 16)

11. Dot product in linear spaces. Norm of a vector. Metric properties: distances and angles. Projection onto a subspace. Orthogonal bases. Orthogonalization. Equations of lines and planes. (Chernyak, ch. 16, Simon & Blume, ch. 10)

Topic-by-hour plan

	Topic	Total hours		In-class hours		Self-study	
		For spec. БдиФ, ЭиФ, ЭиМ	For spec. Э	Lectures	Seminars	For spec. БдиФ, ЭиФ, ЭиМ	For spec. Э
1.	Systems of linear equations	9	12	2	2	5	8
2.	Vectors and linear spaces	9	12	2	2	5	8
3.	Fundamental set of solutions	9	12	2	2	5	8
4.	Linear dependence and rank	9	12	2	2	5	8
5.	Determinants	9	12	2	2	5	8
6.	Inverse matrix	9	12	2	2	5	8
7.	Complex numbers	9	12	2	2	5	8
8.	Eigenvalues and eigenvectors	9	12	2	2	5	8
9.	Quadratic forms	9	12	2	2	5	8
10.	Geometry of linear spaces	9	12	2	2	5	8
	Total	90	120	20	20	50	80