Japan—Russia winter school (15.01.2012 – 02.02.2012)
will take place at HSE Department of Mathematics, room 212.

Speakers:

Natalia Goncharuk: **Rotation numbers and moduli of elliptic curves .**
(3 lectures).

Alexander Efimov: **Homotopy finiteness of DG categories from algebraic geometry .**
(3 lectures).

Kei Irie: **Symplectic homology.**
(3 lectures)

Maxim Kazarian: **Characteristic classes of singularities.**
(3 lectures)

Yoshikata Kida: **Introduction to orbit equivalence for group actions .**
(2 lectures)

Vladlen Timorin: **The Hurwitz sums of squares formulas.**
(4 lectures)

Shintaro Yanagida: **Stability conditions of sheaves and complexes on algebraic variety.**
(3 lectures)

Schedule:

January 15 (Tue)	15:30 Timorin – 1	17:00 Kazarian -1
January 16 (Wed)	15:30 Timorin – 2	17:00 Efimov - 1
January 17 (Thu)	15:30 Kazarian - 2	17:00 Efimov - 2
January 18 (Fri)	15:30 Efimov – 3	
January 21 (Mon)		17:00 Kida - 1
January 22 (Tue)	15:30 Irie - 1	17:00 Timorin-3
January 23 (Wed)	15:30 Timorin – 4	17:00 Kida - 2
January 24 (Thu)	15:30 Kazarian – 3	17:00 Irie – 2
January 25 (Fri)	15:30 Yanagida – 1	
January 29 (Tue)	15:30 Goncharuk – 1	17:00 Irie -3
January 30 (Wed)	15:30 Goncharuk – 2	17:00 Yanagida – 2
January 31 (Thu)		
February 1 (Fri)	15:30 Yanagida - 3	
February 2 (Sat)		17:00 Goncharuk - 3

Abstracts:

Natalia Goncharuk: **Rotation numbers and moduli of elliptic curves .**
(3 lectures)

Abstract: The course is devoted to two notions related to circle diffeomorphisms: the classical notion of rotation number originated in the works of Poincare and the notion of complex rotation number suggested by V. Arnold.

Take the cylinder of height h; gluing its bottom and top boundaries via some circle diffeomorphism, we obtain a torus. For an analytic circle diffeomorphism, this torus has a natural structure of complex manifold. How does this complex manifold depend on h, in particular how does it behave as h tends to 0? (Formally, we consider the complex rotation number, that is, the modulus of the corresponding elliptic curve).

The aim of the course is to answer this question. The answer was obtained in 2001 -- 2012 by V.Moldavskis, Yu.Ilyashenko, X.Buff and me.

The first lecture will concern rotation numbers and Arnold tongues. This lecture is devoted to classical results and self-contained, so you may choose to attend only this lecture.

In the second lecture, we will discuss moduli of elliptic curves and complex rotation numbers, and I will formulate several results on complex rotation numbers. We will prove some of them using a simple trick named Buff construction.

In the third lecture, I will give the definition of quasiconformal maps and apply the techniques of the quasiconformal maps to complex rotation numbers.

Alexander Efimov: **Homotopy finiteness of DG categories from algebraic geometry .**
(3 lectures).

Abstract: We will explain that for any separated scheme of finite type over a field of characteristic zero, the DG category $D^b_{coh}(X)$ is homotopically finitely presented. This uses categorical resolution of singularities by Kuznetsov and Lunts, and confirms a conjecture of Kontsevich.

We will also explain how to show the analogous result for coherent matrix factorizations on such schemes.

Kei Irie: **Symplectic homology.**
(3 lectures)

Abstract: Symplectic homology is a version of Hamiltonian Floer homology, which is defined for open symplectic manifolds satisfying a certain convexity condition. Although it is rather easy to define, symplectic homology is a powerful tool to study questions in symplectic and contact topology.

We will begin from a review of classical Hamiltonian Floer homology, and explain some basic results in symplectic homology. We will also present some applications, especially those concerning periodic orbits of Hamiltonian systems.

Maxim Kazarian: **Characteristic classes of singularities.**
(3 lectures).

Abstract: In these lectures we present an introduction to the theory of universal expressions, known as Thom polynomials, for the cohomology classes Poincare dual to the degeneracy loci of differential geometry objects in the parameter space of a family of such objects - configurations, varieties, morphisms of vector bundles, singularities and multisingularities of holomorphic mappings etc. It provides a power and universal tool for the solution of many enumerative problems in the complex projective geometry.

In the first lecture we will overview the Chern classes of vector bundles: possible definitions, relations, methods of computations and first applications.

In the second lecture the Thom polynomials will be introduced; we will prove the existence theorem and discuss some tools of their computations.

The third (last) lecture will be devoted to the generalization of Thom polynomials to the case of multisingularities.

Yushikata Kida: **Introduction to orbit equivalence for group actions.**

(2 lectures)

Abstract: 1st lecture:
Introduction to orbit equivalence for group actions, I: Actions of the integer group

I give two introductory talks on orbit equivalence theory for probability-measure-preserving actions of discrete countable groups. In the first lecture, I explain a story toward understanding actions of the integer group, mainly contributed by Dye around 1960. This is nowadays a classical topic. Further development on actions of amenable groups are also explained.

2nd lecture:
Introduction to orbit equivalence for group actions, II: An aspect of rigidity

In recent years, many people obtained various kinds of rigidity results on orbit equivalence.
In the second lecture, I explain rigidity for actions of higher rank Lie groups by Zimmer, and actions of mapping class groups of surfaces by the lecturer. To understand these results, the notion of virtual groups plays an important role. This was introduced by George W. Mackey in 1960s. I focus on connection between virtual groups and these rigidity results. These two lectures deal with almost independent topics.

Vladlen Timorin: **The Hurwitz sums of squares formulas.**

(4 lectures)

Abstract: Consider a formula
$(x_1^2 + ... + x_r^2)\,(y_1^2 + ... + y_s^2) = z_1^2 + ... + z_n^2,$
in which x_i and y_j are independent variables, and z_k are bilinear functions of x_i and y_j. This is a *Hurwitz formula* of type (r,s,n). Multiplications of complex numbers, quaternions and octonions provide examples of Hurwitz formulas of types (2,2,2), (4,4,4) and (8,8,8). In 1898, A. Hurwitz posed the following problem: describe all triples (r,s,n), for which there exists a Hurwitz formula of type (r,s,n). This problem is still open. We will discuss concepts from Representation Theory, Geometry and Topology related to the Hurwitz problem.

Shintaro Yanagida: **Stability conditions of sheaves and complexes on algebraic variety.**

(3 lectures)

Abstract. I will give some introductory talks on the several notions of stability conditions in the algebraic geometry.

The 1st talk deals with the classical Mumford-Gieseker stability, which is defined with repsect to the coherent sheaves on the algebraic curves or surfaces.

The 2nd talk is the introduction of Bridgeland's stability conditions on triangulated categories. I will also explain some constructions of those stability conditions in the case of bounded derived categories of coherent sheaves.

In the 3rd talk, I will explain some advanced topics on Bridgeland's stability conditions: the wall/chamber structures on the space of stability conditions, the behavior under Fourier-Mukai transform and its consequence.