«Winning by Losing»:
Incentive Incompatibility in Multiple Qualifiers

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Plan

- Problem of manipulability of football matches outcomes
- Real-world examples
- Literature review
- Formal model
- Possible fixes
Do the competing teams have perverse incentives to lose a game deliberately? «Yes», if

- team is bribed
- ranking rule or distribution of prizes is not monotonous
- the game is coalitional

What if a tournament is fair? In a single tournament — «No». But in multiple tournaments...
Standings as of May 8, 2012:

<table>
<thead>
<tr>
<th>Place</th>
<th>Team</th>
<th>Pts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Zenit St.Petersburg</td>
<td>85</td>
</tr>
<tr>
<td>2</td>
<td>CSKA Moscow</td>
<td>73</td>
</tr>
<tr>
<td>3</td>
<td>Spartak Moscow</td>
<td>72</td>
</tr>
<tr>
<td>4</td>
<td>Dynamo Moscow</td>
<td>71</td>
</tr>
<tr>
<td>5</td>
<td>Anzhi Makhachkala</td>
<td>70</td>
</tr>
<tr>
<td>6</td>
<td>Lokomotiv Moscow</td>
<td>66</td>
</tr>
<tr>
<td>7</td>
<td>Rubin Kazan’</td>
<td>65</td>
</tr>
<tr>
<td>8</td>
<td>Kuban’ Krasnodar</td>
<td>60</td>
</tr>
</tbody>
</table>


Cup final (May 9): Rubin – Dynamo.

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1Credits to Dr. Andrei Brichkin who initially noticed this drawback.
Two major European international tournaments: UEFA Champions League and UEFA Europa League

Berths are distributed according to the following rules:

1. Teams that are ranked 1st and 2nd in the Russian national championship qualify for the Champions League.
2. Teams that are ranked 3rd to 5th in the national championship qualify for the Europa League.
4. If cup winner is ranked 1st or 2nd in the national championship, then this team qualifies for the Champions League, and the cup runner-up qualifies for the Europa League.
5. If Cup winner is ranked 3rd to 5th in the national championship, then the team ranked 6th in the national championship qualifies for the Europa League.
Suppose that Dynamo wins the Russian Cup and beats Kuban’, and Rubin – CSKA is a draw. Anzhi – Zenit game is irrelevant.

<table>
<thead>
<tr>
<th>Lokomotiv’s win</th>
<th>Draw</th>
<th>Lokomotiv’s loss</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>№</strong></td>
<td><strong>Team</strong></td>
<td><strong>Pts</strong></td>
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<td>Lokomotiv</td>
<td>69</td>
</tr>
<tr>
<td>7</td>
<td>Rubin</td>
<td>66</td>
</tr>
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<td>8</td>
<td>Kuban</td>
<td>60</td>
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</tbody>
</table>

Lokomotiv has all the incentives to lose!

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▶ (Harary, Moser, 1966) Ranking teams = Aggregation of voters’ preferences
▶ (Arrow, 1963) Several highly desired properties of aggregation of voters’ preferences rules. Arrow’s impossibility theorem.
▶ (Rubinstein, 1985) Similar approach for the problem of ranking participants in the round-robin tournament
▶ (Gibbard, 1973; Satterthwaite, 1975; Duggan, Schwartz, 2000) Under «good enough» aggregation rules there always exists a voter who can profitably deviate from his truly preferences
▶ (Russell, Walsh, 2009) Coalitional manipulating in cups and round-robin competitions
Tournament is a pair \((\mathcal{X}, v(x, y))\), where \(\mathcal{X}\) is a non-empty finite set of the teams and \(v(x, y)\) is a function which satisfies the following three conditions:

1) \(v(x, y)\) is defined on the set \((\mathcal{X} \times \mathcal{X}) \setminus \{(x, y) | x = y\}\);
2) image of \(v(x, y)\) is a subset of the set \((-1, 0, 1)\);
3) for each \(x_0, y_0 \in \mathcal{X}, x_0 \neq y_0\), the equality \(v(x_0, y_0) = -v(y_0, x_0)\) holds.
**Example.** Consider a tournament $T = (\mathcal{X}, \nu_0)$, where $\mathcal{X} = \{A, B, C, D\}$ and characteristic function $\nu_0$ is given by the following table:

<table>
<thead>
<tr>
<th></th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>-</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>$B$</td>
<td>-1</td>
<td>-</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$C$</td>
<td>1</td>
<td>-1</td>
<td>-</td>
<td>-1</td>
</tr>
<tr>
<td>$D$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>
Ranking method $S = S(\nu)$ is a rule that orders the participating teams in accordance with the results of all matches $\nu$.

If $|\mathcal{X}| = n$, then $S(\nu) = (s_1(\nu), \ldots, s_n(\nu))$, where $s_i(\nu)$ is the place assigned to $i$-th team by the ranking method $S$. 

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If $s_i(\nu) \neq s_j(\nu)$ for any $i \neq j$, we say that the set $S(\nu)$ is strictly totally ordered.
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Ranking method $S = S(\nu)$ is well-defined iff for any characteristic function $\nu$ the set $S(\nu)$ is strictly totally ordered.
Let $\mathcal{X} = \{A, B, C, D\}$ and $S = S(v)$ be the following ranking method:
1) Victory = 3 points, Draw = 1 point, Defeat = 0 points;
2) More points $\Rightarrow$ ranked higher;
3) Same number of points $\Rightarrow$ matches between these teams are considered;
4) Initial seeding: $A \succ B \succ C \succ D$. 
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In particular, $S(v_0) = D \succ B \succ A \succ C$. 
$N^1_v(i)$, $N^0_v(i)$ and $N^{-1}_v(i)$ = numbers of wins, draws and losses of team $i$ respectively.
Ranking method $S$ satisfies \textbf{monotonicity} property, iff for any characteristic function $\nu$ and for any two teams $x, y \in \mathcal{X}$ such that

\[ N^1_\nu(x) \geq N^1_\nu(y), \quad N^1_\nu(x) + N^0_\nu(x) \geq N^1_\nu(y) + N^0_\nu(y), \quad (1) \]

where at least one of the inequalities in (1) is strict,

\[ s_x(\nu) < s_y(\nu) \]

holds.
Theory – definitions

Ranking method $S$ satisfies **monotonicity** property, iff for any characteristic function $v$ and for any two teams $x, y \in \mathcal{X}$ such that

$$N_v^1(x) \geq N_v^1(y), \quad N_v^1(x) + N_v^0(x) \geq N_v^1(y) + N_v^0(y), \quad (1)$$

where at least one of the inequalities in (1) is strict,

$$s_x(v) < s_y(v)$$

holds.

All reasonable ranking methods satisfy monotonicity property.

’More wins $\Rightarrow$ ranked worse’ doesn’t
Theory – definitions

- One international tournament and \( N \) domestic tournaments take place, \( N \geq 2 \).
- Tickets to international tournament are the only prizes in domestic tournaments.
- \( \mathcal{X} = \{1, 2, ..., K\}, \ K \geq 1 \).
- \( b_i \) is the number of tickets into international tournament laying on the line in tournament \( i, \ i = 1, ..., K \).
What if a certain team gets the ticket into international tournament more than once?

In the extreme case there will be only $\max_i b_i$ contested tickets instead of $\sum_i b_i$.

Redistribution rule must be defined
Example

- 2 domestic tournaments, 1 international tournament
- 3 participating teams
- 2 tickets to international tournament, $b_1 = b_2 = 1$
- **Redistribution rule**: if one team wins both tournaments (it means that there is one vacant ticket), then the second ticket is given to the team that finished on the second place in the first tournament.

**Redistribution rule** is a labelled tree like this:
**Theorem.** Let the following five conditions hold simultaneously:

1) $N \geq 2$;
2) $b_i \geq 1$ for each $i = 1, \ldots, N$;
3) $K > \max \left( \sum_i b_i, 3 \right)$;
4) for each $i = 1, \ldots, N$ ranking method $S_i$ is well-defined;
5) for each $i = 1, \ldots, N$ ranking method $S_i$ satisfies monotonicity property.

Then for any ranking methods $S_1(v), \ldots, S_N(v)$ and for any redistributing rule $R$ there exist such characteristic functions $v_1, \ldots, v_N, w$ and $i, 1 \leq i \leq N$, that the following four conditions hold simultaneously:
Theorem

i) there exists the collection \((x_0, y_0)\), such that \(v_i(x_0, y_0) = 1\) and \(w(x_0, y_0) = -1\);

ii) for any collection \((x, y)\), different from \((x_0, y_0)\), holds the equality \(w(x, y) = v_i(x, y)\);

iii) according to the standings
\[S_1(v_1), ..., S_{i-1}(v_{i-1}), S_i(v_i), S_{i+1}(v + 1), ..., S_N(v_N)\] team \(x\) gets a ticket to international tournament;

iv) according to the standings
\[S_1(v_1), ..., S_{i-1}(v_{i-1}), S_i(w), S_{i+1}(v + 1), ..., S_N(v_N)\] team \(x\) doesn’t get a ticket to international tournament.

In other words, for each «good» ranking methods and redistribution rule it is possible to give an example of tournament results, when a team prefers to lose
Idea of the proof. Example

- 2 domestic tournaments, 1 international tournament
- 4 participating teams
- 2 tickets to international tournament, $b_1 = b_2 = 1$
- **Redistribution rule**: if one team wins both tournaments, then the second ticket is given to the team that finished on the second place in the first tournament.

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<table>
<thead>
<tr>
<th>Tournament 1</th>
<th>Tournament 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>A</td>
<td>Win</td>
</tr>
<tr>
<td>B</td>
<td>Loss</td>
</tr>
<tr>
<td>C</td>
<td>Loss</td>
</tr>
<tr>
<td>D</td>
<td>Loss</td>
</tr>
<tr>
<td>A</td>
<td>?</td>
</tr>
<tr>
<td>B</td>
<td>?</td>
</tr>
<tr>
<td>C</td>
<td>Draw</td>
</tr>
<tr>
<td>D</td>
<td>Loss</td>
</tr>
</tbody>
</table>

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Generalizations

1. Most of European football national championships are run in two rounds on the home-away basis

2. Sometimes teams compete for tickets to several tournaments, not to one. Example: Champions League and Europa League.

3. Play-off tournaments
   - No incentives to lose in a play-off tournament
   - If there is one play-off tournament (Cup) and one round-robin tournament (Championship) it could be profitable to lose in round-robin tournament in order to push a certain team that is successful in both tournaments higher and to gain from redistribution
   - However, it is possible only if redistribution favours play-off tournament
Advice to UEFA

- How to prevent cases like Russia-2011/2012?

- Just make redistribution rule always favouring championship, not cup. Avoid the rules like this:

  4. If cup winner is ranked 1st or 2nd in the national championship, then this team qualifies for the Champions League, and the cup runner-up qualifies for the Europa League.
Thank you