A Multidimensional Model for Analyzing Democratic Development in Central and Eastern Europe

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Abstract. Various indices and ratings describing democratic processes in countries around the world have been developed by international organizations (such as Freedom House) and analytical centers (such as the one affiliated with the journal Economist). The main drawback of such ratings is that they only provide a linear ordering of countries by averaging a multitude of criteria. Such approach does not make it obvious which particular problems exist in which countries and thus does not help comparing democratic processes in different countries.
In this paper, we propose a multidimensional model for ratings based on a mathematical discipline of formal concept analysis, which deals, in particular, with automated taxonomy construction from object–attribute data. In our case, every node of a taxonomy would group countries similar in certain aspects, while at the same time providing a description of these aspects. The aim is not to question the existing ratings, but rather to provide a neutral instrument for uncovering the structure of the data underlying these ratings. The proposed representation is much more informative than linear ratings, since it shows the commonalities and differences in the democratic development of various countries. In addition, it provides a solid ground for discussing, comparing, and criticizing ratings. It can also help formulate theoretical hypotheses on the evolution of democracy, thereby advancing scientific discovery.
We illustrate the proposed representation with the case study of countries in Central and Eastern Europe and the former Soviet Union.

1 Introduction

There has long been a need for an objective analytical toolset to be used in the study of democracy that would reduce the impact of personal judgments and preferences on the research output. Different schools of political thought have different views on how to understand democracy and how to evaluate progress and failure in the democratic development [11, 3, 4]. Striving to unify these different views, political scientists often resort to statistical data, which can be

analyzed mathematically. There have been attempts to quantify the essentially "qualitative" data such as evaluations of the functioning of government, political freedom, civil liberties, etc., based on both "hard" data (e.g., legislative bills and official reports) and on expert assessment of political process. Quantified and averaged, this data becomes ratings—in particular, ratings of democratic development in countries around the world.

One of the first and most popular such ratings is "Freedom in the World" issued by Freedom House on a yearly basis [6]. This rating provides long-term statistics of democratic development from 1973 up to now. It is widely used by analytical organizations and in academia (for example, see [19]).

Other ratings of freedom and democracy include those issued by the Heritage Foundation, the Economist, USAID, World Bank, etc.; see also [15] and [13].

Ratings often get criticized for how they aggregate data on different aspects of democracy. To quote [17]:

> [...] the sum of a civil liberty score of 4 and a political liberty score of 2 is the same as the sum of a civil liberty score of 2 and a political liberty score of 4 even though the substantive interpretation of these different combinations is different.

Indeed, averaging the values of different parameters may often be misleading. In our work, we propose a representation based on formal concept analysis [8] that avoids mixing together incomparable factors by unfolding linear rankings into hierarchical structures. In Section 2, we describe the data behind the "Freedom in the World" rating that will be used to illustrate our approach. Section 3 introduces main tools of formal concept analysis. We then show how it can be applied to the "Freedom in the World" rating in Section 4 and how it helps analyze democratic development in temporal perspective in Section 5.

2 The "Freedom in the World" Rating and the Underlying Data

In this paper, we use data available from Freedom House as part of the "Freedom in the World" rating for 2006–2009 [6]. Freedom House assigns every country a score in each of the following seven categories:

1. Electoral Process
2. Political Pluralism and Participation
3. Functioning of Government
4. Freedom of Expression and Belief
5. Associational and Organizational Rights
6. Rule of Law
7. Personal Autonomy and Individual Rights

Each score is obtained by summing up points awarded with respect to three or four related questions. As a result, Electoral Process, Functioning of Government, and Associational and Organizational Rights are evaluated on a scale

of 0 to 12, while the other four categories are evaluated on a scale of 0 to 16. Then, the first three (Electoral Process, Political Pluralism and Participation, and Functioning of Government) and the last four (Freedom of Expression and Belief, Associational and Organizational Rights, Rule of Law, and Personal Autonomy and Individual Rights) scores are added together to obtain cumulated scores on, respectively, political rights and civil liberties, which are then scaled down and averaged to get a combined score for each country.

For 2009, the Central and Eastern European countries and countries of the former Soviet Union are ranked as follows (a smaller number after the name of a country corresponds to a better score):

Table 1. Combined average ratings from the 2009 edition of "Freedom in the World".

Free	Partly Free	Not Free
Czech Republic (1.0)	Albania (3.0)	Azerbaijan (5.5)
Estonia (1.0)	Macedonia (3.0)	Kazakhstan (5.5)
Germany (1.0)	Montenegro (3.0)	Russia (5.5)
Hungary (1.0)	Bosnia-Herzegovina (3.5)	Tajikistan (5.5)
Lithuania (1.0)	Georgia (4.0)	Belarus (6.5)
Poland (1.0)	Moldova (4.0)	Turkmenistan (7.0)
Slovakia (1.0)	Kyrgyzstan (4.5)	Uzbekistan (7.0)
Slovenia (1.0)	Armenia (5.0)	
Latvia (1.5)		
Bulgaria (2.0)		
Croatia (2.0)		
Romania (2.0)		
Serbia (2.5)		
Ukraine (2.5)		

While good for getting a general impression, such a table does not provide insight into concrete problems existing in various countries. An interested researcher must go back to the original data to find out why a country occupies this or that position in the table. Fortunately, starting from 2006, Freedom House provides access to more complete information in the form shown in Table 2. Although this table contains data on individual parameters, it is not immediately helpful if one wants to to get an idea of similarities and differences between the countries. We show how concept lattices from formal concept analysis can transform data from Table 2 into a more informative representation than Table 1, at the same time, simplifying comparative analysis.

Table 2. Aggregate and sub-category scores for the 2009 "Freedom in the World" rating. **PR:** Political Rights; **CL:** Civil Liberties; **Status:** F—Free; PF—Partly Free; NF—Not Free; **A:** Electoral Process; **B:** Political Pluralism and Participation; **C:** Functioning of Government; **D:** Freedom of Expression and Belief; **E:** Associational and Organizational Rights; **F:** Rule of Law; **G:** Personal Autonomy and Individual Rights.

	PR	CL	Status	A	B	C	D	E	F	G
Albania	3	3	PF	8	11	7	13	8	10	9
Armenia	6	4	PF	4	4	3	8	5	5	9
Azerbaijan	6	5	NF	2	3	3	6	3	4	8
Belarus	7	6	NF	0	3	1	3	1	2	5
Bosnia-Herzegovina	4	3	PF	8	10	5	11	7	10	10
Bulgaria	2	2	F	12	15	8	14	11	11	11
Croatia	2	2	F	12	14	9	14	12	10	13
Czech Republic	1	1	F	12	15	11	16	12	14	15
Estonia	1	1	F	12	15	12	16	12	14	14
Georgia	4	4	PF	6	6	6	11	7	5	10
Germany	1	1	F	12	15	12	15	12	15	15
Hungary	1	1	F	12	15	10	16	12	13	14
Kazakhstan	6	5	NF	2	3	2	7	4	4	8
Kyrgyzstan	5	4	PF	4	5	4	9	5	5	7
Latvia	2	1	F	12	15	7	16	12	12	13
Lithuania	1	1	F	12	16	9	16	11	14	13
Macedonia	3	3	PF	7	10	7	11	7	8	10
Moldova	4	4	PF	9	8	5	10	6	8	9
Montenegro	3	3	PF	9	9	6	13	10	8	11
Poland	1	1	F	12	16	10	16	12	13	14
Romania	2	2	F	12	14	8	14	11	12	12
Russia	6	5	NF	2	3	3	8	4	4	7
Serbia	3	2	F	9	13	7	14	11	9	13
Slovakia	1	1	F	12	15	10	16	12	12	14
Slovenia	1	1	F	12	15	11	14	12	14	13
Tajikistan	6	5	NF	2	4	2	6	4	4	5
Turkmenistan	7	7	NF	0	1	0	2	0	1	2
Ukraine	3	2	F	10	13	5	13	11	10	11
Uzbekistan	7	7	NF	0	0	0	1	0	0	2

3 A Short Introduction into Formal Concept Analysis

Formal concept analysis (FCA) [8] provides tools for understanding the structure of data given as a set of objects with certain descriptions (e.g., in terms of attributes they posses), which is done by representing the data as a hierarchy of concepts, or more exactly, a concept lattice (in the sense of lattice theory [2]). Every concept has extent (the set of objects that fall under the concept) and intent (a set of attributes or features that together are necessary and sufficient for an object to be an instance of the concept). Concepts are ordered in terms of being more general or less general (i.e., covering more objects or fewer objects).

The concept lattice, being a rather universal structure, provides a wealth of information about the relations among objects and attributes, which made possible applications in areas ranging from history and sociology [14] to epistemic community detection [16] and social network analysis [7] to machine learning [9] and ontology construction [18]. Indeed, it can help in processing a wide class of data types (for example, any data represented as a table). Besides structural representation of data, the concept lattice provides a framework in which various data analysis and knowledge acquisition techniques can be formulated.

We briefly introduce necessary mathematical definitions [8] and then explain them less formally. Given a *(formal) context* $\mathbb{K} = (G, M, I)$, where G is called a set of *objects*, M is called a set of *attributes*, and the binary relation $I \subseteq G \times M$ specifies which objects have which attributes, the derivation operators $(\cdot)^I$ are defined for $A \subseteq G$ and $B \subseteq M$ as follows:

$$A^I = \{m \in M \mid \forall g \in A : gIm\} \qquad B^I = \{g \in G \mid \forall m \in B : gIm\}$$

In words, A^I is the set of attributes common to all objects of A and B^I is the set of objects sharing all attributes of B.

If this does not result in ambiguity, $(\cdot)'$ is used instead of $(\cdot)^I$. The double application of $(\cdot)'$ is a closure operator, i.e., $(\cdot)''$ is extensive, idempotent, and monotonous. Therefore, sets A'' and B'' are said to be *closed*.

A *(formal) concept* of the context (G, M, I) is a pair (A, B), where $A \subseteq G$, $B \subseteq M$, $A = B'$, and $B = A'$. In this case, we also have $A = A''$ and $B = B''$. The set A is called the *extent* and B is called the *intent* of the concept (A, B).

A concept (A, B) is a *subconcept* of (C, D) if $A \subseteq C$ (equivalently, $D \subseteq B$). The concept (C, D) is then called a *superconcept* of (A, B). We write $(A, B) \leq (C, D)$ and define the relations $\geq$, $<$, and $>$ as usual. If $(A, B) < (C, D)$ and there is no (E, F) such that $(A, B) < (E, F) < (C, D)$, then (A, B) is a *lower neighbor* of (C, D) and (C, D) is an *upper neighbor* of (A, B); notation: $(A, B) \prec (C, D)$ and $(C, D) \succ (A, B)$.

The set of all concepts ordered by $\leq$ forms a lattice, which is denoted by $\underline{\mathfrak{B}}(\mathbb{K})$ and called the *concept lattice* of the context $\mathbb{K}$. The relation $\prec$ defines edges in the *covering graph* of $\underline{\mathfrak{B}}(\mathbb{K})$.

Speaking less formally, the context makes precise the scope of the discussion by specifying the domain to which it applies (listing all the objects of this domain) and defining the terms in which it is going to be discussed (listing the

attributes to be used in object descriptions). For example, if we wish to analyze democratic development, one thing we need to do is to specify the geographical (e.g., Central and Eastern Europe) or otherwise restricted (e.g., the third-world countries) area under consideration and to divide this area into individual entities subject to democratic development. In this paper, we talk about countries, but we could instead choose to talk about regions (such as Balkan countries as a whole, CIS countries, etc.) or—perhaps, less easily in this case—about certain areas within each country, thus, controlling the level of granularity. These entities are the objects of our context.

We also need to identify and fix a set of parameters of democratic development to be used in our discussion. These are the attributes of the context, and they can also be different with respect to granularity. For instance, we could use four parameters, such as (1) Freedom of Expression and Belief, (2) Associational and Organizational Rights, (3) Rule of law, and (4) Personal Autonomy and Individual Rights to evaluate civil liberties in each country (as does the Freedom House in its "Freedom in the World" rating), or we could use just a single parameter for the same purpose (this is the approach adopted by the Economist Intelligence Unit in its Democracy Index [5]). Besides selecting a level of granularity for parameters, we may also decide to take or not to take a particular parameter into account (an example is political culture, which features as a separate parameter in the Democracy Index of the Economist Intelligence Unit, but not in the "Freedom in the World" rating).

A formal concept is usually visualized by means of a cross-table, where rows correspond to objects, columns correspond to attributes. An otherwise empty cell contains a cross if the respective object has the respective attribute (see Fig. 1 for an example).

A formal concept of a formal context is a mathematization of the old philosophical tradition (see *Logic of Port Royal* [1]) of characterizing a concept through of its extension (or extent) and intension (or intent), so that the extension consists of precisely all objects having all attributes of the intension and the intension contains exactly all attributes shared by all objects of the extension. To give an (oversimplified) example, one can hypothesize a concept whose extent consists of all countries with low level of personal autonomy and individual rights. Assuming that the latter is an attribute of our context, the intent of this concept will contain this very attribute, but also all other attributes shared by such countries in the context—e.g., "inadequate enforcement of rule of law" if every country with low level of personal autonomy and individual rights in our context exhibits this trait, too.

This leads us to the notion of an *implication*, which is, formally, an expression $A \to B$, where $A, B \subseteq M$ are attribute subsets. It *holds* in the context is $A' \subseteq B'$, i.e., every object of the context that has all attributes from A also has all attributes from B. A simple example of an implication is

$$\{\text{"low level of personal autonomy and individual rights"}\} \to \{\text{"inadequate enforcement of rule of law"}\},$$

which may or may not hold in a particular context. A is called the *premise* and B the *conclusion* of the implication $A \to B$. Neither A nor B has to be single-element, although they both are in the above example.

Note that concepts and implications are inherently context-dependent. In particular, the extent of a concept depends on our choice of attributes used to describe the objects, while the intent depends on the objects included in the context. One concept is *more general* than another one if the extent of the former covers all objects from the extent of the latter and some other objects. Clearly, the intent of a more general concept should be narrower (i.e., should contain fewer attributes) than the intent of a less general concept: a larger set of object shares a smaller set of attributes. This generality order is captured by the subconcept–superconcept $\leq$ relation defined above. This relation has some special properties, and, mathematically, is a partial order, which means that two different concepts cannot both be more general than the other, but they can be incomparable. In addition, this partial order is a lattice: every two concepts have a unique least general generalization and a unique most general specification.

Concept lattices are visualized with line diagrams, where every node corresponds to a concept and more general concepts are placed above less general ones. Two concepts are connected with a line if one is more general than the other and there is no concept between the two. We will see examples of line diagrams in the next section.

4 A Multidimensional Rating Based on Concept Lattices

To build a concept lattice corresponding to the "Freedom of the World" rating, we use the sub-category scores provided by Freedom House (see Table 2). This data is numerical, which we need to transform into a formal context before proceeding. In formal concept analysis, numerical (and some other types of) data is handled by so-called many-valued contexts, which are then scaled into normal one-valued formal contexts.

Mathematically, a *many-valued context* is a quadruple (G, M, W, I), where G and M are object and attribute sets, respectively; W is a set of attribute values; and $I \subseteq G \times M \times W$ is a ternary relation satisfying the following condition:

$$(g, m, w) \in I \text{ and } (g, m, v) \in I \Rightarrow w = v.$$

In our case, G is the set of countries under consideration, M is the set of sub-categories (Electoral Process, Functioning of Government, etc.), and W is the set of sub-category scores (integer numbers from 0 to 16). The relation I simply specifies the scores of every country for every parameter.

For further analysis of a many-valued context, we must transform every many-valued attribute into a series of one-valued attributes by means of *plain conceptual scaling*. For a many-valued attribute m, we define a scale $SS_m = (G_m, M_m, I_m)$, where G_m is the set of all possible values of the attribute m, i.e., $\{w \mid w \in W \text{ and } \exists g \in G : (g, m, w) \in I\} \subseteq G_m$. The relation I_m translates every

value of m into a subset of M_m. Then, the many-valued context (G, M, W, I) is replaced by the one-valued context

$$(G, \bigcup_{m \in M} \{m\} \times M_m, J),$$

where $(g, (m, n)) \in J$ if and only if there is $w \in W$ such that $(g, m, w) \in I$ and $(w, n) \in I_m$.

To give an example, we may replace the many-valued attribute "Functioning of Government", which takes values from the integer interval 0..16 by two one-valued attributes: "Functioning of Government ≤ 4" and "Functioning of Government ≤ 8" with obvious semantics. This amounts to using the scale shown in Fig. 1. By using this scale, we split all countries into three subcategories according to the value of the "Functioning of Government" parameter: those with the value between 0 and 4, between 5 and 8, and between 9 and 12 (these subcategories roughly correspond to poor, satisfactory, and adequate implementation of the parameter). With such scaling, one-valued attributes replacing the many-valued attribute can be regarded as problems in the implementation of the corresponding parameter. Thus, a country with the value between 0 and 4 will have both new one-valued attributes, while the country with the value between 9 and 12 will have none.

Functioning of Government	Functioning of Government ≤ 4	Functioning of Government ≤ 8
0..4	×	×
5..8		×
9..12		

Fig. 1. A scale for "Functioning of Government". Each row describes several objects, each object corresponding to a value of the many-valued attribute "Functioning of Government". For example, the row labeled with "0..4" corresponds to the values from 0 to 4.

We use similar scales for the other six parameters, except that, for parameters evaluated in the range 0..16, we use an additional one-valued attribute "$\ldots \leq 12$", and, in so doing, split the countries into four subcategories, which is proportionate to the number of questions used by Freedom House to calculate the scores (three questions have been used to calculate the score for parameters evaluated in the range 0..12 and four questions for parameters evaluated in the range 0..16).

The line diagram of the resulting concept lattice is shown in Fig. 2. The diagram is divided into three parts corresponding to the "Free", "Partly Free", and "Not Free" categories used by Freedom House. The higher positions in the diagram correspond to a higher rating, but more than that can be inferred from the diagram. Every node (a concept) in the diagram is described extensionally, by a group of countries, and intensionally, by attributes shared by all the countries in the extent of this node. The names of the countries in the extent of a node can be read off from the diagram considering the labels immediately below this node and below all nodes that can be reached from this node by the downward arcs. Conversely, the set of attributes forming the intent of a node consists of labels immediately above this node and those above nodes that can be reached from this node by upward arcs. (We use slightly abbreviated parameter names: "Political Pluralism" instead of "Political Pluralism and Participation", etc.)

Such a diagram provides a better view of various countries in terms of how they score with respect to different parameters as compared to the combined average ratings of Table 1. For example, Montenegro and Albania have the same combined rating of 3.0. One can see from the diagram that they both have a smaller score in Political Pluralism than Bulgaria, Romania, and Ukraine, the lowest-rated Free countries. However, they differ in that in Albania, other political rights related to Electoral Process, as well as civil liberties related to Associational Rights, seem to be implemented worse than in Montenegro, whereas Montenegro is less successful in enforcing Rule of Law. Macedonia is below both Montenegro and Albania, which means that it has all their problems (as well as additional issues with Freedom of Expression). Kyrgyzstan has a better combined average rating than Armenia (4.5 vs. 5.0). The lattice diagram substantiates this fact by showing that Armenia seems to enjoy less Freedom of Expression and Belief and has quite noticeable problems with Political Pluralism and Participation in comparison to Kyrgyzstan, but it also shows that, despite its higher average rating, Kyrgyzstan has more problems with Individual Rights than does Armenia.

On a macro-level, the concept lattice reveals the structure behind the rating. In particular, it justifies the division of the countries into the Free, Partly Free, and Not Free categories by clearly showing that the score of every Partly Free country with respect to every parameter is at least as good as the score of every Not Free country and at most as good as the score of every Free country with respect to the same parameter. The diagram shows the structure within each of the three categories by highlighting the similarities and differences between the countries, which is not at all obvious from the combined ratings.

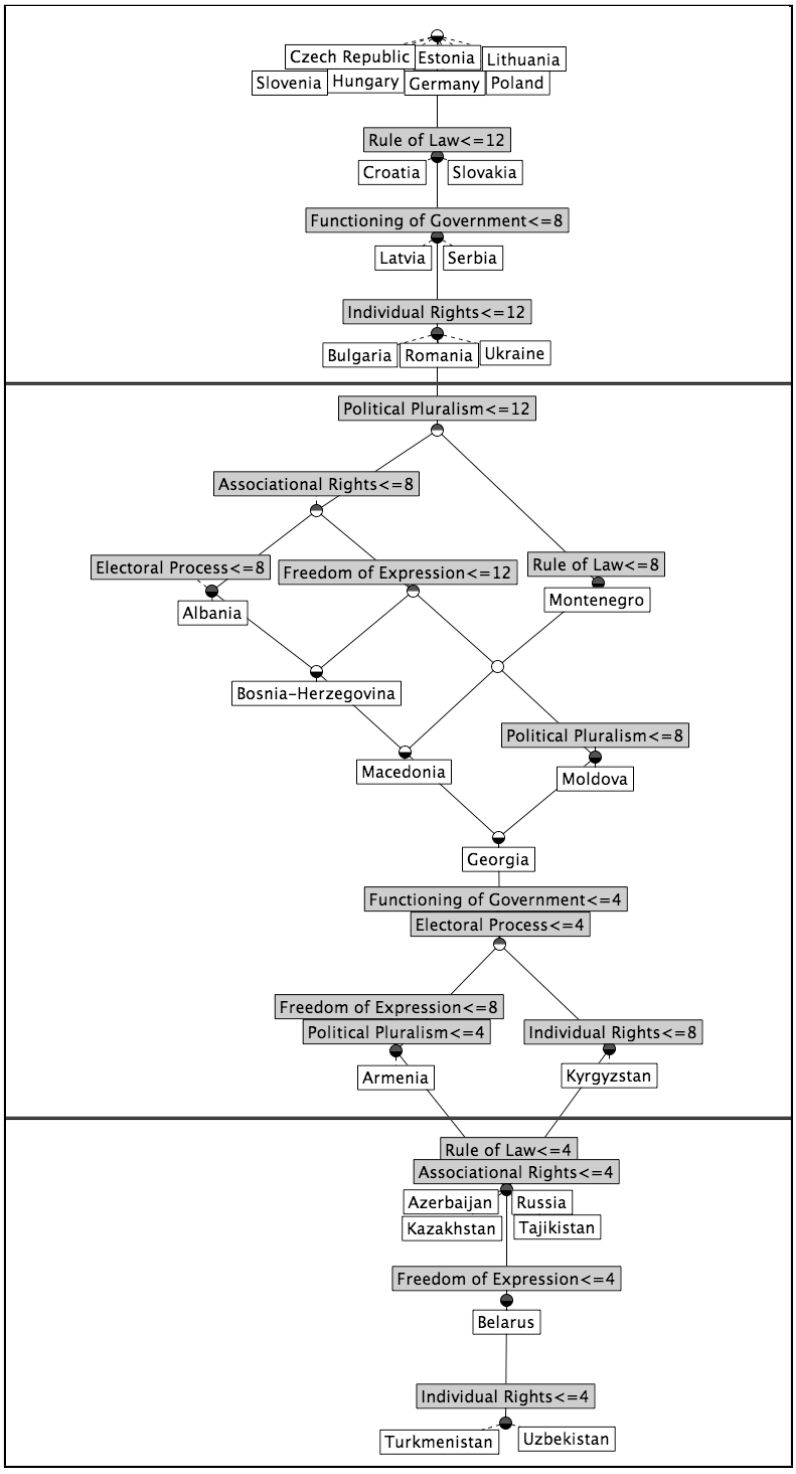

Fig. 2. The line diagram of the concept lattice of the 2009 "Freedom in the World" rating divided into three parts corresponding to the Free (top), Partly Free (middle), and Not Free (bottom) categories.

However, the diagram in Fig. 2 is partly a result of our choice of scales (see Fig. 1). Had we chosen different scales (e.g., had we decided to use exactly three one-valued attributes for every many-valued attribute), we would have obtain a different picture. Our choice of scales seems reasonable, but we should bear in mind that some information has been lost due to scaling. For example, Bosnia-Herzegovina has lower rating than Macedonia according to Table 1, but the diagram in Fig. 2 suggests the opposite. This happened because our scales apparently emphasized the difference in parameter values between the countries for parameters in which Bosnia-Herzegovina is superior over Macedonia and downplayed these differences for parameters in which Macedonia is superior.

This may not be a problem if we are aware of such effects and happy with the chosen level of granularity. If we still want to avoid any loss of information as compared to Table 2, all we need is to use more granular scales, namely, ordinal scales of the form $(W, W, \leq)$. When using such a scale, a many-valued attribute m taking values from the integer interval $0..n$ is replaced by a series of one-valued attributes $m \leq 0, m \leq 1, m \leq 2, \ldots, m \leq n$. (We may actually use $m = 0$ instead of $m \leq 0$ if we know that the attribute cannot take negative values.)

The diagram of the concept lattice obtained with such scales is shown in Fig. 3. It looks way too complicated when printed, but it is easy to browse with interactive tools like Concept Explorer [20], which we used to produce all the diagrams in this paper. Still, to get more manageable structures, we can build separate lattices for Free, Partly Free, and Not Free countries (we know from the preceding analysis of Fig. 2 that such division is justified by the data). The three diagrams of these lattices are shown in Figs. 4, 5, and 6. The labels at the top and bottom of the diagrams specify maximum and minimum parameter values for corresponding groups of countries. These diagrams contain exactly the same information as Table 2, but they also clearly visualize similarities and differences between the countries. Looking at Fig. 5, we see that Macedonia is no longer under Bosnia-Herzegovina, since the latter has lower scores in Functioning of Government (≤ 5), but neither is Macedonia above Bosnia-Herzegovina, since, despite its higher combined average rating, it has more problems with Rule of Law (as we could have seen already in Fig. 2) and Electoral Process.

5 Dynamic Analysis

Concept lattice diagrams are useful for tracing the dynamics of ratings and comparing ratings from different organizations. Figure 7 presents the line diagram of the concept lattice corresponding to the Freedom House rating of 2006. A quick examination of the two diagrams corresponding to 2006 and 2009 immediately reveals that, for example, Slovenia was able to overcome whatever problems it had with Individual Rights, whereas Latvia moved down due to some new issues with Functioning of Government. Even more noticeable is the fall of Bulgaria, which was among the top countries in 2006, but apparently did not so well with respect to Rule of Law, Functioning of Government, and Individual Rights in

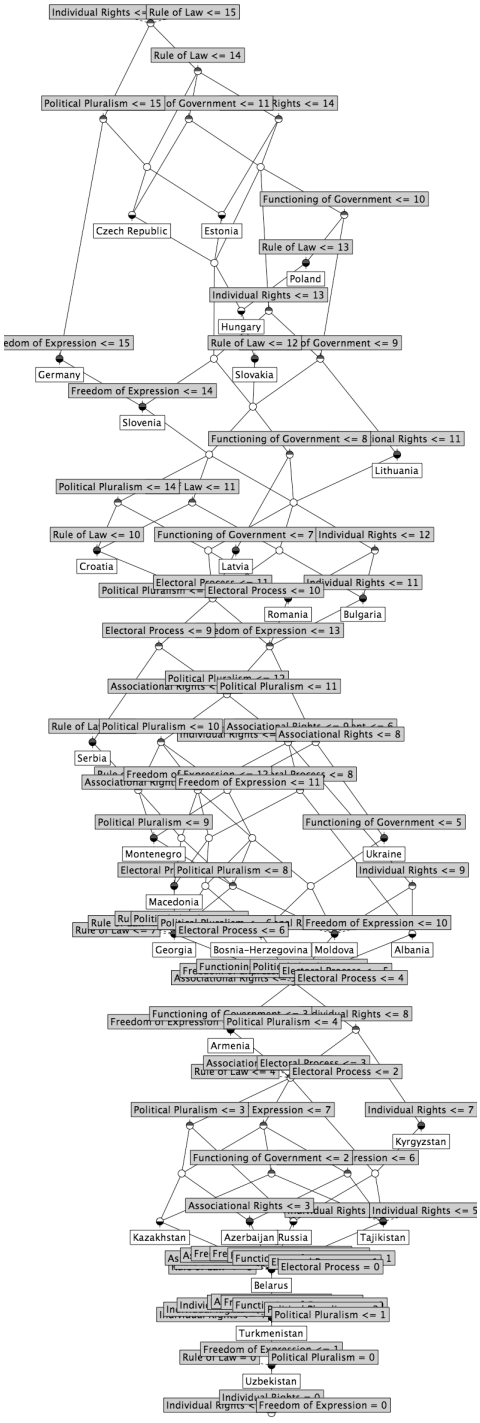

Fig. 3. The line diagram of the 2009 "Freedom in the World" rating: higher granularity—complete information.

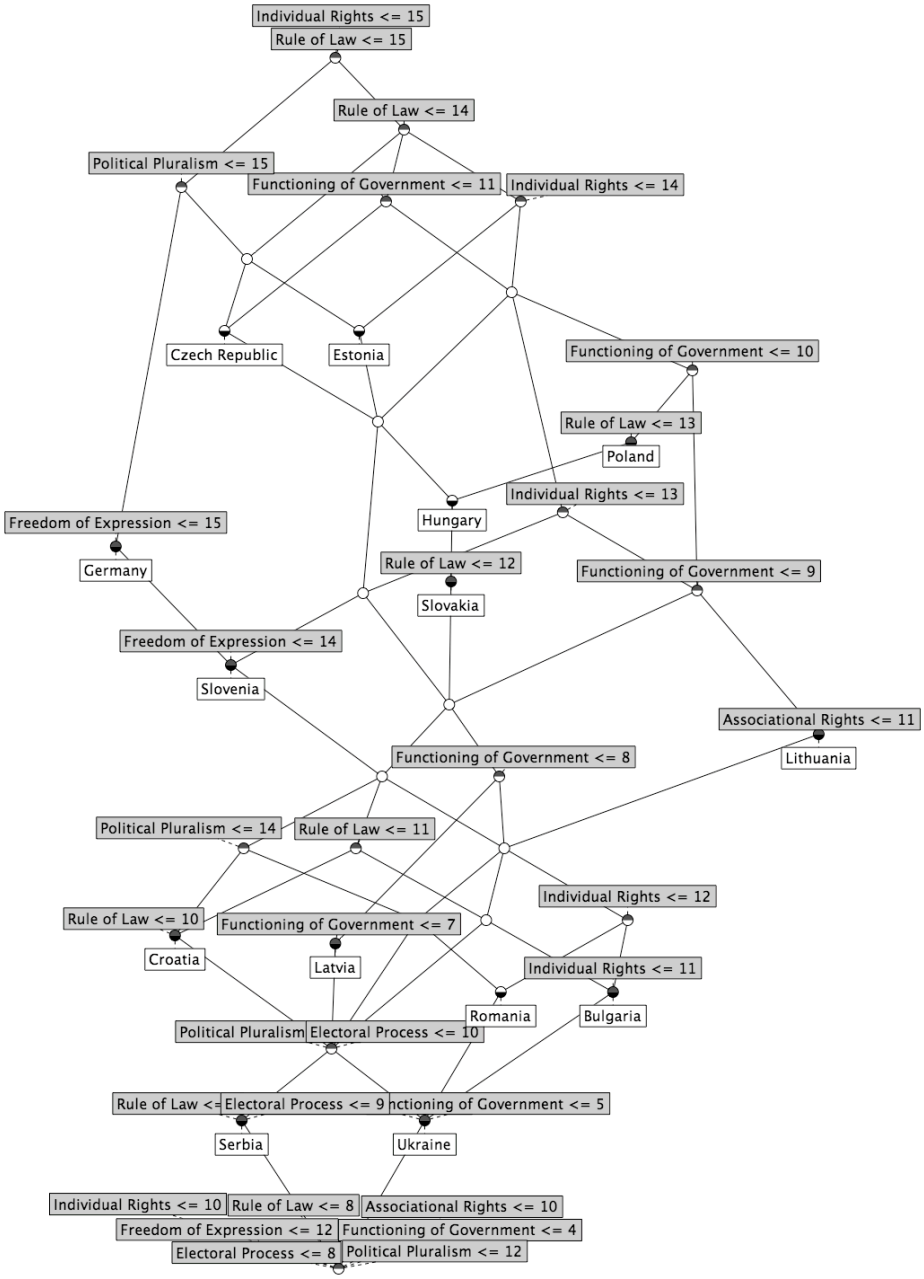

Fig. 4. The line diagram of the Free countries from the 2009 "Freedom in the World" rating.

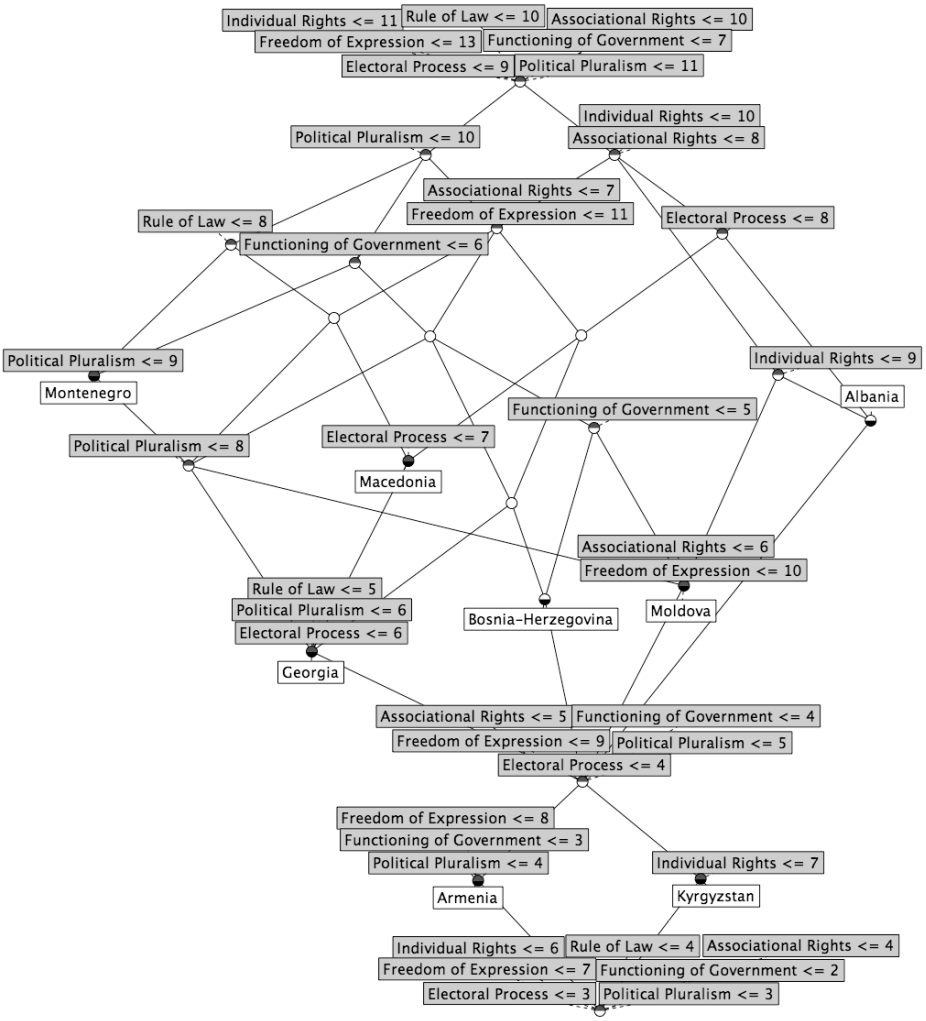

Fig. 5. The line diagram of the Partly Free countries from the 2009 "Freedom in the World" rating.

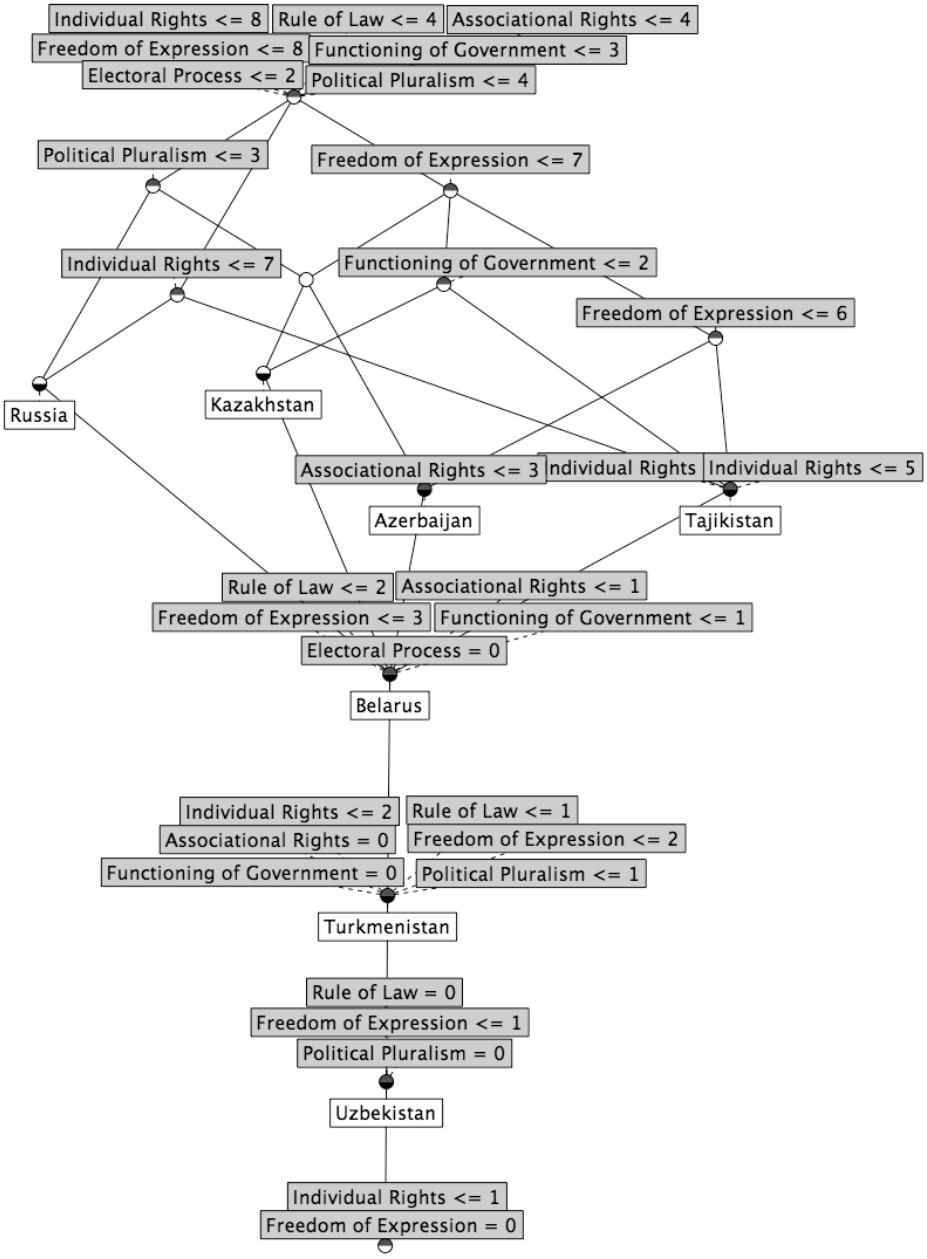

Fig. 6. The line diagram of the Not Free countries from the 2009 "Freedom in the World" rating.

2009. One can also see that problems with Electoral Process and Political Pluralism, which together put Georgia behind Bosnia-Herzegovina and Macedonia in 2009, did not exist (or were less serious) in 2006. As for Russia, further restricting Associational and Organizational Rights and reducing Political Pluralism from 2006 on, put it at the same level as Azerbaijan, Kazakhstan, and Tajikistan in 2009.

It is also remarkable that countries in the Free and Not Free categories are linearly ordered in the 2009 version of the lattice-based rating, which was not the case in 2006. Partly, this is an effect of the level of granularity we have chosen for our scales, but it does reflect the structure of the data behind the "Freedom of the World" rating. Let us look at it in more detail.

The Free countries of 2009 can be concisely described as countries where Political Pluralism and Participation is (more or less) fully implemented (the value of the corresponding parameter is greater than 12). All such countries seem to have (virtually) no problems with Electoral Process, Freedom of Expression and Belief, and Associational and Organization Rights. Some of them do have some problems with the other three parameters, though: problems with Personal Autonomy and Individual Rights imply problems with Functioning of Government, which, in turn, imply problems with Rule of Law (cf. FCA implications defined in Section 3).

In 2006, the situation was a bit different. A Free country could be defined as one properly implementing Associational and Organization Rights and Freedom of Expression and Belief, while probably having some issues with the other five parameters. Ukraine, in particular, had issues with all the five; the other Free countries scored well on Electoral Process and Political Pluralism and Participation. The relation between the remaining three attributes were similar to 2009, except that Slovenia did not exhibit problems with Rules of Law or Functioning of Government, despite a lower score in Individual Rights.

Taking a look at the countries in the Not Free category, we can see that, in 2006, it consists of countries with noticeable violations of Individual Rights and relatively restricted Freedom of Expression (the values of the corresponding parameters are at most 8 out of 16), as well as poorly organized Electoral Process (at most 4 out of 12). Still, these countries were different in the implementation of Associational Rights and Political Pluralism (in Russia better than elsewhere) and Rule of Law (in Azerbaijan better than elsewhere). In 2009, countries that are Not Free are characterized by all the same problems, but they can be more succinctly described as countries with pour implementation of the Rule of Law and Associational Rights, since now they all share these characteristics. (The same goes for Political Pluralism, but in 2009, it is not a distinctive feature of Not Free countries, since Armenia, a Partially Free country, did equally badly in this aspect.)

This analysis suggests that, between 2006 and 2009, the situation has become better in Free countries and worse in Not Free countries, but also that it has become sort of more uniform in each category. It is not that all countries within one 2009 category are exactly the same, but if a country is better than another

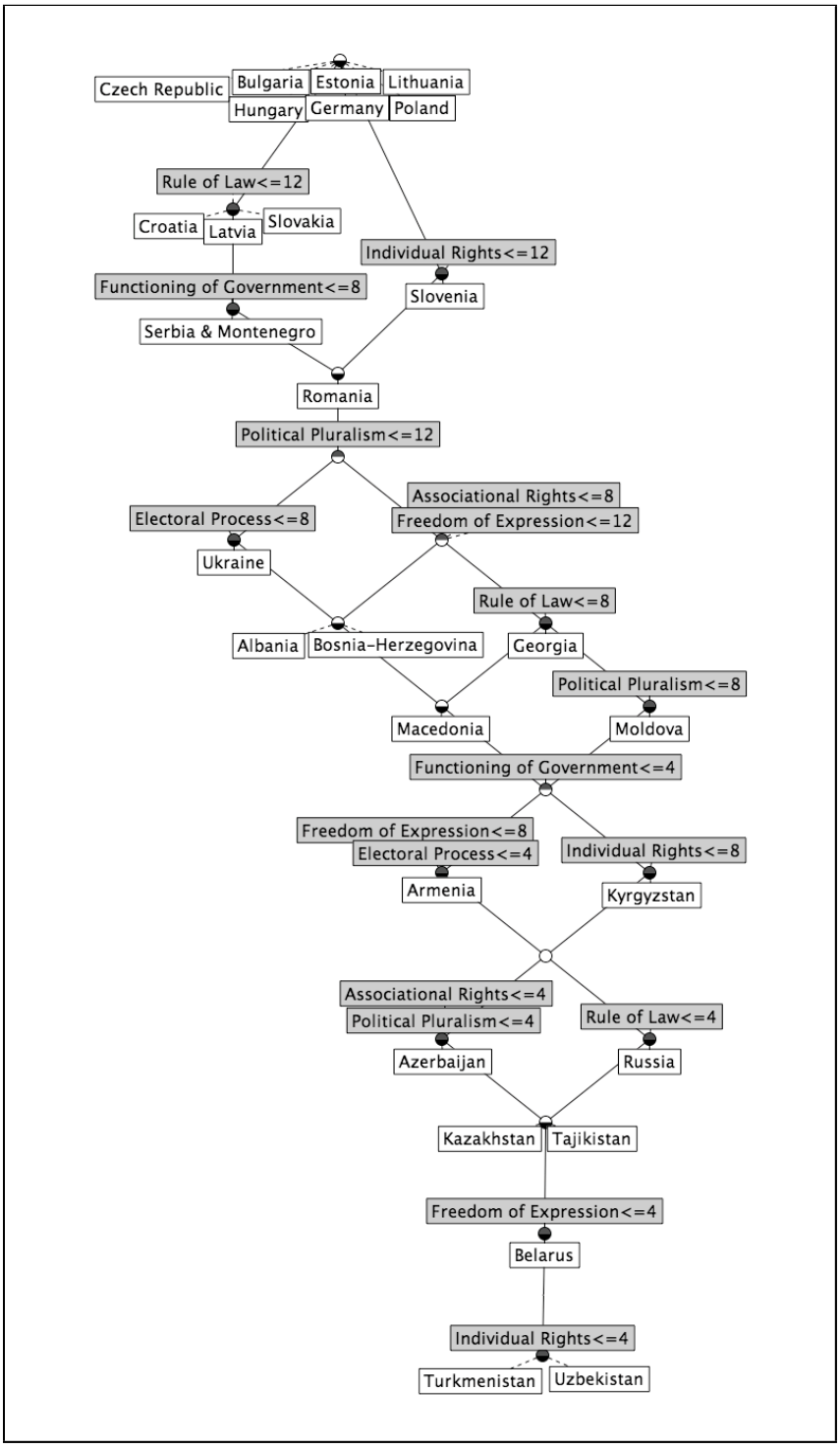

Fig. 7. The line diagram of the concept lattice of the 2006 "Freedom in the World" rating.

country in one aspect, it is at least as good in any other aspect. This was not the case in 2006 and it is still not the case in the 2009 Partially Free category, where one can observe much more diversity (even though the Free category has almost twice as many countries as the Partially Free category).

However, one must not forget that our diagrams reflect the true state of affairs only to the extent to which it is reflected by the data behind the underlying rating, "Freedom in the World". Hence, the observed "linearization" in the Free and Not Free parts of the rating might be a result of a bias shift in data collection, rather than of actual changes in the countries under consideration. The concept lattice diagram reveals the effects of such shift and thus provides grounds for the evaluation of its validity.

6 Conclusion

In this paper, we showed that concept lattices provide a multidimensional model for the analysis of democratic development in countries around the world. Concept lattices arrange countries into a hierarchy according to problems they have in various aspects of democracy. By providing a better interface to the knowledge hidden in data than do linear rankings, the concept lattice does not have the weaknesses of the latter pointed out by, e.g., Giannone [10], who, referring to the report in [12], suggests that

> [...] the index by FH has been used as a tool for measuring democracy, good governance, and human rights, thus producing a conceptual stretching which is a major cause of 'losses in connotative precision' [...] an instrument used to measure everything, in the end, is not able to discriminate against anything.

Instead of averaging the values of parameters, which are not always related in an obvious way, the concept lattice offers a taxonomical representation effectively discriminating between factors that define democracy, while, at the same time, revealing their interconnections.

Being in the first place a highly practical tool for exploring the data, the concept lattice can also help formulate theoretical hypotheses, thus, supporting the process of scientific discovery. For example, the concept lattices in Figs. 2 and 6 suggest a particular order in which parameters deteriorate with the decline of democracy. The two fastest—and, thus, crucial for holding the system together—are Functioning of Government and Electoral Process. These parameters refer to the features that constitute the framework of political process in the country. Their decline is closely followed by a regress in Associational and Organizational Rights, Political Pluralism and Participation, and Rule of Law, i.e., parameters related to the activity of civil society and citizen participation. The last to be restricted are the features related more to a personal sphere than to political activities: Personal Autonomy and Individual Rights and Freedom of Expression and Beliefs. Of course, to verify this hypothesis it would be necessary to study data from different sources related to other countries and other time periods.

We also demonstrated how lattice-based representation can help analyzing dynamics of the democratic development in individual countries or in a geographical area as a whole by comparing lattices corresponding to the 2006 and 2009 versions of the "Freedom in World" rating. A related application of lattices is to compare ratings coming from different sources, such as Freedom House and the Economist Intelligence Unit. Different organizations will most likely use different sets of parameters and may even give different meanings to identically named parameters, which complicates comparison. Nevertheless, the duality between the extensional and intensional characterization of lattice nodes should help us identify similarities and differences between two ratings and the meanings of the parameters used therein. We plan to develop this approach in our further research.

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