Optimization and Modeling in Energy Systems

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2013
“activity, operation”

ἐνέργεια : energeia

“Air, earth, water and fire are ever existing elements beginning and end of the Universe.”

Empedocles, pre - Socratic philosopher (c. 490 BC - 430 BC)
Dynamics of global energy systems

- Changes in oil and gas production and trade flows (shale gas and oil, new fields in USA / Canada, oil production in Iraq, changes in global economy and geopolitical balance)
- Renewable energy (solar, wind, biofuels etc.)
- Focus on energy efficiency / sustainable energy systems (climate changes)
- $CO_2$ emissions remain at record high
- Issues with Fossil Fuel Subsidies
- Over 1 billion people have no access to electricity
- Energy/Water/Environmental issues
- Advances in Technology/Modeling/Optimization
World Energy Outlook 2012

The surge in unconventional oil & gas production has implications well beyond the United States

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1 WEO-2012
World Energy Outlook 2012

Iraq oil production

Iraq oil exports

Iraq accounts for 45% of the growth in global production to 2035; by the 2030s it becomes the second-largest global oil exporter, overtaking Russia

1WEO-2012
Rising supplies of unconventional gas & LNG help to diversify trade flows, putting pressure on conventional gas suppliers & oil-linked pricing mechanisms
Introduction

Smart Grid must predict and intelligently respond to the behavior and actions of power users
- Electricity demand is growing worldwide
- Making the grid more flexible
- Security concerns
- Network expansion problems

Energy Systems are Interdependent
- Increased use of natural gas for electricity generation
- Liquified Natural Gas terminals
- Natural gas transportation and distribution systems
- Long-term planning horizon for expansion planning

Hydro-Thermal Scheduling
- Uncertainties in weather, demand, and prices
- Scenario reduction
- CO2 emissions constraints
Outline

1 Introduction

2 Smart Grid
   - Islanding
   - Reliability Analysis
   - Stochastic Unit Commitment Problem
   - Expansion Planning

3 Hydro-Thermal Scheduling

4 Activities
   - Publications
   - Books
   - Energy Systems Journal
Outline

1. Introduction

2. Smart Grid
   - Islanding
   - Reliability Analysis
   - Stochastic Unit Commitment Problem
   - Expansion Planning

3. Hydro-Thermal Scheduling

4. Activities
   - Publications
   - Books
   - Energy Systems Journal
Recently, the number of massive blackouts has increased.

Potential reasons for these blackouts:

- system limits, weak conditions, unexpected events, hidden failures, human errors, intentional attacks, natural disasters, etc.
- two main reasons: security and stability issues

<table>
<thead>
<tr>
<th>Blackout</th>
<th>People</th>
<th>Location</th>
<th>Date(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005 Java-Bali Blackout</td>
<td>100M</td>
<td>Indonesia</td>
<td>2005-08-18</td>
</tr>
<tr>
<td>1999 Southern Brazil blackout</td>
<td>97M</td>
<td>Brazil, south and southeastern</td>
<td>1999-03-11</td>
</tr>
<tr>
<td>2009 Brazil and Paraguay blackout</td>
<td>60M</td>
<td>Brazil and Paraguay</td>
<td>2009-11-10/2009-11-11</td>
</tr>
<tr>
<td>Northeast Blackout of 2003</td>
<td>55M</td>
<td>North America, northeastern</td>
<td>2003-08-14/2003-08-15</td>
</tr>
<tr>
<td>2003 Italy blackout</td>
<td>55M</td>
<td>Italy</td>
<td>2003-09-28</td>
</tr>
<tr>
<td>Northeast Blackout of 1965</td>
<td>30M</td>
<td>North America, northeastern</td>
<td>1965-11-09</td>
</tr>
</tbody>
</table>
Security refers to the degree of risk in its ability to survive imminent disturbances (contingencies) without interruption of customer service.

- ability to withstand the effects of contingencies
- keep the power flows and bus voltages within acceptable limits despite changes in load or available resources
- avoidance of cascading outages leading to blackout

Power system security analysis
- five states: normal, alert, emergency, extreme emergency and restorative
- planning and operating criteria: $N - 1$, $N - k$ contingency analysis; automatic generation control, transmission line switching, load shedding
- SCADA: Supervisory Control And Data Acquisition for security assessment
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Power Grid Islanding

- Splitting a large power system into subsystems
  - most parts of the system can operate in an acceptable condition
  - each grid island is a self-sufficient subnetwork
Splitting a large power system into subsystems
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1. Islanding for self-healing strategy
- large disturbances, such as simultaneous loss of several generating units or major transmission lines
- catastrophic failure, by vulnerability analysis
- control actions, to limit the extent of the disturbance
- facilities the restoration process:
  - islanding with low generation-load imbalance in each island;
  - smaller islands with slightly reduced capacity and being restored quickly;
  - the extent of disruption is limited
Power Grid Islanding

- Splitting a large power system into subsystems
  - most parts of the system can operate in an acceptable condition
  - each grid island is a self-sufficient subnetwork

- (2) Islanding for distributed generation system
  - renewable energy resources connected to the existed system
  - centralized generation becomes distributed generation
  - an islanding operation occurs when the DG continues supplying power into the grid after power from the main utility is interrupted
DC Optimal Power Flow Model

- **Linear programming model**

\[
\begin{align*}
\min_{P_G, P_S, P_{ij}, \theta} & \quad \sum_{i \in I} (C_{Gi} P_{Gi} + C_{Si} P_{Si}) \\
\text{s.t.} & \quad P_{ij} = B_{ij} (\theta_i - \theta_j), \forall (i, j) \in L \\
& \quad P_{Gi} + \sum_{j < i} P_{ji} = (P_{Di} - P_{Si}) + \sum_{j > i} P_{ij}, \forall i \in I \\
& \quad -P_{ij_{\text{max}}} \leq P_{ij} \leq P_{ij_{\text{max}}}, \forall (i, j) \in L \\
& \quad 0 \leq P_{Gi} \leq P_{Gi_{\text{max}}}, \forall i \in I \\
& \quad 0 \leq P_{Si} \leq P_{Di}, \forall i \in I
\end{align*}
\]
Objective

- The $K$-islanding problem is to separate a power grid into $K$ components.
- $\bigcup_k I_k = I$, $I_k \cap I_{k'} = \emptyset$ for $k \neq k'$, $i^k \in I_k$.
- Each component, an induced graph by $I_k$.
- Objective: minimizing the generating and load shedding cost.

$$\min_{P_G, P_S, P_{ij}, \theta, x, y, z} \sum_{i \in I} (C_{Gi} P_{Gi} + C_{Si} P_{Si})$$
Complete Islanding Constraints

- **DC-OPF constraints:**

  \[
  \begin{align*}
  P_{ij} &= B_{ij}(\theta_i - \theta_j)z_{ij}, \forall (i, j) \in L \\
  -P_{ij_{\text{max}}} &\leq P_{ij} \leq P_{ij_{\text{max}}}, \forall (i, j) \in L \\
  P_{G_i} + \sum_{j<i} P_{ji} &= (P_{D_i} - P_{S_i}) + \sum_{j>i} P_{ij}, \forall i \in I \\
  0 &\leq P_{G_i} \leq P_{G_{i_{\text{max}}}}, \forall i \in I \\
  0 &\leq P_{S_i} \leq P_{D_i}, \forall i \in I
  \end{align*}
  \]

- approximate active power flow on transmission lines
- the maximum power flow on each line
- the power balance at each bus, where the served satisfied at bus \( i \) is \( (P_{D_i} - P_{S_i}) \)
- the maximum generating output
- the limitation of load shedding by the maximum load
- \( z_{ij} = 1 \), the constraint is the same as in standard DC-OPF since line \( (i, j) \) is inside of an island; \( z_{ij} = 0 \), the constraint becomes \( P_{ij} = 0 \) since line \( (i, j) \) is between two islands and is removed.
Complete Islanding Constraints

- **Graph partitioning constraints:**

\[
\begin{aligned}
\sum_{k=1}^{K} x_{ik} &= 1, \forall i \in V \\
\sum_{i \in I} 1^g_i x_{ik} &\geq 1, \sum_{i \in I} 1^d_i x_{ik} \geq 1, \forall k \\
z_{ij} &= \sum_{k} x_{ik} x_{jk}, \forall (i, j) \in L
\end{aligned}
\]

- every node must belong to exactly one island
- every island must have at least one generator and one load consumer
- if two buses \(i\) and \(j\) are in the same island, there exists exactly one \(k' (1 \leq k \leq K)\) such that \(x_{ik'} = x_{jk'} = 1, x_{ik} = x_{jk} = 0\) for all other \(k\)s, and thus \(z_{ij} = 1\). Otherwise, \(z_{ij} = 0\).
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  &z_{ij} = \sum_{k} x_{ik} x_{jk}, \forall (i, j) \in L
  \end{align*}
  \]

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  - every island must have at least one generator and one load consumer
  - if two buses $i$ and $j$ are in the same island, there exists exactly one $k' (1 \leq k' \leq K)$ such that $x_{ik'} = x_{jk'} = 1, x_{ik} = x_{jk} = 0$ for all other $k$s, and thus $z_{ij} = 1$. Otherwise, $z_{ij} = 0$. 
Complete Islanding Constraints

- Graph partitioning constraints:

\[
\begin{align*}
K \sum_{k=1}^{K} x_{ik} &= 1, \quad \forall i \in V \\
\sum_{i \in I}^{g} 1_{i}^{g} x_{ik} &\geq 1, \quad \sum_{i \in I}^{d} 1_{i}^{d} x_{ik} \geq 1, \quad \forall k \\
z_{ij} &= \sum_{k} x_{ik} x_{jk}, \quad \forall (i, j) \in L
\end{align*}
\]

- every node must belong to exactly one island
- every island must have at least one generator and one load consumer
- if two buses \(i\) and \(j\) are in the same island, there exists exactly one \(k' (1 \leq k \leq K)\) such that \(x_{ik'} = x_{jk'} = 1\), \(x_{ik} = x_{jk} = 0\) for all other \(k\)s, and thus \(z_{ij} = 1\). Otherwise, \(z_{ij} = 0\).
Complete Islanding Constraints

- Connectivity constraints to ensure that every island is connected:
  - single commodity flow model to ensure connectivity

\[
\begin{align*}
\text{Island } k \\
\text{Island } K \\
\vdots \\
\text{Island } 1 \\
\vdots \\
\text{Island } k \\
\vdots \\
\text{Island } K
\end{align*}
\]
IEEE-30-Bus System

- 30 buses, 41 lines
- 6 generators
- total generation capacity 130 MW, and total load demand 137.5 MW
### Complete Islanding

- **By GI-DC-OPF model**

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>22.5</td>
<td>Island 1: 1-30</td>
<td>130.0</td>
<td>130.0</td>
<td>137.5</td>
<td>94.5%</td>
</tr>
<tr>
<td>2</td>
<td>1,13</td>
<td>22.5</td>
<td>Island 1: 1,2,5-8,15,21-30</td>
<td>70.0</td>
<td>70.0</td>
<td>76.9</td>
<td>91.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Island 2: 3,4,9-14,16-20</td>
<td>60.0</td>
<td>60.0</td>
<td>60.6</td>
<td>99.0%</td>
</tr>
<tr>
<td>3</td>
<td>1,8,13</td>
<td>22.5</td>
<td>Island 1: 1,2,5-7,9-11,14,15,18-24</td>
<td>85.0</td>
<td>85.0</td>
<td>87.3</td>
<td>97.4%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Island 2: 8,25-30</td>
<td>15.0</td>
<td>15.0</td>
<td>16.5</td>
<td>90.9%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Island 3: 3,4,12,13,16,17</td>
<td>30.0</td>
<td>30.0</td>
<td>33.7</td>
<td>89.0%</td>
</tr>
<tr>
<td>4</td>
<td>1,8,11,13</td>
<td>37.5</td>
<td>Island 1: 1,3,4</td>
<td>10.0</td>
<td>15.0</td>
<td>10.0</td>
<td>100.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Island 2: 2,5-8,21,22,24-30</td>
<td>55.0</td>
<td>55.0</td>
<td>65.5</td>
<td>84.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Island 3: 9-11,16,17,19,20</td>
<td>30.0</td>
<td>30.0</td>
<td>30.0</td>
<td>100.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Island 4: 12-15,18,23</td>
<td>30.0</td>
<td>30.0</td>
<td>32.0</td>
<td>93.8%</td>
</tr>
<tr>
<td>5</td>
<td>1,5,8,11,13</td>
<td>37.5</td>
<td>Island 1: 1,3,4</td>
<td>10.0</td>
<td>15.0</td>
<td>10.0</td>
<td>100.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Island 2: 5,7</td>
<td>15.0</td>
<td>15.0</td>
<td>22.8</td>
<td>65.8%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Island 3: 8,25-30</td>
<td>15.0</td>
<td>15.0</td>
<td>16.5</td>
<td>90.9%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Island 4: 2,6,9-11,17,19-24</td>
<td>55.0</td>
<td>55.0</td>
<td>55.9</td>
<td>98.4%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Island 5: 12-16,18</td>
<td>30.0</td>
<td>30.0</td>
<td>32.3</td>
<td>92.9%</td>
</tr>
<tr>
<td>6</td>
<td>1,2,5,8,11,13</td>
<td>112.5</td>
<td>Island 1: 1,3</td>
<td>2.4</td>
<td>15.0</td>
<td>2.4</td>
<td>100.0%</td>
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<td></td>
<td>Island 2: 2,4</td>
<td>7.6</td>
<td>25.0</td>
<td>7.6</td>
<td>100.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Island 3: 5,7</td>
<td>15.0</td>
<td>15.0</td>
<td>22.8</td>
<td>65.8%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Island 4: 8,25-30</td>
<td>15.0</td>
<td>15.0</td>
<td>16.5</td>
<td>90.9%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Island 5: 6,9-11,21-24</td>
<td>30.0</td>
<td>30.0</td>
<td>35.2</td>
<td>85.2%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Island 6: 12-20</td>
<td>30.0</td>
<td>30.0</td>
<td>53.0</td>
<td>56.6%</td>
</tr>
</tbody>
</table>
Complete Islanding

Figure: IEEE-30-Bus network with two islands

\((K = 2, \text{Root Buses } 1, 13)\)
Complete Islanding

Figure: IEEE-30-Bus network with three islands

\[(K = 3, \text{Root Buses 1,8,13})\]
Complete Islanding

Figure: IEEE-30-Bus network with four islands
Extensions

More considerations and constraints:

- adding security and stability constraints to our current model. For example, the voltage constraints on each bus to ensure they work normally.
- adding the generation and load demand balance constraints. For each island, there is a limit on load shedding to prevent blackouts.
  \[ r(0 < r \leq 1) \]: total generation of an island should be larger than \( r \) times of its total load consumption, while a small part of unsatisfied demand is allowed
  \[ \sum_{i \in I} P_{G_i} x_{ik} \geq r \cdot \sum_{i \in I} (P_{D_i} - P_{S_i}) x_{ik}, \forall k \]
- adding physical location constraints. For example, physically close buses should be divided into one island to reduce transmission cost.
  \[ C = \{(i,j) \in L : \text{bus } i \text{ and bus } j \text{ should be in the same island}\}\]: the set for buses should be within the same island
  \[ x_{ik} = x_{jk}, \forall (i,j) \in C, \forall k \]
Extensions

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- adding security and stability constraints to our current model. For example, the voltage constraints on each bus to ensure they work normally.
- adding the generation and load demand balance constraints. For each island, there is a limit on load shedding to prevent blackouts.
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    \]
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    \[ x_{ik} = x_{jk}, \forall (i, j) \in C, \forall k \]
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Contingency analysis is a key function in the Energy Management System (EMS).

- a set of unexpected events happening within a short duration
- failures of buses (generators, substations, etc) or transmission and distribution lines

$N - 1$ contingency, not sufficient to model the application in reality to evaluate the vulnerabilities of power grids

$N - k$ contingency: reflecting a larger variation of vulnerabilities, a substantial computational burden for analysis

Two steps: contingency selection and evaluation
\[ z(\delta, \sigma) = \min_{g,s,f,\theta} \sum_{i \in I} (h_i g_i + f_i s_i) \]

s.t. \[ f_{ij} = b_{ij}(\theta_i - \theta_j)(1 - \delta)(1 - \sigma_{ij}), \forall (i, j) \in L \]

\[ -\bar{F}_{ij} \leq f_{ij} \leq \bar{F}_{ij}, \forall (i, j) \in L \]

\[ \sum_{j} f_{ji} + g_i = \sum_{j} f_{ij} + (D_i - s_i), \forall i \in I \]

\[ 0 \leq g_i \leq \tilde{G}_i(1 - \delta), \forall i \in I \]

\[ 0 \leq s_i \leq D_i, \forall i \in I \]

- \( \delta, \sigma \): selected of \( k \) failures on buses and/or lines
- generating and load shedding cost for economic analysis
N – k OPF Model

\[ z(\delta, \sigma) = \min_{g, s, f, \theta} \sum_{i \in I} (h_i g_i + r_i s_i) \]

s.t.
\[ f_{ij} = b_{ij} (\theta_i - \theta_j)(1 - \delta_j)(1 - \delta_j)(1 - \sigma_{ij}), \forall (i, j) \in L \]
\[ -\overline{F}_{ij} \leq f_{ij} \leq \overline{F}_{ij}, \forall (i, j) \in L \]
\[ \sum_{j} f_{ji} + g_i = \sum_{j} f_{ij} + (D_i - s_i), \forall i \in I \]
\[ 0 \leq g_i \leq \tilde{\alpha}_i (1 - \delta_i), \forall i \in I \]
\[ 0 \leq s_i \leq D_i, \forall i \in I \]

- approximate active power flow on transmission lines by considering failures on two ending buses and the line
\[ z(\delta, \sigma) = \min_{g, s, f, \theta} \sum_{i \in I} (h_i g_i + r_i s_i) \]

subject to:

\[ f_{ij} = b_{ij}(\theta_i - \theta_j)(1 - \delta)(1 - \delta)(1 - \sigma_{ij}), \forall (i, j) \in L \]

\[-\bar{F}_{ij} \leq f_{ij} \leq \bar{F}_{ij}, \forall (i, j) \in L \]

\[ \sum_{j} f_{ji} + g_i = \sum_{j} f_{ij} + (D_i - s_i), \forall i \in I \]

\[ 0 \leq g_i \leq \bar{g}_i(1 - \delta), \forall i \in I \]

\[ 0 \leq s_i \leq D_i, \forall i \in I \]

- line capacity limits
**N – k OPF Model**

\[ z(\delta, \sigma) = \min_{g, s, f, \theta} \sum_{i \in I} (h_i g_i + r_i s_i) \]

**Subject to:**

\[ f_{ij} = b_{ij}(\theta_i - \theta_j)(1 - \delta_j)(1 - \delta_j)(1 - \sigma_{ij}), \forall (i, j) \in L \]

\[ -\bar{F}_{ij} \leq f_{ij} \leq \bar{F}_{ij}, \forall (i, j) \in L \]

\[ \sum_j f_{ji} + g_i = \sum_j f_{ij} + (D_i - s_i), \forall i \in I \]

\[ 0 \leq g_i \leq \bar{g}_i(1 - \delta_i), \forall i \in I \]

\[ 0 \leq s_i \leq D_i, \forall i \in I \]

- power balance at each bus, where the served satisfied at bus \( i \) is \( D_i - s_i \)
\[ z(\delta, \sigma) = \min_{g,s,f,\theta} \sum_{i \in I} (h_i g_i + r_i s_i) \]

\[ s.t. \quad f_{ij} = b_{ij}(\theta_i - \theta_j)(1 - \delta_i)(1 - \delta_j)(1 - \sigma_{ij}), \forall (i, j) \in L \]

\[ -\tilde{F}_{ij} \leq f_{ij} \leq \tilde{F}_{ij}, \forall (i, j) \in L \]

\[ \sum_{j} f_{ji} + g_i = \sum_{j} f_{ij} + (D_i - s_i), \forall i \in I \]

\[ 0 \leq g_i \leq \tilde{g}_i (1 - \delta_i), \forall i \in I \]

\[ 0 \leq s_i \leq D_i, \forall i \in I \]

- generating output limits
$N - k$ OPF Model

$$z(\delta, \sigma) = \min_{g,s,f,\theta} \sum_{i \in I} (h_i g_i + r_i s_i)$$

s.t.  
$$f_{ij} = b_{ij}(\theta_i - \theta_j)(1 - \delta_i)(1 - \delta_j)(1 - \sigma_{ij}), \forall (i, j) \in L$$

$$-\overline{F}_{ij} \leq f_{ij} \leq \overline{F}_{ij}, \forall (i, j) \in L$$

$$\sum_{j} f_{ji} + g_i = \sum_{j} f_{ij} + (D_i - s_i), \forall i \in I$$

$$0 \leq g_i \leq \overline{g}_i(1 - \delta_i), \forall i \in I$$

$$0 \leq s_i \leq D_i, \forall i \in I$$

- the limitation of load shedding by the maximum load
Interdiction Analysis

- Interdiction: worst case scenario analysis
- A group of terrorists will attack the power grid with limited resources to maximize the disruption

\[
\begin{align*}
\max_{\delta, \sigma} & \quad \min_{g, s, f, \theta} \sum_{i \in I} (h_i g_i + r_i s_i) \\
\text{s.t.} & \quad f_{ij} = b_{ij} (\theta_i - \theta_j)(1 - \delta)(1 - \delta_j)(1 - \sigma_{ij}), \forall (i, j) \in L \\
& \quad - \bar{f}_{ij} \leq f_{ij} \leq \bar{f}_{ij}, \forall (i, j) \in L \\
& \quad \sum_j f_{ji} + g_i = \sum_j f_{ij} + (D_i - s_i), \forall i \in I \\
& \quad 0 \leq g_i \leq \bar{g}_i (1 - \delta_i), \forall i \in I \\
& \quad 0 \leq s_i \leq D_i, \forall i \in I \\
& \quad \sum_{i \in I} \delta_i + \sum_{(i,j) \in L} \sigma_{ij} = k \\
& \quad \delta_i, \sigma_{ij} \in \{0, 1\}, \forall i \in I, (i, j) \in L
\end{align*}
\]
Selection of $k$ Buses

assume all failures, or attacks happen on buses, $\sum_{i \in I} \delta_i = k$.

- The *random failure*: selects $k$ buses for failure with equal probability;
- The *degree* based method: selects $k$ buses starting with the highest degree bus, till the $k$th highest degree;
- The *maximum-traffic* and *minimum-traffic* methods: the maximum-traffic method selects $k$ buses with highest $T_i$s, while the minimum-traffic method selects $k$ buses with lowest $T_i$s, where from OPF model

$$T_i = |g_i - (D_i - s_i)| + \sum_{j:j \in I} |b_{ij}(\theta_i - \theta_j)|$$
Selection of $k$ Buses

- The *node betweenness* method: finds $k$ buses with highest node betweenness. The node betweenness for bus $i$ is defined as

\[
C_B(i) = \sum_{s \neq i \neq t \in I} \frac{n_{st}(i)}{n_{st}}
\]

where $n_{st}$ is the number of shortest paths from $s$ to $t$, and $n_{st}(i)$ is the number of shortest paths from $s$ to $t$ that pass through a bus $i$.

- Floyd-Warshall algorithm, Johnson’s algorithm, Brandes’ algorithm
Selection of $k$ Buses

- The *critical node detection* problem (CNP) method: detects a set of vertices in a graph whose deletion results in the graph having the minimum pairwise connectivity between the remaining vertices. It is NP-hard and can be formulated as a mixed integer linear problem like:

\[
\begin{align*}
\min & \quad \sum_{i,j:v_i,v_j \in V} x_{ij} \\
\text{s.t.} & \quad x_{ij} + \delta_i + \delta_j \geq 1, \forall (i, j) \in L \\
& \quad x_{ij} + x_{jt} - x_{it} \leq 1, \forall i, j, t \in I \\
& \quad x_{ij} - x_{jt} + x_{it} \leq 1, \forall i, j, t \in I \\
& \quad -x_{ij} + x_{jt} + x_{it} \leq 1, \forall i, j, t \in I \\
& \quad \sum_{i:i \in I} \delta_i = k \\
& \quad \delta_i, x_{ij} \in \{0, 1\}, \forall i, j \in I
\end{align*}
\]

- Arulselvan, Commander, Elefteriadou, Pardalos (2009)
The *edge betweenness* method: finds $k$ edges with highest edge betweenness. The edge betweenness is adapted from node betweenness, and it can be expressed as

$$C_{(i,j)B} = \sum_{s,t \in l} \frac{n_{st}(i,j)}{n_{st}},$$

where $n_{st}$ is the number of shortest paths from bus $s$ to bus $t$, and $n_{st}(i,j)$ is the number of shortest paths from bus $s$ to $t$ that pass through line $(i,j)$. 
Interdiction Analysis

- Interdiction: worst case scenario analysis
- A group of terrorists will attack the power grid with limited resources to maximize the disruption

\[
\begin{align*}
\max & \quad \delta, \sigma \\
\min & \quad g, s, t, \theta \\
\end{align*}
\]

\[\sum_{i \in I} (h_i g_i + r_i s_i)\]

s.t.  \[f_{ij} = b_{ij} (\theta_i - \theta_j)(1 - \delta_i)(1 - \delta_j)(1 - \sigma_{ij}), \forall (i, j) \in L\]
\[-\bar{f}_{ij} \leq f_{ij} \leq \bar{f}_{ij}, \forall (i, j) \in L\]
\[\sum_j f_{ji} + g_i = \sum_j f_{ij} + (D_i - s_i), \forall i \in I\]
\[0 \leq g_i \leq \bar{g}_i (1 - \delta_i), \forall i \in I\]
\[0 \leq s_i \leq D_i, \forall i \in I\]
\[\delta_i + \sum_{(i, j) \in L} \sigma_{ij} = k\]
\[\delta_i, \sigma_{ij} \in \{0, 1\}, \forall i \in I, (i, j) \in L\]
Case Study

Figure: Generating and load shedding cost vs. failed buses (RTS-96 System)
Case Study

Figure: Generating and load shedding cost vs. failed lines (RTS-96 System)
Conclusions

- random failures, degree attack, maximum-traffic attack, minimum-traffic attack and betweenness attack
- the critical node detection method only works with the contingencies consisting of failures only on buses
- the interdiction model can select contingencies consisting both buses and lines
- interdiction always select the most crucial components
- the maximum-traffic method and critical node detection method select the second most crucial buses, while the minimum-traffic method finds the least crucial ones
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   - Expansion Planning

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4 Activities
   - Publications
   - Books
   - Energy Systems Journal
Unit Commitment in Electrical Power Generation
- A key optimization problem in power system operations and control (short-term)
- Minimizing total generation cost
- Technical constraints (minimum on/off, ramping, reserves, capacity, etc.)
- Mixed Integer Nonlinear Programs with binary variable for on/off status

Approaches to Unit Commitments
- Priority list
- Dynamic programming
- Lagrangian relaxation
- Branch-and-bound based MILP algorithms
- Benders decomposition, etc.
Introduction

- Uncertainties related to the Deregulation of Power Market
- Uncertainties due to the High Penetration of Renewable Energy
- Uncertainties of infrastructure stability (generator, transmission line, failure)
- Generalized Unit commitment
  - Commitment of units
  - Economic Dispatch
  - Operating Reserves
  - Power Transmission
Approaches to handle uncertainties in unit commitment problems

- Reserve requirements for operating units
- Stochastic programming models (two-stage and multi-stage)
- Robust Optimization models (given uncertainty levels)
Features of the model

- Two-stage stochastic optimization problem for day-ahead scheduling
- First stage unit commitment
- Second stage economic dispatch and power transmission
- Network constraints and power loss calculation
- Chance constraints for risk control
  Bounds on risk measures such as VaR and CVaR
Piecewise linear fuel cost function

\[ F(p) = \begin{cases} 
  c_1 & \text{if } p \leq \Delta_1 \\
  c_2 & \text{if } \Delta_1 < p \leq \Delta_2 \\
  c_3 & \text{if } \Delta_2 < p \leq \Delta_3 \\
  c_4 & \text{if } \Delta_3 < p \leq \Delta_4 \\
  c_5 & \text{if } \Delta_4 < p 
\end{cases} \]

**Figure:** Piecewise Linear Approximation of the Fuel Cost Function
Why pay for things which are very unlikely to happen?

Extremely “bad” scenarios with very small probability.

Probabilistic constraints:
- Determine how “bad” it is;
- Determine how “unlikely” it is.

Limiting VaR (Value at Risk) or CVaR (Conditional Value at Risk).
Take Risk and Have It Under Control

Suppose $L(x,y)$ is the loss function of random variable $x$ and decision variable $y$.

- **Value at Risk (VaR):**

$$\text{VaR}_\theta = \inf \{ l \in \mathbb{R} : P(L(x, y) \geq l) \leq 1 - \theta \}$$

- **Conditional value at Risk (CVaR):**

$$\text{CVaR}_\theta = E(L(x, y) | L(x, y) \geq \text{VaR}_\theta)$$
VaR and CVaR

\[ F_L(l) \]

1

\[ \theta \]

\[ L_{\text{min}} \]

\[ \text{VaR}_\theta \]

\[ \text{CVaR}_\theta \]

\[ L_{\text{max}} \]
Cost V.S. Risk

Figure: Minimal Cost V.S. Limit of CVaR

![Graph showing Cost V.S. Limit of CVaR](image-url)
Handling a large number of scenarios in the SMIP

- The problem becomes difficult to solve when there is a large number of scenarios.
- The structure makes the problem highly decomposable.
- Using Benders decomposition to approximate the second stage.
- Second stage becomes many separable problems in parallel.
Master Problem

\[
\begin{align*}
\text{Min} & \quad \sum_{t=1}^{T} \sum_{g \in G} (SU_{gt}v_{gt} + SD_{gt}w_{gt}) \\
\text{s.t.} & \quad (u, v, w) \in U \\
& \quad y^t(\xi) \geq 0, \quad \forall t \in T, \xi \in \Xi, \\
& \quad \eta^t + \sum_{\xi \in \Xi} \frac{Pr(\xi)}{1 - \theta} y^t(\xi) \leq \bar{\phi}, \forall t \in T.
\end{align*}
\]

Optimality cuts
Feasibility cuts
Generating Optimality Cuts

Need to solve the following subproblem of scenario $\xi$,

\[
\begin{align*}
\text{Min} & \quad \sum_{t=1}^{T} \sum_{g \in G} F_i \left( p^\xi_{gt} \right) \\
\text{s.t.} & \quad (p(\xi), s(\xi), f(\xi), d(\xi), \beta(\xi)) \in F(\hat{u}) \\
& \quad d_t^i(\xi) + x_t^i(\xi) \geq D_t^i(\xi), \quad t = 1, \ldots, T, \forall i \in N, \\
& \quad \sum_{i \in N} L_i^t(\xi) x_i^t(\xi) \leq \hat{\eta}^t + \hat{y}^t(\xi), \quad \forall t \in T,
\end{align*}
\]

The optimal dual solution can help constructing an Optimality cut.
Generating feasibility Cuts

Need to solve the following violation testing problem of scenario $\xi$,

$$\text{Min} \quad \sum_{t=1}^{T} \left[ \alpha^t + \sum_{g \in G} \delta^t_i \right]$$

s.t.

$$\delta^t_i + \sum_{g \in \{G_i\}} s^t_{gt}(\xi) \geq R_{Si}^t(\xi), \quad t \in T, \forall i \in N,$$

$$d^t_i(\xi) + x^t_i(\xi) \geq D^t_i(\xi), \quad t = 1, \ldots, T, \forall i \in N,$$

$$-\alpha^t + \sum_{i \in N} L^t_i(\xi)x^t_i(\xi) \leq \hat{\eta}^t + \hat{y}^t(\xi), \quad \forall t \in T,$$

The optimal dual solution is actually an extreme ray of the dual to the subproblem, which helps construct a feasibility cut.
Conclusions & Future Research

- Stochastic security constrained unit commitment with CVaR constraint
- Benders’ decomposition for large number of scenario
- More extensive numerical experiments
- Modeling of quick-start generators
- Modeling of demand response, etc.
- Long-term power system expansion planning with embedded stochastic unit commitment
- New emerging technologies, in particular in solar energy, can drastically change the dynamics of energy systems.
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Electricity and Natural Gas Network Expansion Problem

- Considers demand for both electricity and gas
- Accounts for expansion of
  - LNG terminals
  - Gas Distribution Network
  - Electricity Network
- Captures uncertainty in the forecasted future electricity and gas demand
Natural gas release less greenhouse gas than oil and coal while giving a fair amount of energy when burnt. More and more gas-fired power plants are built to protect the environment. (Gives rise to Stochastic Unit Commitment problems.)

- From 1980 to 2007, the world's demand of natural gas has doubled, 52.9 to 108 trillion cubic feet (EIA 2010).
- The demand is predicted to increase by 44% more until 2035 to around 156 TCF (EIA 2010).
- Natural gas-fired electricity production increases by 2.1% per year from 3.9 trillion kilowatthours in 2007 to 6.8 trillion kilowatthours in 2035 (EIA 2010).
Natural Gas Introduction

- Natural gas release less greenhouse gas than oil and coal while giving fair amount of energy when burnt. More and more gas fired power plants are built to protect environment. (Give the rise to Stochastic Unit Commitment problems.)
- From 1980 to 2007, the world’s demand of natural gas has doubled, 52.9 to 108 trillion cubic feet (EIA 2010).
- The demand is predicted to increase by 44% more until 2035 to around 156 TCF (EIA 2010).
- Natural gas-fired electricity production increases by 2.1% per year from 3.9 trillion kilowatthours in 2007 to 6.8 trillion kilowatthours in 2035 (EIA 2010)
World Gas Consumption Trend

Figure: World gas consumption in billion cubic feet (DOE)
Electricity Generation

**Figure**: Electricity generation capacity additions by fuel type, 2009-2035 (gigawatts) (EIA)
According to EIA International Energy Outlook 2010,
- World average RTP (Reserve To Production) ratio is about 60 years.
- Central and South America RTP is about 46 years.
- 72 and 68 years are for Russia and Africa.
- More than 100 years for Middle East.
- US production rate is about 20 TCF per year and its estimated reserves are about 1747.47 TCF. (RTP: 87 years.)

In national level, how to analyze the whole electricity and natural gas systems by considering transmission networks and LNG locations together?
Our Modeling Aims

- System Level (linear model with line reverse for gas network)
- Capable of handling different demand and supply patterns
- Gas transportation network expansion and LNG location due to imbalanced reserves and different economic growths in the world.
- Electricity generation and transmission network capacity expansion to satisfy growing demand.
- Meet the electricity and gas demands minimizing the costs.
The CVaR Risk Management Constraints

\[
\sum_{(i,j) \in A^+_i} f_{ij}(\xi) - \sum_{(j,i) \in A^-_i} (1 - l_{ji}) f_{ji}(\xi) + s_i(\xi) = d^G_i(\xi) + d^{GP}_i(\xi) - \mu_i(\xi),
\]
\[\forall i \in N_G \setminus N_R, \xi \in \Xi\]

\[
\mu_i(\xi) = 0, \quad \forall i \in N_G \setminus N_R, \xi \in \Xi
\]

\[
\bar{\mu}_i \geq \mu_i(\xi) \geq 0, \quad \forall i \in N_R, \xi \in \Xi
\]

\[
\sum_{i \in N_G} \mu_i(\xi) \leq \eta + w(\xi), \quad \forall \xi \in \Xi
\]

\[
w(\xi) \geq 0, \quad \forall \xi \in \Xi
\]

\[
\eta + \sum_{\xi \in \Xi} \frac{Pr(\xi)}{1 - \zeta} w(\xi) \leq \phi
\]
Problem Formulation

- The resulting problem is a two stage stochastic program
- First stage corresponds to investment decisions
- Second stage corresponds to operational constraints (generation and transmission decisions)
- Use Benders decomposition to solve the problem
Restricted Master Problem

\[ \text{[RMP]}: \]

Min \[ \sum_{(i,j) \in A_G} \sum_{k \in K_{ij}} c_{ij}^k \alpha_{ij}^k + \sum_{i \in N_{LNG}} \sum_{k \in K_i} c_i^k \beta_i^k \]
\[ + \sum_{(i,j) \in A_{ELC}} p_{ij} x_{ij} + \sum_{i \in N_{GEN}} \sum_{k \in K_i} r_i^k y_i^k + \sum_{\xi \in \Xi} \text{Prob}(\xi) \pi(\xi) \]

s.t.
\[ u_{ij} = u_{ij} + \sum_{k \in K_{ij}} \Delta_{i,j}^k \alpha_{ij}^k, \quad \forall (i, j) \in A_G, \]
\[ v_i = v_i + \sum_{k \in K_i} \Delta_{i}^k \beta_i^k, \quad \forall i \in N_{LNG}, \]
\[ G_i = g_{i}^{\max} + \sum_{k \in K_i} \Delta_{i}^k y_i^k, \quad \forall i \in N_{GEN}, \]
\[ \pi(\xi) + a_k^1(\xi) u + a_k^2(\xi) v + a_k^3(\xi) x + a_k^4(\xi) G \geq a_k^5(\xi), \quad \forall k \in K(\xi), \xi \in \Xi, \]
\[ \alpha_{ij}^k \in \{0, 1\}, \quad \forall k \in K_{ij}, (i, j) \in A, \quad \beta_i^k \in \{0, 1\}, \quad \forall k \in K_i, i \in N_{LNG}, \]
\[ x_{ij} \in \{0, 1\}, \quad \forall (i, j) \in A_{ELC}, \quad y_i^k \in \{0, 1\}, \quad \forall k \in K_i, i \in N_{GEN} \]
Subproblems

The subproblem of scenario $\xi$, given the first stage solution $(\hat{u}, \hat{v}, \hat{x}, \hat{y})$:

$$[\text{SP}(\xi)]:$$

Min Generation and Transportation Cost
subject to
Flow balance Constraints,
Physical Limits Constraints,
Risk Management constraints.
Conclusions and Future Work

- Mixed integer programming model
  - Electricity and Gas Network expansion planning
  - LNG terminal location
  - Generators Capabilities
- Risk constraints (VaR / CVaR)
- MILP implemented in C++ and solved by CPLEX® 12.2, decomposition algorithm is being developed
- Numerical comparison for large-scale problems
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Hydro-Thermal Power Systems

Figure: Itaipu, Brazil

Figure: Coal plant

Figure: Gas plant
Water Balance

\[
\text{Inflow} = a_t \\
\text{Storage} = v_t \\
\text{Release} = u_t \\
\text{Losses, spillage} = s_t
\]
...linear programming methods (to) be extended to include the case of uncertain demands for the problem of optimal allocation of a carrier fleet to airline routes to meet an anticipated demand distribution...

George B. Dantzig

*Linear Programming under Uncertainty*

Management Science, 1:3 & 4, 197–206, 1955
What Exactly is Uncertain?

Such an energy system is subject to different uncertainties:

- stochastic fuel prices,
- stochastic electricity demand,
- stochastic (water) inflows,

and in the liberalized market in addition also:

- stochastic electricity spot prices,
- stochastic CO2 prices.
What Exactly is Uncertain?

Such an energy system is subject to different uncertainties:

- stochastic fuel prices,
- stochastic electricity demand,
- stochastic (water) inflows,

and in the liberalized market in addition also:

- stochastic electricity spot prices,
- stochastic CO2 prices.
What is the Problem?

Figure: Hydro Scheduling Tradeoff
Multi-Stage Stochastic Optimization

\[ z := \min c_1(u_1) + \min \mathbb{E}_{\omega_2 \in \Omega_2} \left[ c_t(u_t(\omega_t)) + \ldots + \right. \]

\[ + \min \mathbb{E}_{\omega_t \in \Omega_t} \left[ c_t(u_t(\omega_t)) \right] + \ldots + \]

\[ + \min \mathbb{E}_{\omega_T \in \Omega_T} \left[ c_T(u_T(\omega_T)) \right] \ldots \] (1)
Linear Operational Constraints

- **Electricity Demand**
  - load blocks

- **Hydro**
  - reservoir security constraints
  - limits on total outflow
  - peak modulation constraints in run-of-the-river plants
  - run-of-the-river plants generation
  - irrigation for hydro reservoirs
  - initial fill-up of reservoirs
  - tailwater elevation
  - risk aversion
Linear Operational Constraints (cont’d)

- **Thermal**
  - piecewise linear cost
  - must-run thermal plants
  - fuel consumption limits
  - fuel consumption rate limit
  - minimum generation constraint for a set of thermal plants
  - multiple fuels
  - unit commitment

- **Generation Reserve**
  - spinning reserve
  - generation reserve
Linear Operational Constraints (cont’d)

- **Power Transmission Network**
  - interconnection model
  - linearized power flow model
  - transmission losses

- **Natural Gas Network**
  - production limits
  - pipeline flow limits
  - supply and demand balance
The considered energy system has the following characteristics:

1. hydro-dominated power system,
2. mid-term to long-term optimization horizon,
3. all operational constraints can be linearized, and
Is the “Hydro-Thermal Scheduling World” Linear?

No!

…but piecewise linear is a very good approximation!

D.D. Wolf and Y. Smeers
_The Gas Transmission Problem Solved by an Extension of the Simplex Algorithm_
Management Science, 46, 1454–1465, 2000

R. Rubio-Barros, D. Ojeda-Esteybar, and A. Vargas,
Energy Carrier Networks: Interactions and Integrated Operational Planning
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Energy Carrier Networks: Interactions and Integrated Operational Planning
**Assumption**: Perfect competition (absence of market power)

- centrally dispatched = market-based dispatch
- allows *fundamental modeling* of prices

---

G. Gross and D. Finlay  
*Generation supply bidding in perfectly competitive electricity markets*  
Computational & Mathematical Organizations Theory, 6, 83–98, 2000

P. Lino, L.A.N. Barroso, M.V.F. Pereira, R. Kelman, and M.H.C. Fampa  
*Bid-Based Dispatch of Hydrothermal Systems in Competitive Markets*  
Solution Methods

Classification with respect to inflow uncertainty methodology:

1. *deterministic* models,
2. *scenario-based* methods,
3. *sampling-based* methods.

W. Yeh
*Reservoir management and operations models: A state of the art review*
Water Resources Research, 21, 1797–1818, 1985

J. Labadie
*Optimal operation of multireservoir systems: State-of-the-art review*
Journal of Water Resources Planning and Management, 130, 93–111, 2004
Scenario-Based Methods

Idea

Scenario-based methods generate **up-front** a set of realizations of the random space. The realizations are then used to generate the **extensive form** of the stochastic program. These are then typically solved **exactly**.

- typically LP problems
- (very) large-scale mathematical programs
- solution quality depends on the approximation of the realizations to the original, stochastic program
- reach limitations for multi-stage problems
- “scenario tree” or “fan”
Scenario-Based Methods (cont’d)

- **Advantage**: various uncertainties (correlated and uncorrelated) can be incorporated into the model; *e.g.*, hydro inflows, electricity spot prices, contract prices, electricity demand, and fuel prices.

  - scenario generation; (Wallace and co-workers)

  - J. Dupačová, G. Consigli, and S. Wallace
    *Scenarios for multistage stochastic programs*

  - K. Høyland and S. W. Wallace
    *Generating Scenario Trees for Multistage Decision Problems*
    Management Science, 47, 295–307, 2001

  - K. Høyland, M. Kaut and S.W. Wallace
    *A heuristics for generating scenario trees for multistage decision problems*
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  *A heuristics for generating scenario trees for multistage decision problems*
  Computational Optimization and Applications, **23**, 169–185, 2003
Limitations

In order to capture the correlation among the inflows of the reservoirs, a large scenario tree may be required, leading to very large scale deterministic equivalent programs.

S.-E. Fleten and S.W. Wallace

*Delta-Hedging a Hydropower Plant Using Stochastic Programming*

Sampling-Based Methods

**Idea**

Sampling-based methods generate samples of the random space **on-the-fly** and solve the resulting problems **approximately**.

- typically Dynamic Programming methods
- statistical convergence results
- may possess “Curse of Dimensionality”
- very popular for hydro-thermal scheduling
The major lines of research for sampling-based methods towards hydro-thermal scheduling is driven by the methods of

- **Stochastic Dynamic Programming** (SDP)
- **Stochastic Dual Dynamic Programming** (SDDP)

B.F. Lamond and A. Boukhtouta

*Optimizing long-term hydro-power production using markov decision processes*  
When solving the One Stage Dispatch Problem, one encounters (at least) the following two challenges:

1. the (conditioned) distribution of $\omega$ is not known and expected to be continuous, and

2. One Stage Dispatch Problem cannot be solved computationally for the whole continuum of reservoir levels $v_t$. 
When solving the One Stage Dispatch Problem, one encounters (at least) the following two challenges:

1. the (conditioned) distribution of $\omega$ is not known and expected to be continuous, and

2. One Stage Dispatch Problem cannot be solved computationally for the whole continuum of reservoir levels $v_t$. 
**Stochastic Dynamic Programming (SDP) and Stochastic Dual Dynamic Programming (SDDP)** overcome these two challenges in the following way:

1. These inflows are modeled as a linear autoregressive model via a continuous Markov Process.

2. The set of reservoir levels is discretized into $M$ values. The function $z_t$ is then approximated either via
   - interpolation of the $M$ points (in SDP), or via
   - extrapolation of the $M$ points (in SDDP).
SDP: Expected Future Cost Interpolation

- solve “backwards” in time
- $M$ forward samples
- $L$ backward openings
- discretize storage values vector into $N_1$ values
- discretize inflows into $N_2$ values
SDP: Challenges

1. “curse of dimensionality” 
   \((\mathcal{N}_1 \cdot \mathcal{N}_2)^I\) states in each stage

2. static discretization of state space

3. lack of solution quality measure
The situation with respect to stochastic dynamic programming is that there are, as yet, no widely applicable computational devices other than discrete dynamic programming (DDP). Because of their curse of dimensionality, [...] DDP is not adequate for solving many water resource problems of interest. The largest numerical stochastic dynamic programming solutions [...] are for problems having at most two or three state variables.

S. Yakowitz

*Dynamic programming applications in water resources*

Water Resources Research, 18:4, 673–696, 1982
use information of dual to \textit{underestimate} future cost function

- “Benders cuts”
- backwards pass: $z$
- forward Monte Carlo simulation: $\hat{z}$
- stop when convergence criteria is satisfied
SDDP: Strength

1. **no** curse of dimensionality
2. state space is discretized **dynamically**
3. statistical solution quality measure

M.V.F. Pereira
*Optimal stochastic operations scheduling of large hydroelectric systems*

M.V.F. Pereira and L.M.V.G. Pinto
*Multi-stage stochastic optimization applied to energy planning*
Mathematical Programming, 52, 359–375, 1991
SDP vs. SDDP

**Figure**: Approximation of FCF: SDP versus SDDP
This is a tree; neither a Markov process nor a Markov Chain
Scenario Tree

Figure: Scenario Tree with 4 stages
Unifying Two Worlds: ‘Tree’ vs. ‘Sampling’

- Scenario tree ‘on top’ of the stochastic (dual) dynamic programming.
- For each stage $t$ and state, we need to solve $S_t$ one stage dispatch problems.
- Instead of $M$ cuts for the future cost function, we obtain $M \cdot S_t$.
- Computational complexity increases with the size of the tree.

Works also in combination with electricity demand uncertainty.
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Case Study I: Panama

**Fortuna**
- Min. Storage [hm$^3$]: 4.67
- Max. Storage [hm$^3$]: 172.30
- $\overline{\text{Ø Production}}$ [MW m$^3$/sec.]: 6.67
- Capacity [MW]: 300.00

**Canjilone**
- Min. Storage [hm$^3$]: 0.06
- Max. Storage [hm$^3$]: 0.21
- $\overline{\text{Ø Production}}$ [MW m$^3$/sec.]: 1.02
- Capacity [MW]: 120.00

**Estrella**
- Min. Storage [hm$^3$]: 34.63
- Max. Storage [hm$^3$]: 38.94
- $\overline{\text{Ø Production}}$ [MW m$^3$/sec.]: 3.05
- Capacity [MW]: 47.20

**Los Valle**
- Min. Storage [hm$^3$]: –
- Max. Storage [hm$^3$]: –
- $\overline{\text{Ø Production}}$ [MW m$^3$/sec.]: 2.36
- Capacity [MW]: 54.76

**Bayano**
- Min. Storage [hm$^3$]: 1784.71
- Max. Storage [hm$^3$]: 4965.23
- $\overline{\text{Ø Production}}$ [MW m$^3$/sec.]: 0.42
- Capacity [MW]: 260.00

**Figure**: Hydro-electric system of Panama
Case Study I: Panama (cont’d)

Demand [GWh]

Figure: Electricity demand scenarios.
Case Study I: Results

**Expected Value Solution (EVS)**
- “ignore demand uncertainty and use expected electricity demand”
- $158.653$ million

**Value of Stochastic Solution (VSS)**
- EVS - stochastic solution value
- $158.653 - 157.374 = 1.279$ [million]
- $0.81\%$ EVS

**Value of Perfect Information**
- “how much am I willing to pay”
- $157.374 - 156.721$ [million] = $653,429
Case Study II: Costa Rica

Arenal
Min. Storage [hm³]: 722.75
Max. Storage [hm³]: 2266.17
Ø Production [MWm³/sec.]: 1.62
Capacity [MW]: 175.40

Corobici
Min. Storage [hm³]: –
Max. Storage [hm³]: –
Ø Production [MWm³/sec.]: 1.78
Capacity [MW]: 174.00

Sandillal
Min. Storage [hm³]: 0.00
Max. Storage [hm³]: 4.80
Ø Production [MWm³/sec.]: 0.32
Capacity [MW]: 32.00

Rio Macho
Min. Storage [hm³]: –
Max. Storage [hm³]: –
Ø Production [MWm³/sec.]: 3.73
Capacity [MW]: 120

Cachi
Min. Storage [hm³]: 3.53
Max. Storage [hm³]: 51.47
Ø Production [MWm³/sec.]: 1.90
Capacity [MW]: 105

Bot Joya
Min. Storage [hm³]: –
Max. Storage [hm³]: –
Ø Production [MWm³/sec.]: 0.93
Capacity [MW]: 50.00

Figure: Hydro-electric reservoir system of Costa Rica; excluding additional 24 run-of-the-river plants
Case Study II: Results

Expected Value Solution (EVS)
= “ignore demand uncertainty and use expected electricity demand”
= $102.089 million

Value of Stochastic Solution (VSS)
= EVS - stochastic solution value
= $102.089 - $100,947 = $1.142 [million]
= 1.12% EVS

Value of Perfect Information
= “how much am I willing to pay”
= $100,947 - $100,032 [million] = $915,870
Outline

1 Introduction

2 Smart Grid
   - Islanding
   - Reliability Analysis
   - Stochastic Unit Commitment Problem
   - Expansion Planning

3 Hydro-Thermal Scheduling

4 Activities
   - Publications
   - Books
   - Energy Systems Journal
S. Rebennack, N. Iliadis, J. Kallrath, and P. M. Pardalos
*Short Term Portfolio Optimization for Discrete Power Plant Dispatching*
IEEE PES GM proceedings, Calgary, Canada, pp. 1-6, 2009.

S. Rebennack, N. Iliadis, M. V.F. Pereira, and P. M. Pardalos,
*Electricity and CO2 Emissions System Prices Modeling and Optimization*
IEEE PowerTech conference proceedings, Bucharest, Romania, pp. 1-6, 2009

Q. P. Zheng, S. Rebennack, N. Iliadis, and P. M. Pardalos
*Optimization Models in the Natural Gas Industry*

S. Rebennack, J. Kallrath, and P. M. Pardalos
*Energy Portfolio Optimization for Electric Utilities: Case Study for Germany*
Q. P. Zheng and P. M. Pardalos
Stochastic and Risk Management Models and Solution Algorithm for Gas Transmission Network Expansion and LNG Terminal Location Planning

S. Rebennack, B. Flach, M. V.F. Pereira, and P. M. Pardalos,
Stochastic Hydro–Thermal Scheduling under CO2 Emission Constraints

N. Fan, H. Xu, F. Pan, and P.M. Pardalos
Economic analysis of the N-k power grid contingency selection and evaluation by graph algorithms and interdiction methods

N. Fan, D. Izraelevitz, F. Pan, and P.M. Pardalos
A mixed integer programming approach for optimal power grid intentional islanding
Publications – Books

*Electrical Power Unit Commitment: Models and Algorithms*
Q. P. Zheng and P. M. Pardalos
Springer, to appear in 2013

*Handbook of Wind Power Systems*
V. Pappu, S. Rebennack, P. M. Pardalos, N. Iliadis, M. V. F. Pereira (eds.)
Springer, to appear in 2013

*Handbook of CO₂ in Power System*
Q. P. Zheng, S. Rebennack, P. M. Pardalos, N. Iliadis, M. V. F. Pereira (eds.)
Springer 2012
Publications – Books

*Handbook of Networks in Power Systems*
A. Sorokin, S. Rebennack, P. M. Pardalos, N. Illiadis, M. V. F. Pereira (eds.)
Two volumes, Springer 2012

*Energy, Natural Resources and Environmental Economics*
E. Bjorndal, M. Bjorndal, P. M. Pardalos, M. Ronnqvist, (eds.)
Springer 2010

*Handbook of Power Systems*
S. Rebennack, P. M. Pardalos, M. V. F. Pereira, N. Illiadis (eds.)
Springer 2010

*Optimization in the Energy Industry*
J. Kallrath, P. M. Pardalos, S. Rebennack, and M. Scheidt (eds.)
Springer 2009
Energy Systems Journal
Optimization, Modeling, Simulation, and Economic Aspects
Editor-in-Chief: Panos M. Pardalos
Published by Springer

Applies mathematical programming, control, and economic approaches to energy systems topics, and is especially relevant in light of challenges facing humanity.
“Mortals are immortals and immortals are mortals, the one living the others’ death and dying the others’ life.”

“They say that Euripides gave Socrates a copy of Heraclitus’ book and asked him what he thought of it. He replied: “What I understand is splendid; and I think what I don’t understand is so too - but it would take a Delian diver to get to the bottom of it.” (Diogenes Laertius, Lives of Philosophers, II 22).
The END!

http://www.ise.ufl.edu/pardalos/

Questions, Comments, Suggestions?