Trade Patterns and Export Pricing Under Non-CES Preferences*

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Abstract

We develop a two-factor, two-sector trade model of monopolistic competition with variable elasticity of substitution. Firm profit and firm size may increase or decrease with market integration depending on the degree of asymmetry between countries. The country in which capital is relatively abundant is a net exporter of the manufactured good, while both firms' size and profits are lower in this country than in the country where capital is relatively scarce. By contrast, the pricing policy adopted by firms does not depend on capital endowment and country asymmetry. It is determined by the nature of preferences: when demand elasticity increases (decreases) with consumption, firms practice dumping (reverse-dumping).

Keywords: two-factor trade model; monopolistic competition; capital asymmetry; variable markups.

Highlights:

- We study a two-factor, two-sector trade model, allowing variable markups.
- Price policy and markups depend on increasing or decreasing demand elasticity.

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- Dumping (reverse dumping) is observed with increasing (decreasing) demand elasticity.
- We examine capital price and firm size behavior under globalization.
- Asymmetries in capital endowment and population affect capital price and firm size.

1 Introduction

New trade theories have raised new and important questions. How do asymmetry between countries and trade liberalization affect firms' size, trade flows, and price policies? How do they affect countries' specialization and do factor-owners benefit from trade liberalization? Apart from a few exceptions, these questions have been addressed in the Dixit-Stiglitz model of monopolistic competition (Helpman and Krugman, 1987; Feenstra, 2004). Yet, it is now well known that this model is unable to replicate facts that are well documented in the empirical trade literature: (i) markups vary with market size (Syverson, 2007); (ii) firm size are affected by the market size where firm locates (Campbell and Hopenhayn, 2005); (iii) firms price-discriminate across destinations (Bernard et al., 2007; Manova and Zhang, 2009; Martin, 2009; Schott, 2001); and (iv) firms located in countries endowed with a large amount of human or physical capital charge higher prices (Schott, 2004; Hummels and Klenow, 2005; Hallak and Schott, 2008).

In this paper, we revisit some of the questions addressed in new trade theories using a setting whose aim is to assess the compatibility of the corresponding results with the recent empirical evidence as well as their robustness. To this end, we develop a new model that has the following two distinctive features: preferences display variable elasticity of substitution and countries have capital endowments that differ from the relative population size. Specifically, we consider a trade setting with two countries that are asymmetric in endowments, namely, capital-rich Home and capital-poor Foreign, while consumers share non-CES preferences. This allows us to deal with questions that have been left aside in many existing theoretical contributions: (i) what happens when the capital/population differs across countries, (ii) how does trade liberalization affect firms' size and profits, and (iii) do and how firms price discriminate across countries?

Our main results may be summarized as follows. First, we find that the country with the higher (smaller) capital/population ratio is a *net exporter* of the manufacturing (agricultural) good. In other words, partial specialization of countries takes place and a Krugman-type home market effect (Krugman, 1980) is observed. This result is in the spirit of the Heckscher-Ohlin theory of trade. In addition, we show that both the capital price and firm size are smaller in the country with the higher capital/population ratio. In other words, the relative abundance of capital makes the capital-owners worse-off and leads to a larger number of smaller firms.

Second, unlike in the CES case, we show that trade liberalization affects *firms' size*. Specifically, the size of a firm is now variable and determined by the interaction between the following three effects: the standard *competition effect*, which stems from better accessibility to local markets by foreign competitors; the standard *market-access effect* due to better accessibility of foreign markets to the domestic firms; and the *iceberg trade cost effect*, which measures the additional output needed to deliver one unit of output abroad. When the difference in population is large, the size of firms in the more (less) populated country shrinks (expands) with trade opening. Indeed, the market access effect for firms in the smaller (larger) country overcomes (is dominated by) the competition effect when the foreign market is larger (smaller) than the domestic market. By contrast, when the difference between the two populations is small, trade liberalization shifts the size of firms in both countries in the same direction. However, how firm size varies with trade costs is unclear.

This indeterminacy finds its origin in the definition of a firm size that includes the quantity of output needed for the firm to export. When it is recognized that a firm often hires a carrier to ship its output, its size is equal to the total consumption of the firm product. In this event, the iceberg cost effect mentioned above disappears. Defining the *net size of a firm* as its total sales rather than total output, we are able to show that trade liberalization always leads firms to grow when the difference between the two populations is small. This suggests that the iceberg trade cost assumption leads to an artificial definition of a firm's size and to results that may be driven by this peculiar modeling strategy.

Firms' profits obey a similar logic. Two cases may arise. In the first one, the bigger country is very large. We then show that the competition effect overcomes the market-access effect, which implies that trade liberalization lowers firms' profits. In the smaller country, the effect is opposite. Thus, in the larger country firms lobby their government in favor of a tougher trade policy that protects them against the entry of foreign products. By contrast, producers in the smaller country lobby in favor of trade liberalization to access the foreign market. In the second case, countries have similar population sizes and firms' profits move in the same direction in both countries. This is because the market access and competition effects are more or less the same in each country. However, profits can increase or decrease. As a consequence, market integration can make firm-owners better- or worse-off.

Last, we show that the price of a domestic variety in the capital-poor country is higher (lower) than the one in the capital-rich country when the demand elasticity is increasing (decreasing). Under the same condition of the demand elasticity, the price of an imported variety in the capital-poor country exceeds that in the capital-richer country. Furthermore, unlike the CES, we show that, depending on the behavior of demand elasticity, firms' pricing exhibits richer behaviors such as *dumping* (Brander and Krugman, 1983) or *reverse dumping* (Greenhut et al., 1985). When the elasticity of demand increases (decreases), firms practice dumping (reverse dumping) in both countries. In other words, the behavior of demand elasticity is the only driving force for dumping or reverse dumping to arise.

The model is presented in Section 2. The main results are derived and discussed in Section 3, while Section 4 concludes.

2 Trade Model

We assume that the world economy includes two countries named Home and Foreign. To simplify the aggregate demands of capital owners and workers, we assume two sectors called (traditionally) "manufacturing" and "agriculture," with the latter used as numeraire. Manufacturing includes one differentiated good; agriculture includes one homogeneous good.

Lower-tier utilities defined on differentiated products are general and embedded in an upper-tier quasi-linear utility. Though admittedly restrictive, we want to argue that there are at least two sensible reasons for using a quasi-linear setting. First, in a general equilibrium model with non-homothetic preferences, we would face the wage non-equalization problem. Thus, income effects would interfere with the various effects we focus on. Thus, using quasi-linear preferences reduces drastically the role of supply-side restrictions and allows focusing on product and capital markets, abstracting from potentially complicated labor-market-based ingredients. Second, we stress that using quasi-linear preferences for studying international trade issues is far from being a novelty. For example, using quasi-linear preferences Grossman and Helpman (1994) studied the role of political campaign contributions to influence government decision on the trade policy. Feenstra (2004, ch. 7) combined quasi-linear preferences with Ricardian technology to study the impact of trade policies. Last, Melitz and Ottaviano (2008) used quasi-linear preferences to investigate in great details the impact of firm heterogeneity on the nature and type of trade.[1]

The two production factors are called "labor" and "capital" although there can be alternative interpretations: skilled and unskilled labor, etc.

The consumer side includes L identical consumers, each of them either a worker or/and a capital owner. There is a total mass K of capital endowment in the world. Workers supply one unit of labor, whereas capital owners supply one unit of capital, both inelastically. Thus the world economy has a total population L, a total capital endowment K, and a some total labor endowment that will play no role in our analysis. θ and $(1-\theta)$ are the share of agents in Home and Foreign, and λ and $(1-\lambda)$ are the share of capital endowment in these countries. We assume that the Home country has a larger supply of capital, i.e., $\lambda \geqslant \frac{1}{2}$.

The differentiated good is represented by continuum of varieties indexed by $i \in [0, N]$, where N is the mass of varieties. An infinite-dimensional consumption vector is $X^j = (x_k^{ij})_{k\in[0,\,N^i],\,i,\,j\in\{H,\,F\}}$ where x_k^{ij} is the individual consumption of variety k produced in country i and consumed in country j. Let p_k^{ij} be the price of x_k^{ij}.

Consumers share similar preferences in both countries and producers have similar technologies. We follow Ottaviano et al. (2002) and assume quasi-linear preferences of consumers. The absence of the income effect is a drawback of using quasi-linear preferences but we need this assumption to isolate the impact of differences in factor endowments

[1] Note also that the analysis undertaken by Dinopoulos et al. (2011) of standard trade theory under quasi-linear preferences suggests that this simplifying assumption does not fundamentally affect the qualitative nature of the results.

from the influence of income differential. Preferences are defined for differentiated varieties and a homogeneous good following utility function $V(m) + A$. Here m is "aggregate" consumption of the differentiated good, and A stands for the consumption level of the homogeneous good. Utility derived from the consumption of each variety of differentiated good m is defined by "elementary" utility function $u(x_k^{ij})$. Utility maximization problems in Home and Foreign are as follows:

$$\max_{X^H, A^H} \left[V\left(\int_0^{N^H} u(x_k^{HH})dk + \int_{N^H}^{N^H+N^F} u(x_k^{FH})dk\right) + A^H \right], \text{ s.t. } \int_0^{N^H} p_k^{HH} x_k^{HH} dk + \int_{N^H}^{N^H+N^F} p_k^{FH} x_k^{FH} dk + p_a A^H \le E^H$$

$$\max_{X^F, A^F} \left[V\left(\int_0^{N^H} u(x_k^{HF})dk + \int_{N^H}^{N^H+N^F} u(x_k^{FF})dk\right) + A^F \right], \text{ s.t. } \int_0^{N^H} p_k^{HF} x_k^{HF} dk + \int_{N^H}^{N^H+N^F} p_k^{FF} x_k^{FF} dk + p_a A^F \le E^F,$$

where p_a is the price of the agriculture good, E^j, $j \in \{H, F\}$ is income. For a pure worker, $E = 1$, whereas the income of pure capital owners in Home and Foreign equals the interest rates $E = \pi^H$, $E = \pi^F$, respectively. (With quasi-linearity, we need no assumptions of such separated ownership or any mixed ownership of capital.) Both utility functions $u(\cdot)$ and $V(\cdot)$ are thrice continuously differentiable, strictly increasing (at least at some zone of equilibria $[0, \tilde{x})$) and strictly concave with $u(0) = 0$. Unlike Dixit and Stiglitz (1977) and Behrens and Murata (2007), we do not assume a specific form of function $u(\cdot)$.

The first-order condition for the consumer's problem implies the inverse demand function $\mathbf{p}$ for variety k:

$$\mathbf{p}(x_k^{HH}, \mu^H) \equiv \frac{u'(x_k^{HH})}{\mu^H}, \quad \mathbf{p}(x_k^{FH}, \mu^H) \equiv \frac{u'(x_k^{FH})}{\mu^H}, \tag{1}$$

$$\mu^H \equiv \frac{1}{V'(m^H)}, \quad m^H \equiv \int_0^{N_H} u(x_k^{HH})dk + \int_{N_H}^{N_H+N_F} u(x_k^{FH})dk, \tag{2}$$

$$\mathbf{p}(x_k^{FF}, \mu^F) \equiv \frac{u'(x_k^{FF})}{\mu^F}, \quad \mathbf{p}(x_k^{HF}, \mu^F) \equiv \frac{u'(x_k^{HF})}{\mu^F}, \tag{3}$$

$$\mu^F \equiv \frac{1}{V'(m^F)}, \quad m^F \equiv \int_0^{N_H} u(x_k^{HF})dk + \int_{N_H}^{N_H+N_F} u(x_k^{FF})dk. \tag{4}$$

Here $\mu^i > 0$ denotes the *marginal utility of expenditure for manufacturing*, because it is the country's Lagrange multiplier of the "budget constraint" in the sub-optimization problem

$$m^*(E_m^i) \equiv \max_{x: px^i \le E_m^i} m_i(x); \qquad \max_{E_m^i + A^i \le E} V(m^*(E_m^i)) + A^i, \tag{5}$$

where m_i is the satisfaction from manufacturing, and endogenous $E_m > 0$ is the expenditure for it. (The multiplier of the real budget, as is standard, equals 1.) Thus, μ^H diminishes all prices in (1) and is thereby interpreted as the *intensity of Home competition in manufacturing.* These intensities, μ^H, μ^F, may differ in Home and Foreign, being positively related to satisfaction from varieties m_H, m_F.

On the production side, as it standard, the agricultural sector produces a homogeneous good under perfect competition and with constant returns to scale. The marginal production cost equals one unit of labor, hence, price $p_a \equiv 1$. The manufacturing sector presents homogeneous firms. Each manufacturing firm incurs a fixed cost of one unit of capital and a marginal cost amounting to one unit of labor. Thus, the total production cost equals $C(q) = \pi + wq$, where π is the price of capital (interest rate) and y stands for output. Our approach to these questions differs from that in Helpman and Krugman's (1987) classical book. They assume some substitution between labor and capital as well as general equilibrium — but standard CES utility — that shadows the price effects explored in the next subsection. We, however, reject the assumption of general equilibrium to get a tractable model of price effects stemming from variable elasticity of substitution.

Total demand (output) q_k^H of Home firm k and output q_k^F of Foreign firm k are given by

$$q_k^H \equiv \theta L x_k^{HH} + (1-\theta)\tau L x_k^{HF},$$

$$q_k^F \equiv (1-\theta) L x_k^{FF} + \theta\tau L x_k^{FH},$$

where $\tau > 1$ is the "iceberg-type" trade cost for the manufactured good; in contrast, the agricultural good requires zero trade cost.

Labor is intersectorally mobile, and this leads to the same wages in both sectors, normalized without loss of generality to $w = 1$. Then total production cost of output q becomes

$$C(q) = \pi + q.$$

Each firm produces one unique variety, and each is produced by a single firm. Furthermore, we assume that the number of firms N is large enough to disregard the impact of each firm on the market. This means that each firm perceives current μ^j, $j = \{H, F\}$, which is an aggregate market statistic analogous to the price index under CES preferences.

Home and Foreign firms maximize profits

$$\max_{x^{HH},x^{HF}} \left[(p_k^{HH}(x_k^{HH},\mu^H)-1)\theta L x_k^{HH}+(p_k^{HF}(x_k^{HF},\mu^F)-\tau)(1-\theta)Lx_k^{HF}-\pi^H\right], \tag{6}$$

$$\max_{x^{FF},x^{FH}} \left[(p_k^{FF}(x_k^{FF},\mu^F)-1)(1-\theta)L x_k^{FF}+(p_k^{FH}(x_k^{FH},\mu^H)-\tau)\theta Lx_k^{FH}-\pi^F\right], \tag{7}$$

where π^H and π^F are capital prices in Home and Foreign.

To assist with further analysis, we introduce a specific function that plays a critical role in what follows:

$$r_u(z)=-\frac{u''(z)z}{u'(z)}. \tag{8}$$

On the one hand, r_u is the *elasticity of the inverse-demand* function for variety i. On the other hand, $r_u(z)$ can be treated as the "relative love for variety" (RLV). (For more discussion on this, see Vives, 1999; and Zhelobodko et al., 2012.) We assume that $r_u(x)<1$, at least for some interval of x values. This restriction is both natural and helpful in further analysis. In particular, $r_u(z)$ for the widely-used CES-function ($u(z)=z^\rho$) is a constant: $r_u(z)=1-\rho$. For CARA-function ($u(z)=1-e^{-\rho z}$), $r_u(z)$ increases linearly, but may decrease for some other functions. Mostly, we assume utilities that generate increasing inverse demand elasticity, which seems more natural (see Krugman, 1979; Vives, 1999).

To guarantee concavity of profit function, we assume that

$$-\frac{zu'''(z)}{u''(z)}<2$$

always holds. Under this assumption, the solution for each producer's problem is the same and unique (see Appendix A). It allows us to disregard producer's index k and study only the symmetric outcomes.

Using the first-order condition for the producer's problem, we characterize the symmetric profit-maximizing prices as

$$p^{HH}=\frac{1}{1-r_u(x^{HH})}, \qquad p^{FH}=\frac{\tau}{1-r_u(x^{FH})} \tag{9}$$

$$p^{FF}=\frac{1}{1-r_u(x^{FF})}, \qquad p^{HF}=\frac{\tau}{1-r_u(x^{HF})}, \tag{10}$$

and markup as

$$M^{ij} = \frac{p^{ij} - 1}{p^{ij}} = r_u(x^{ij}) \in (0, 1). \tag{11}$$

For proof, see Appendix A.

We next consider the capital market balance. Since capital is immobile among countries, the mass of firms in each country is predetermined by the country's capital share:

$$N^H = \lambda K, \qquad N^F = (1 - \lambda)K. \tag{12}$$

Equilibrium. Consider equilibrium when both countries produce both differentiated and homogeneous goods. We define *symmetric trade equilibrium* as a bundle that satisfies consumers' maximization problem (1), (3); producers' maximization problem (6), (7); capital balance (12); and zero-profit condition:

$$(p^{HH}(x^{HH}) - 1)\theta L x^{HH} + (p^{HF}(x^{HF}) - \tau)(1 - \theta)L x^{HF} = \pi^H, \tag{13}$$

$$(p^{FF}(x^{FF}) - 1)(1 - \theta)L x^{FF} + (p^{FH}(x^{FH}) - \tau)\theta L x^{FH} = \pi^F. \tag{14}$$

Note that, in this paper, we focus only on equilibria with positive manufacturing and agricultural production in both countries. We call them *completely diversified equilibria.* A full characterization of existence conditions for such equilibria, i.e., determination of the exact range of parameter values suitable for complete diversification, was not addressed in this paper. We indicate that two kinds of inequalities must hold: (1) the amount of labor required by manufacturers should be smaller than the country's labor endowment; and (2) the income of each type of agents (workers and capital owners) should be sufficient to buy both agricultural and manufacturing goods under current prices. Positivity of capital endowment is not sufficient to guarantee such properties. However, it is obvious that, for any utilities and factor endowments, sufficient labor and sufficient (additional) income *exist* to ensure complete diversification.

To investigate our trade equilibrium, we can rearrange the equilibrium conditions in terms of consumption variables only and state equilibrium uniqueness. (See Appendix B for details.)

Proposition 1. *(i) The equilibrium individual consumption bundle* (x^{HH}, x^{FH}) *in Home country is the solution to the system*

$$\frac{u'(x^{HH})[1 - r_u(x^{HH})]}{u'(x^{FH})[1 - r_u(x^{FH})]} = \frac{1}{\tau} \tag{15}$$

$$V'\left[\lambda K u(x^{HH}) + (1 - \lambda)\, K u(x^{FH})\right] \cdot u'(x^{HH})[1 - r_u(x^{HH})] = 1, \tag{16}$$

Foreign consumption $(x^{FF},\, x^{HF})$ is found from a similar system resolved independently from (15), (16).

(ii) Consumption levels are independent of labor endowments.

(iii) There is at most one solution $(x^{HH},\, x^{FH},\, x^{HF},\, x^{FF})$ to these equilibrium equations.

Proof: See Appendix B.

The first equilibrium equation essentially says that the ratio of marginal revenues[2] of local and foreign producers equals the ratio of their transportation costs. The second equation compares the marginal utility of income spent on manufacturing goods to the marginal utility from agriculture (substitution between manufacturing and agricultural goods). In studying comparative statics, it is often be useful create one equation from the equations (15) and (16), using function G:

$$G(x^{HH},\, \lambda,\, K,\, \tau) \equiv V'(\lambda K u(x^{HH}) + (1-\lambda) K u(z(x^{HH},\, \tau))) \cdot MR(x^{HH}) = 1, \tag{17}$$

where $MR(x) \equiv u'(x)[1 - r_u(x)]$ is the marginal revenue, $z(x^{HH},\, \tau) \equiv \overset{-1}{MR}(\tau \cdot R(x^{HH}))$ is a solution to equation (15). This inverse function is well-defined since the marginal revenue is strictly decreasing in x. Moreover, it is easy to show (Appendix B) that $z(x,\, \tau)$ increases in x. Totally differentiating (17) w.r.t. $\lambda \in [0.5, 1]$ and $\tau \in [1, \infty)$, we obtain comparative statics (how the trade equilibrium changes with capital asymmetry and trade costs).

3 The impact of capital asymmetry and trade liberalization

This section studies the impact of countries' asymmetry in factor endowments on trade. We first explain how market size and capital endowment change the equilibria in the simplest setting — a closed economy.

3.1 Opening trade: Transition from autarky to free trade equilibrium

At least since Krugman (1979), an increase in a country's population (L) is often interpreted as a transition from autarky (infinite trade cost) to free trade (zero trade cost). In our setting, it may induce an increase in the mass of consumers L or/and an increase in capital endowment K. We study the impacts of independent variations in both K and L on a closed economy, showing what happens to consumption and prices after a "jump" from autarky to integration. These two states are just the *two endpoints of the globalization path*, studied in the next subsection.

The equilibrium price is given by the monopoly pricing formula,

$$p = \frac{1}{1 - r_u(x)}. \tag{18}$$

[2]Since total revenue is given by $xu'(x)$, it is readily verified that the marginal revenue equals $u'(x)[1 - r_u(x)]$.

The number of firms in the economy is fixed at $N = K$, for the per-firm capital requirement is normalized to one. The closed economy counterpart of the equilibrium conditions (16) is a single equation,

$$V'(Ku(x)) \cdot MR(x) = 1. \tag{19}$$

Since $MR(\cdot)$ and $V'(\cdot)$ are both decreasing, (19) has a unique solution x^*. Note that x^* is *independent of the population* L. This result is a by-product of three essential ingredients of our modeling strategy: quasi-linear utility, constant marginal costs, and two non-substitutable production factors.

Plugging x^* into (18), we pin down the equilibrium price p^*. It remains to determine the capital price π^*. The assumption about free entry implies that $\pi = Lx(p-1)$. Using (18), we come to

$$\pi = L\frac{xr_u(x)}{1-r_u(x)}. \tag{20}$$

Equations (18) to (20) define a unique symmetric equilibrium for the closed economy case. We now turn to comparative statics of the equilibrium with respect to K and L.

Consumption. Differentiating (19) with respect to K, we obtain

$$\frac{dx}{dK} = -\frac{x}{K}\frac{r_V}{r_V\varepsilon_{u(x)} - \varepsilon_{MR}} < 0, \tag{21}$$

since $r_V = -\frac{V''(m)}{V'(m)}m > 0$, $\varepsilon_{u(x)} = \frac{u'(x)}{u(x)}x > 0$, and $\varepsilon_{MR} = -\frac{r_u(x)(2-r_{u'}(x))}{1-r_u(x)} < 0$ is the elasticity of marginal revenue. (See Appendix C for details.)

Thus, *an increase in capital supply K decreases the individual consumption level, whereas with an increase in population L, individual consumption remains unchanged.* Why does x shrink as more firms enter? When the mass K of firms/varieties increases exogenously, the market crowding effect is at work, i.e., the consumer's expenditure for the manufacturing good is split among more varieties (all the varieties are consumed by strict concavity of u). On the other hand, it can easily be shown that the expenditure $E_m(K) \equiv Kp(x)x$ for manufacture increases less than proportionally (or even decreases) in K. (See Appendix C.) Thus the market expansion effect triggered by an increase in K is generically insufficient to dominate the market crowding effect. As a result, x decreases.[3]

Price and demand elasticity. The behavior of prices is more involved, being governed by demand elasticity — defined in (8) and (18). Clearly, the *inverse* demand elasticity $r_u(x)$ increases/decreases if and only if the elasticity of the direct demand $\varepsilon(p) \equiv \frac{p}{x}\frac{dx}{dp}$ increases/decreases, although these two magnitudes are inverse to each other at a given point $\varepsilon(p) = 1/r_u(x(p))$. The reason is that $x(p)$ decreases. That is why we use, here and later in the paper, the term

[3]The only exception is the limiting case when V is linear. Then, $E_m(K)$ is proportional to K, and the two effects balance each other exactly. Consequently, x remains unchanged.

increasingly elastic demand (IED) as a synonym for $r'_u(x) > 0$, and decreasingly-elastic demand (DED) as a synonym for $r'_u(x) < 0$. Naturally, CES utility is the borderline case or iso-elastic demand, i.e., $r_u(x) = 1 - \rho$, $r'_u(x) \equiv 0$.

Differentiating the pricing equation (18) with respect to K, we obtain

$$\frac{dp}{dK} = \frac{r'_u(x)}{(1 - r_u(x))^2} \cdot \frac{dx}{dK} \lesseqqgtr 0 \;\Leftrightarrow\; r'_u \gtreqqless 0. \tag{22}$$

We observe that, *under* ***increasing****/decreasing demand elasticity (**IED**/DED case), the equilibrium price* ***decreases****/increas* *with transition from autarky to free trade because of a capital supply shock*, and *the markup $M \equiv \frac{p-c}{p}$ changes in the same direction. Both equilibrium price and markup are generically independent of the population L and are independent from K under CES utility.*

Prices are independent of the market size L for the same reasons that consumption levels are. As for the price effect (22), economic intuition suggests that price p should *decrease* with the number of competitors K. However, this occurs only under increasing inverse demand elasticity. Seemingly surprising, the price increase under decreasing demand elasticity is, however, typical in monopoly theory. With the downward shift of the inverse demand schedule (1) caused by increasing μ, what happens to price is governed by the demand elasticity. Rather flat demand schedules, called *sub-convex* in Mrazova and Neary (2012) are those with $r'_u(x) > 0$ (IED). By contrast, *super-convex* demands are those having $r'_u(x) < 0$ (DED). Here, the increasing number of competitors looks natural because the firms compensate for their *very sharp* decrease in output by increasing markup. In the middle is the (degenerate) borderline case of iso-elastic CES demands that brings zero price effects. Some theorists suppose super-convex demands with $r'_u(x) < 0$ less realistic than sub-convex ones, because the price-increasing effect of competition seems to be rare in reality. (See related reasoning in Krugman, 1979; Mrazova and Neary, 2012; Zhelobodko et al., 2012.) The opposite view (Bertoletti and Epiphani, 2012) is that super-convex demands are quite plausible for several reasons.

In any case, both classes of utilities are worth studying. The analysis of trade that follows also shows the importance of distinguishing between price-decreasing and price-increasing effects governed by IED or DED classes of demand.

Capital price (interest rate). Transition from autarky to free trade means increasing either market size L, capital supply K, or both. Whether the capital price increases or decreases depends on the structure of changes in factor endowments. We show now that capital price always decreases (increases) with capital supply (market size), regardless of the type of demand.

The return on a unit of capital (on the fixed cost) is the price margin multiplied by output: $\pi = (p-1)Lx$. Should capital price shrink when capital supply K increases? Differentiating π with respect to capital supply yields

$$\frac{\partial \pi}{\partial K} = -\frac{\varepsilon_{MR}}{1 - r_u(x)} \frac{\partial x}{\partial K} < 0, \tag{23}$$

since $\varepsilon_{MR} < 0$, $0 < r_u(x) < 1$, and $\frac{\partial x}{\partial K} < 0$. (See Appendix C for details.)

This derivative is obviously negative under **both** IED and DED cases. Under IED, the result is clear: both individual consumption and price go down, which, in turn, leads to a decrease in capital price. In the DED case, the price increase is always outweighed by a stronger decrease in individual consumption.

A similar question relates to L. Differentiating equation (20) with respect to L, we find that the impact of market size on capital price π is unambiguously positive:

$$\frac{\partial \pi}{\partial L} = \frac{x r_u(x)}{1 - r_u(x)} > 0.$$

Intuitively, since individual consumption does not depend on the number of consumers, firm size (equilibrium output) and profits both increase with the number of consumers. Thus, *the capital price always increases with the population (number of consumers) and decreases with industry size (capital endowment).*

Welfare. Transition to free trade from autarky changes the welfare of two agent types, workers and capital owners (a consumer may play both roles simultaneously).

First, we consider the changes in the worker's welfare. From (19), we see that *the equilibrium utility of each worker does not depend on the population size* because the manufacturing consumption x does not change and income does not change.

As for the impact of K, under **IED** (in particular, under CES, which is the limiting case), each worker should benefit from additional capital: the price decreases (or remains constant) and a broader variety becomes available for a lower price (by the revealed-preference argument). *So, under IED, the worker's utility is not affected by an increase in market size and increases with capital supply. Consequently, opening up trade increases worker's utility.* However, the outcome in the DED case is less evident: the increasing variety struggles with the decreasing price.

Using the envelope theorem, it is readily verified that the partial derivative of worker's utility with respect to capital supply is given by

$$U'_K = V'(Ku(x)) \left[u(x) - u'(x)x\right] - Kx \cdot \frac{\partial p}{\partial K}.$$

One can see that the first term is positive and related to an increasing number of varieties. The second term is related to the change in price, which increases under the DED case. Which effect is stronger depends on the strength of the price decrease.

Second, we discuss the welfare of pure capital owners who do not own labor. It is clear that the *utility of each capital owner increases with the population*, because consumption x does not change, while the capital price π increases and this additional income can be spent on numeraire.

An increasing capital supply increases variety that struggles with the decrease in such consumers' income. The partial derivative of capitalist's utility with respect to capital supply is

$$U'_K = V'(Ku(x))\left[u(x) - u'(x)x\right] - Kx \cdot \frac{\partial p}{\partial K} + \frac{\partial \pi}{\partial K}.$$

The first and second terms are the same to worker's utility. The third term corresponds to the change in the agent's income that decreases with the capital supply (23). Under a more natural IED case, the first and second terms are positive, whereas the third one is negative. Under a DED case, only first term is positive. So it is more likely that the utility of capital owners increases under the IED case. But, in general, an increase as well as a decrease in capitalists' utility can occur.

We conclude that different effects can take place with a change from autarky to free trade, depending on whether demands belong to the IED or DED class. In the next subsection, we will see that similar effects arise in the case of trade with non-zero finite transportation costs, although any effect arising from additional capital supply in a country is typically *softened* by the existence of its trade partner.

3.2 Trade: impact of capital asymmetry and globalization

Having compared autarky and integration, we now study the trade equilibrium under non-trivial trade cost $1 < \tau < \infty$. We produce comparative statics of consumption levels, prices, firm sizes, and capital prices with respect to two key parameters: the asymmetry in capital endowments and trade cost.

3.2.1 Individual consumptions

To compare the consumption of Home and Foreign varieties, we analyze the monotonicity of the expressions in our equilibrium system (15) and (16). We argue in three steps to get inequality (24) below, using the following conclusions.

(i) Individual consumption of a domestically produced variety in each country is higher than the consumption of any imported variety ($x^{HH} > x^{FH}$, $x^{FF} > x^{HF}$) because, in this model, *various competition effects never outweigh the downward pressure of trade costs* on import consumption.

(ii) Consumption of a domestic variety is smaller in the country with a higher capital endowment ($x^{FF} > x^{HH}$) because *each consumer splits his or her expenditure among a greater mass of varieties.*

(iii) It is obvious that $x^{HH} > x^{HF}$ when the countries are symmetric. Moreover, it remains true even for highly asymmetric capital (when λ is close to 1). Indeed, at the limiting case $\lambda = 1$ (no capital in Foreign), the differentiated goods are produced only in the Home country. As the price for Foreign consumers includes trade costs, we have $x^{HH} > x^{HF}$. On the other hand, it follows immediately from the above results for a closed economy that x^{HH} (x^{HF}) decreases (increases) with λ for all $\lambda \in [1/2,\ 1]$. Hence, regardless of the countries' asymmetry in capital,

$x^{HH}(\lambda) > x^{HF}(\lambda)$.

All these inequalities and other properties of equilibrium consumption can be summarized as follows.

Proposition 2. *(i) Under asymmetry $\lambda > 0.5$, the equilibrium individual consumption of the varieties is ordered as*

$$x^{FF} > x^{HH} > x^{HF} > x^{FH}. \tag{24}$$

(ii) An increasing share λ of Home capital or/and total world capital makes the consumption of both domestic and imported varieties in Home decrease:

$$\frac{dx^{HH}}{d\lambda} < 0, \quad \frac{dx^{FH}}{d\lambda} < 0,$$

$$\frac{dx^{HH}}{dK} < 0, \quad \frac{dx^{FH}}{dK} < 0. \tag{25}$$

(iii) Trade liberalization hampers the consumption of any domestic variety and enhances the consumption of imports, whereas increasing trade costs work in the opposite fashion:

$$\frac{dx^{ii}}{d\tau} > 0, \quad \frac{dx^{ij}}{d\tau} < 0.$$

Proof: See Appendix B.

Therefore, the analysis of the influence of globalization produces no surprises: *the domestic varieties are crowded out by the imported varieties* that become cheaper. Unlike endogenous capital settings, in this model, such an effect occurs even without changes in variety: the range of goods remains the same, but the cost decrease per se is sufficient for crowding. Statement (iii) above also describes crowding: the more competitors there are, the less market share remains for others.

3.2.2 Prices and dumping

Using Proposition 2 and pricing rule (9), (10), we can compare prices and characterize the price behavior of producers in each country.

To discuss this question, we shall introduce the following definition: *dumping* practice by any firm means that its mill price times the trade coefficient exceeds its export price:

$$p^{ii} > \frac{p^{ij}}{\tau},$$

whereas the opposite inequality is called reverse dumping.

Proposition 3. *Domestic varieties are always cheaper than imported ones ($p(x^{ii}) < p(x^{ji})$), and (considering the trade pass-through) three pricing patterns are possible:*

(i) Increasingly elastic demand (IED) yields dumping pricing practiced by Home and Foreign firms, and the dumping by Foreign firms is stronger:

$$p^{FF} > p^{HH} > \frac{p^{HF}}{\tau} > \frac{p^{FH}}{\tau}; \tag{26}$$

(ii) Decreasingly elastic demand (DED) yields reverse dumping used by each firm, and the reverse dumping by Foreign firms is stronger:

$$p^{FF} < p^{HH} < \frac{p^{HF}}{\tau} < \frac{p^{FH}}{\tau}; \tag{27}$$

(iii) Firms in both countries relax dumping (reverse dumping) under trade liberalization. Furthermore, firm in each country weaken dumping and/or reverse dumping in response to an increase in the country's capital share.

Corollary. *Iso-elasic demand (CES) implies proportional export pricing ($p^{ii} = p^{ij}/\tau$).*

Proof: See Appendix D.

Hence, *all Home and Foreign firms adopt the same pricing behavior,* which in the IED situation (the most realistic) amounts to *dumping*. And, in all situations, *the smaller the country, the greater the distortion of its export price.*

To illustrate how (reverse) dumping is enforced or hampered by the trade cost and countries' asymmetry, we consider a numerical example where world capital $K = 1$ and world population $L = 10$. The upper-tier utility is $V(m) = \log m$ and the elementary utility is AHARA: $u(x) = (ax + b)^\rho - b^\rho + lx$.

Figures 1a and 1d show dumping (mill domestic price p^{ii} greater than import price p^{ij} for firms in both countries) because the utility $u(x) = 8\sqrt{x} - \frac{2}{5}x$ ($a = 64$, $b = 0$, $l = -2/5$) here generates an increasingly elastic demand (IED). Similarly, the second line of graphs (figures 1b and 1e) shows the difference in pricing strategies increasing with asymmetry and trade cost; moreover, the effects become stronger for the smaller country. But now reverse dumping takes place, because the utility $u(x) = 8\sqrt{x} + \frac{2}{5}x$ ($a = 64$, $b = 0$, $l = 2/5$) belongs to DED class. In contrast, CES class would generate no effects.

Finally, figures 1c and 1f correspond to the case of non-monotone demand elasticity. Both countries may demonstrate the opposite patterns of dumping (or reverse-dumping) behavior. In those figures, we plot graphs for utility function $u(x) = 4\left[\left(x + \frac{1}{200}\right)^{\frac{1}{4}} - \left(\frac{1}{200}\right)^{\frac{1}{4}}\right] + \frac{2}{5}x$ that demonstrates the first IED property (for small x), and then the DED property. In figure 1c, under $\tau < \tau_0 = 2.41$, the equilibrium price behavior shows reverse dumping. With the trade cost between $\tau_0 < \tau < \tau_1 = 2.54$, the producers from Home (which has a larger capital stock and therefore accommodates more firms) practice dumping, whereas Foreign producers practice reverse dumping. When trade costs

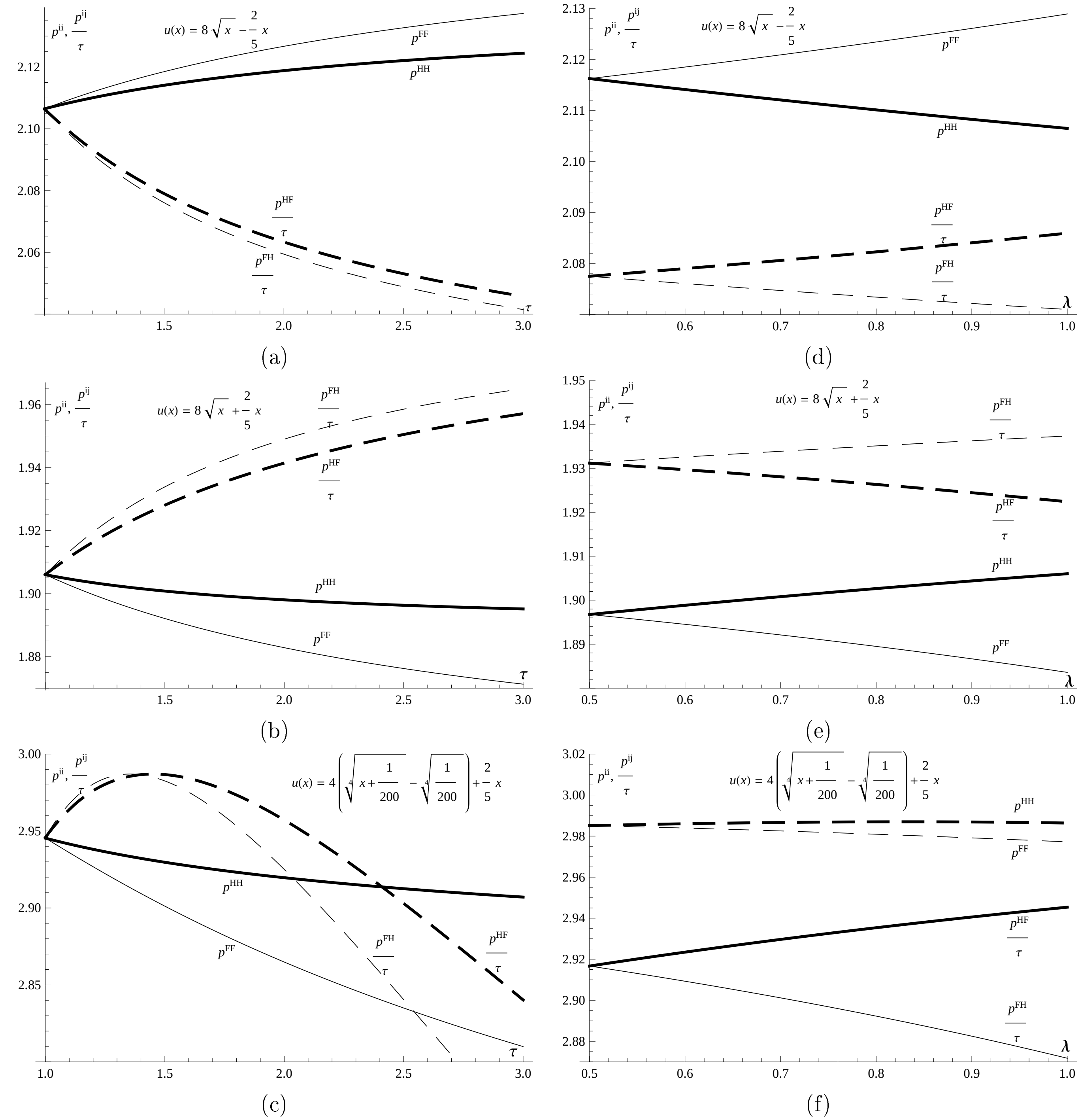


Figure 1: The (reverse) dumping effects depending on elasticity of demand: under $\lambda = 0.6$ [(a) IED case; (b) DED case; (c) non-monotone elasticity of demand]; under $\tau = 1.5$ [(d) IED case; (e) DED case; (f) non-monotone elasticity of demand].

are fairly high ($\tau > \tau_1$), producers from both countries practice dumping.

To sum up, *the pricing patterns chosen by firms depend critically on variable elasticity of substitution* in a way that differs greatly from what we know of the CES-utility case, where non-trivial market segmentation cannot arise.

Thus, in contrast to the conventional wisdom adopted in trade literature, cooperative behavior of the exporters is not the only possible source of the non-proportional pricing. Instead, the demand elasticities and the difference in mill prices (cheaper manufacturing in capital-rich countries) may explain various dumping and reverse dumping effects. (For a similar effect and conclusion in a one-factor trade model see Zhelobodko et al., 2010.)

Now we extend our conclusions on the behavior of equilibrium prices. The following proposition yields a full characterization of prices' comparative statics with respect to λ and τ.

Proposition 4. *(i) Trade liberalization induces a* ***decrease*** *(increase) in the price p^{ii} of any domestic variety under* ***IED*** *(DED), whereas price p^{ij} of any imported variety decreases under DED (remaining ambiguous under IED):*

$$IED \Rightarrow \frac{dp^{ii}}{d\tau} > 0; \quad DED \Rightarrow \frac{dp^{ii}}{d\tau} < 0; \quad DED \Rightarrow \frac{dp^{ij}}{d\tau} > 0.$$

(ii) Growing a country's share of capital (λ for Home, $(1-\lambda)$ for Foreign) makes its prices p^{ii},p^{ji} of domestic and imported goods ***decrease*** *(increase) under* ***IED*** *(DED), in particular,*

$$\text{Under IED: } \frac{dp^{HH}}{d\lambda} < 0, \ \frac{dp^{FH}}{d\lambda} < 0, \ \frac{dp^{FF}}{d\lambda} > 0 \text{ and } \frac{dp^{HF}}{d\lambda} > 0; \tag{28}$$

$$\text{Under DED: } \frac{dp^{HH}}{d\lambda} > 0, \ \frac{dp^{FH}}{d\lambda} > 0, \ \frac{dp^{FF}}{d\lambda} < 0 \text{ and } \frac{dp^{HF}}{d\lambda} < 0. \tag{29}$$

(iii) With an increase in total world capital K, all prices in each country shift in the same direction as reactions (28)-(29) to the country's capital share.

Note that the case of CES preferences is the borderline one between increasing and decreasing elasticity of demand, so any price effects are absent, which contradicts the data.

The reasoning behind point (ii) of Proposition 3 is as follows. An increase in λ invites more firms to enter the Home market, whereas the Foreign country accommodates fewer firms. Consequently, the mass of Home- (Foreign-) produced varieties increases (decreases). Thus, love for variety shifts x^{HH} and x^{FH} downward. Under IED (DED), this makes varieties better (worse) substitutes, and therefore competition on the Home market becomes tougher (weaker). As a result, both p^{HH} and p^{FH} go down (up). With symmetry, the other two prices go in the opposite direction. (For a similar explanation of IED/DED price effects in a closed economy, see Zhelobodko et al., 2012).

As for point (i) of Proposition 5, trade liberalization shifts all domestic prices downward (upward) under IED

(DED). In the former case, the dominant effect works as follows: an increase in competitive pressure from Foreign firms forces local firms to decrease prices. At the same time, prices for the imported varieties, on the one hand, decrease under trade liberalization (*direct import-price effect*). On the other hand, however, this increases demand for imported varieties, which implies that importers acquire more market power and can charge higher markups. This is the *indirect import-price effect*. However, economic intuition suggests that imported prices decrease with trade liberalization. In the latter case (DED), the two effects go in *the same* direction, in other words, imported prices unambiguously decrease and domestic prices increase.

Figure 1 above illustrates price behavior with respect to trade costs and asymmetry in capital endowment between countries. Figure 1a (IED case) shows import prices decreasing with trade liberalization.

To find some empirical justification, we would propose that the price effects discussed above could explain manufacturing price differentials between the developed and developing countries, given that capital is immobile. From this viewpoint, the results about prices obtained above mean that *developed countries should have cheaper high-tech goods than less-developed countries*, with this difference decreasing with globalization. An alternative interpretation of K parameter could be either human capital or skilled labor supply. Of course, such a tendency is not necessarily evidenced in reality, shadowed by other tendencies. A plausible reason for this is the noticeable wage differential between North and South. In our model, the forces generating this wage differential are ruled out by the wage-equalization mechanism, which is widely used in the literature.

3.2.3 Capital price, firm size, and trade flows

In this subsection, we study the impact of asymmetry in countries' capital endowments on capital prices, outputs, and trade flows. Our analysis bears some resemblance to the standard Heckscher-Ohlin story. However, the monopolistic competition approach allows us to highlight new facets of the problem, which are inevitably ruled out under perfect competition.

For convenience, let e^i stand for total exports of manufacturing good from country i:

$$e^H = \lambda K \cdot (1-\theta)L \cdot p^{HF} x^{HF}, \tag{30}$$

$$e^F = (1-\lambda)K \cdot \theta L \cdot p^{FH} x^{FH}. \tag{31}$$

With the agricultural sector serving as an equalizer, the two trade values above need not balance each other. Therefore, we can find who exports more and where the capital price is higher. Studying expressions (30)-(31) and (13)-(14), we can compare the equilibrium capital prices, export volumes, and firm sizes in the two countries. However, from now on we shall distinguish gross firm sizes $q^H \equiv \theta L \cdot x^{HH} + \tau(1-\theta)L \cdot x^{HF}$ measured in physical costs from net firm sizes $y^H \equiv \theta L \cdot x^{HH} + (1-\theta)L \cdot x^{HF}$ measured in outputs which do not include trade costs.

Proposition 5. *(i) When the countries are symmetric in terms of population ($\theta = 1/2$), the country with capital abundance (Home) has a lower capital price π^H and a higher value of exports in manufacturing e^H:*

$$\pi^H < \pi^F, \quad e^H > e^F.$$

(ii) Assume that $u'''(x) > 0$ and $r_{u''}(x) < 3$. Then $q^H < q^F$ and $y^H < y^F$.

Corollary. *Home exports in physical units exceed those of Foreign: $\lambda K \cdot \frac{L}{2} \cdot x^{HF} > (1-\lambda)K \cdot \frac{L}{2} \cdot x^{FH}$.*

Proof: See Appendix E.

Why such inequalities? The market-crowding effect is at work here, whereas the market-access effect is eliminated by our assumptions of quasi-linear utility and similar populations in Home and Foreign. Low output q^H at Home is the consequence of the market-crowding effect[4]. Intuitively, a low capital price at Home is implied by the larger capital supply. A low output by firms, q^H, does not allow the firms to get the benefits from increasing returns to scale. This leads to a decrease in the Home capital price, which is reinforced by tougher competition in the market for capital. More intriguing is the fact that, despite the low q^H, total exports of manufacturing goods from Home are higher. This result has at least two justifications. First, there are more firms at Home. Second, market-crowding effect at the Foreign market is weaker than at Home. Thus partial specialization of countries takes place: the Foreign country becomes more agricultural and the Home country becomes more industrial. Moreover, capital abundance at Home increases the exports from Home and decreases its imports making the world less symmetric.

3.2.4 Firm size under trade liberalization

We now turn to studying how trade liberalization (i.e., a reduction in τ) affects gross firm sizes q^H, q^F and net firm sizes y^H, y^F measured in outputs net of trade costs. We argue that usual interpretation of variables q^i as outputs is not quite realistic. It would mean, that firms do pay for transportation with its production and, thereby, artificially overestimate the real output. Instead, y shows what is really produced and consumed.

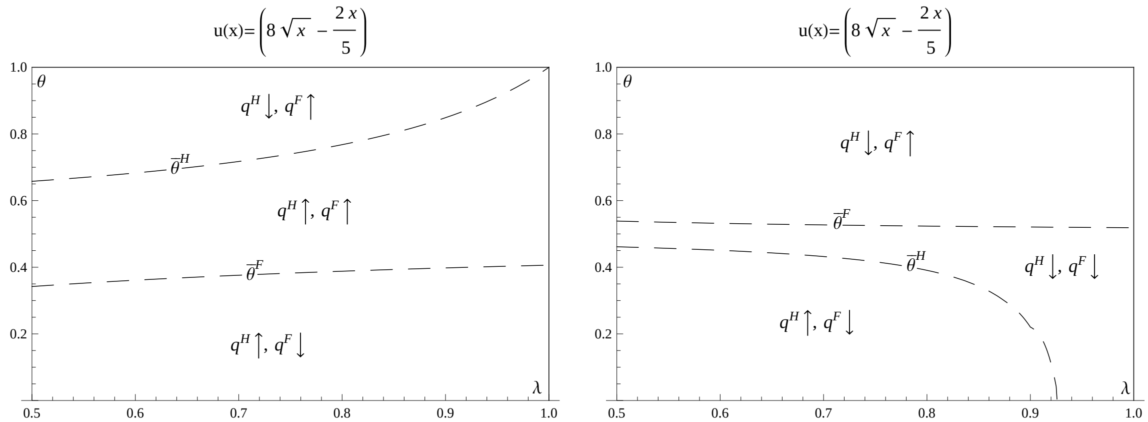
The gross size of a typical Home firm is given by

$$q^H = \theta L x^{HH} + \tau(1-\theta)L x^{HF}.$$

To disentangle the main forces that are at work with a decrease in τ, we decompose dq^H as follows:

$$dq^H = \theta L\, dx^{HH} + \tau(1-\theta)L\, dx^{HF} + (1-\theta)L x^{HF}\, d\tau \tag{32}$$

[4]Additional assumptions for statement $q^H < q^F$ are just technical, satisfied for typical utilities. For instance, AHARA: $u(x) = (x+d)^\rho - d^\rho + lx$ ($\rho < 1$, $d > 0$) yields $u'''(x) = \rho(\rho-1)(\rho-2)(x+d)^{\rho-3} > 0$.



(a) (b)

Figure 2: Firm size behavior: (a) $\bar{\theta}^H(\lambda) < \bar{\theta}^F(\lambda)$; (b) $\bar{\theta}^H(\lambda) > \bar{\theta}^F(\lambda)$.

The first term in (32) is unambiguously negative: trade liberalization leads to a reduction in x^{HH} because of tougher competition with foreign firms. This is the standard *competition effect*.

The second term in (32) is positive: a reduction in trade costs leads to an increase in trade flow. This term can be interpreted as a measure of the *market access effect*.

Finally, the third term in (32) is negative, for $d\tau < 0$. This term arises because lower trade costs mean that firms have to produce less in order to export the same amount. Stated another way, a decrease in τ triggers the *iceberg trade cost effect*.

Comparative statics of firm size with respect to τ depends on whether the market-access effect dominates the other two effects, given the relative country size characteristics θ and λ. The following proposition describes the behavior of gross firm sizes under almost free trade, i.e., when τ is close to one.

Proposition 6. *Assume that trade costs are low, i.e., $\tau \approx 1$. There then exist two threshold values of θ, $\bar{\theta}^H(\lambda)$ and $\bar{\theta}^F(\lambda)$, such that:*

(i) q^H increases with trade liberalization if and only if population share $\theta < \bar{\theta}^H$;

(ii) q^F increases with trade liberalization if and only if population share $\theta > \bar{\theta}^F$;

(iii) the sign of $\bar{\theta}^H(\lambda) - \bar{\theta}^F(\lambda)$ is the same for all $\lambda \in [1/2, 1]$.

Proof : See Appendix F.

Figure 2 illustrates this proposition. In particular, figure 2a was built for upper-tier utility $V(m) = \sqrt{m}$ and figure 2b for $V(m) = \ln(m)$. Both examples use lower-tier utility $u = (ax)^{\rho} \pm lx$, $K = 1$, and $L = 10$. The example proves that all patterns exist: both firm sizes can grow or fall or go in the opposite directions.[5]

Note that log-over-CES preferences yield a limiting case: $\bar{\theta}^H(\lambda) = \bar{\theta}^F(\lambda) = 1/2$ for all λ. This happens because the total expenditures on differentiated products in the countries are proportional to the countries' populations (see

[5] When $\varepsilon_{MR} < -1$, we also obtain a limiting case: thresholds in Figure 2b emerge from our square ($\bar{\theta}^H < 0$ and $\bar{\theta}^F > 1$) and we observe only one pattern when both outputs decrease under trade liberalization.

Appendix F). Hence, the market-access effect dominates the two negative effects triggered by trade liberalization if, and only if, $\theta > 1/2$.

The above analysis was conducted for low trade costs that are close to zero ($\tau \approx 1$). However, using simulations, we have found that the same patterns are in fact *robust* to fairly wide variations of $\tau \in [1, 1.25]$. See Appendix H for a number of examples.

Several comments and interpretations are in order.

First, Proposition 6 essentially says that trade liberalization results in a decrease (increase) in the gross size of firms in a country if the population of this country is sufficiently large (small), exceeding the threshold. The reason is that for firms based in a small country, the market-access effect generates large gains, which dominates the losses resulting from competition and transportation cost effects. Hence, firm sizes increase. For a large country, the argument is reversed. Why the firms located in the country with the higher population reduce their output in response to a decrease in trade costs? On one hand, trade liberalization makes access to the foreign market easier, and they increase output to serve it. On the other hand, output for local consumption decreases due to tougher competition between local and foreign firms. Since the local market is bigger, the decrease in total domestic sales volume exceeds the increase of export volume; therefore, the total sales volume decreases.

Second, it follows immediately from Proposition 6 that, when the population share θ is between the two threshold values (i.e., the population differential between the two countries is relatively small), a decrease in τ shifts q^H and q^F in the same direction. However, firm sizes increase or decrease depending on the sign of $\bar{\theta}^H(\lambda) - \bar{\theta}^F(\lambda)$, which is the same for all λ according to part (iii).[6]

When the countries' populations are close in size to each other, the only difference between cases (a) and (b) in Figure 2 is the output behavior. Case (a) – when both firm sizes increases with trade liberalization – seems more natural. So what is the reason for the reduction in the size of firms in case (b)? Apparently, such a surprising outcome is due mainly to the iceberg trade cost effect. In essence, variables q^i describe gross outputs which would be true if a firm payed for transportation with its production and thus the transporter were a "third country" consuming the commodity alike Home and Foreign. Reduction of this third consumption under globalization is the explanation of surprising reduction in q^i.

Let us get rid of this effect and show the effect of globalization on *net firm sizes* y^H and y^F, which do not include transportation costs:

$$y^H = \theta L x^{HH} + (1-\theta) L x^{HF}.$$

Proposition 7. *Assume that trade costs are low, i.e.,* $\tau \approx 1$. *There then exist two threshold values of* θ, $\tilde{\theta}^H(\lambda)$ *and*

[6] In Appendix F we derive explicit formulas for $\bar{\theta}^H(\lambda)$, $\bar{\theta}^F(\lambda)$, which makes it easy to sketch the plots.

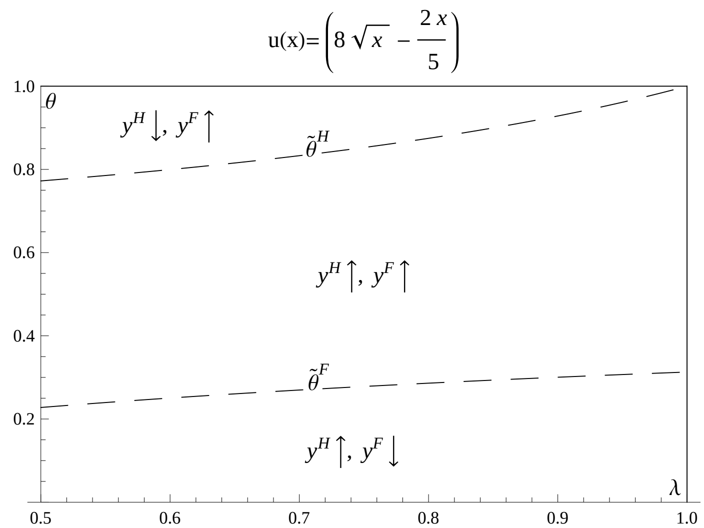


Figure 3: Net firm size behavior.

$\tilde{\theta^F}(\lambda)$, such that:

(i) y^H increases with trade liberalization if and only if population share $\theta < \tilde{\theta^H}$;

(ii) y^F increases with trade liberalization if and only if population share $\theta > \tilde{\theta^F}$.

Proof : See Appendix F.

These findings on net firm sizes are shown in Figure 3, using the same example as Figure 2b.

Here one can see that under sufficiently different countries similarity with gross outputs: for the larger country competition effect dominates but for smaller country market access effect is stronger. However, unlike gross firm sizes, no surprising effects occur under almost equal countries: globalization naturally increases outputs.

One more question of interest is whether trade liberalization eliminates or intensifies dissimilarities between firms in different countries. A possible measure of firm dissimilarities is the differential firm size $(q^H - q^F)$. We find that the difference between the sizes of firms does not depend on upper-tier utility and increases (decreases) when $\varepsilon_{MR} > -1$ $(\varepsilon_{MR} < -1)$. It is easily shown that, even in one given class of familiar lower-tier utility functions (CARA, HARA, quadratic utility), both opportunities can take place: the differential can grow or fall. However, if the lower-tier utility is of the CES type, then $\varepsilon_{MR} = \rho - 1 > -1$; the differential increases.

We conclude that the variable elasticity of substitution is important for outputs as well as for prices, but CES is not a borderline between different patterns.

3.2.5 Capital price under trade liberalization

In this subsection, we analyze capital price behavior under trade liberalization, proceeding in the same way as we studied firm size behavior.

The capital price in Home is given by

$$\pi^H = \theta L(p^{HH} - 1)x^{HH} + (1-\theta)L(p^{HF} - \tau)x^{HF}.$$

Again, we want to disclose the main effects that a decrease in τ triggers. To do this, we decompose $d\pi^H$ as follows:

$$d\pi^H = \theta L\, d\left[(p^{HH} - 1)x^{HH}\right] + (1-\theta)L(p^{HF} - \tau)\, dx^{HF} - (1-\theta)Lx^{HF}\, d\tau + (1-\theta)Lx^{HF}\, dp^{HF} \tag{33}$$

Here we have four effects: three are the same as in the story about firm sizes, and the fourth is a new effect. The first term in (33) is unambiguously negative: trade liberalization leads to a reduction in operating profits from local markets because of tougher competition with foreign firms. This is the standard *competition effect*.

The second term in (33) is positive: a reduction in trade costs leads to an increase in trade flows. This term can be interpreted as a measure of *market access effect*.

The third term in (33) is positive, for $d\tau < 0$. This term arises because lower trade decreases the firm's transportation costs, increasing the firm profit. It is the *iceberg trade cost effect*

Finally, the last term in (33) is positive under DED and could be positive or negative under **IED**. First, trade liberalization immediately decreases the import price, which we call *direct import-price effect*. Second, this increases the individual consumption for imported varieties that, under the IED case, increases import price. This is *indirect import-price effect*.

Capital price behavior under trade liberalization is determined by a trade-off among the four effects named above, given the relative country size characteristics θ and λ. The following proposition contains a full characterization for comparative statics of capital prices when τ is close to one.

Proposition 8. *Assume that trade costs are low, i.e., $\tau \approx 1$. There then exist two threshold values of θ, $\hat{\theta}^H(\lambda)$ and $\hat{\theta}^F(\lambda)$, such that:*

(i) π^H increases with trade liberalization if and only if $\theta < \hat{\theta}^H$;

(ii) π^F increases with trade liberalization if and only if $\theta > \hat{\theta}^F$;

(iii) the sign of $\hat{\theta}^H(\lambda) - \hat{\theta}^F(\lambda)$ is the same for all $\lambda \in [1/2,\ 1]$.

Proof: *See Appendix I.*

We illustrate Proposition 8 in Figure 4 with our examples when the upper-tier utility is $V(m) = log(m)$ and the lower-tier utility is $u = \sqrt{x} \pm \frac{2}{5}x$, $K = 1$ and $L = 10$.

Two comments are in order. First, firms located in the country with the larger population are worse off after trade liberalization: the competition effect on the large local market exceeds the market access effect, for the foreign market is much smaller. Again, the two patterns differ in the behavior of capital prices in the case of countries with almost symmetric populations ($\theta \approx 1/2$).

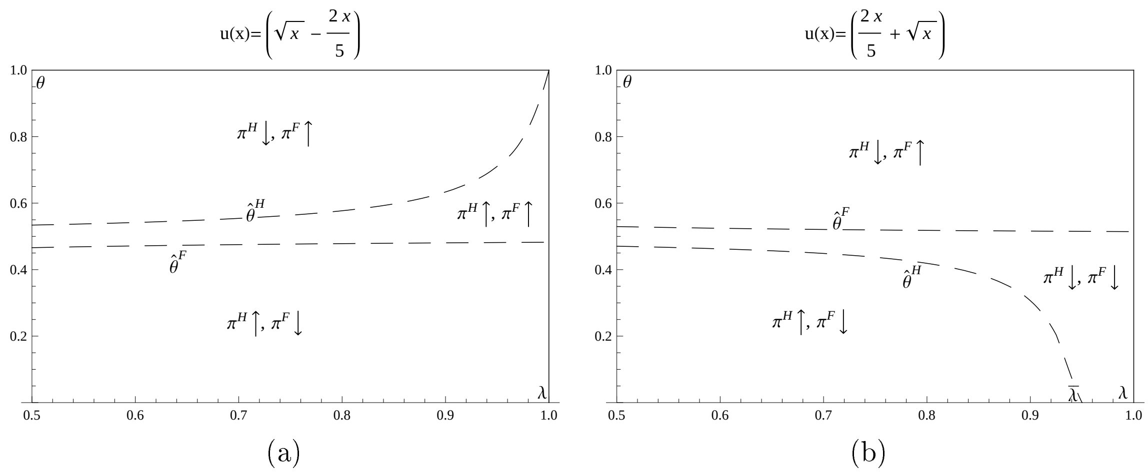


Figure 4: Capital prices behavior : (a) $\hat{\theta}^H < \hat{\theta}^F$, (b) $\hat{\theta}^H > \hat{\theta}^F$.

Second, under relatively same populations we observe the same patterns as for firm sizes, i.e., capital price goes in the same direction in both countries (either decreases or increases). Under trade liberalization firms become better off because of market-access effect and import-price effect. At the same time, firm profits decline because of competition and transportation cost effects. Under fairly same country's populations it seems natural that those four effects almost cancel out each other, and both patterns look fairly natural.

Again, log-over-CES preferences yield a limiting case: $\hat{\theta}^H(\lambda) = \hat{\theta}^F(\lambda) = 1/2$ for all λ. We already mentioned that this feature arises because the total expenditures on differentiated product in the countries are proportional to the countries' populations. So, in this particular case, the market-access effect and transportation effect dominate the negative competition effect (under CES preferences, there is not an import-price effect), triggered by trade liberalization if and only if $\theta > 1/2$.

As in Section 3.2.4, we formulate the proposition for low trade costs close to zero ($\tau \approx 1$) but use a simulation to show that the results are *robust* to fairly wide variations of τ. See Appendix K for a number of examples.

We also study the capital price differential across countries, which is given by

$$\frac{\partial(\pi^H - \pi^F)}{\partial \tau} = (2\theta - 1)Lx.$$

Clearly, the capital price differential decreases with trade liberalization if and only if Home country has both a larger population and a larger capital endowment. One notes that the higher the asymmetry in population size, the faster the difference in capital price decreases in τ.

One might conjecture that, under assumptions of footloose capital, we shall observe a Home Market Effect independent of asymmetry in market and industry size, as well as a form of upper- and low-tier utilities.

4 Conclusion

We develop a new two-factor, two-sector trade model in order to capture the impact of countries' asymmetry in capital and population, as well as variable markups, on trade patterns. The novelty of the approach is combining the Heckscher-Ohlin methodology based on disparities in factor endowments with monopolistic competition. To endogenize markups, we use non-specified quasi-linear utilities that can generate both increasingly and decreasingly elastic demands (IED and DED). We find that, when a country switches from autarky to integration into the free trade world, then under IED (DED), the equilibrium commodity price decreases (increases) with capital supply.

We next examine the implications of globalization in a two-country world with positive finite trade costs. In the model, the Home country has a larger endowment of capital than does the Foreign. The basic result for the closed economy case has several important implications for trade.

First, the domestic variety is always cheaper than the imported one; prices in the Home country are lower (higher) than in the Foreign country under IED (DED), and, firms located in both countries practice (reverse) dumping under IED (DED), both policies reinforced by scarcity of capital. All these differences increase with trade cost and capital asymmetry. Iso-elastic demands generated by the CES utility function are the borderline case, in which the model exhibits degenerate behavior.

Second, globalization triggers two opposing effects: the Foreign market becomes more easily available to Home producers (market-access effect), whereas Home market becomes available to foreigners (competition effect). As a result, firms may face both gains and losses, depending on which of the two effects dominates. The outcome differs for cases of very asymmetric and almost symmetric populations in the countries. When asymmetry is high, the market-access effect unambiguously dominates the competition effect in a country with much lower population, because firms get better access to the much larger foreign market. Hence, the capital price in this lower-population country always increases. The behavior of the capital price in the other country is the reverse. However, we also find that, when countries are almost symmetric in population, trade liberalization shifts capital prices in both countries in the same direction. Whether capital prices increase or decrease depends on which of the two effects suppresses the other. The pattern for the size of firms is quite similar and follows the same logic.

Third, depending on the interpretation of production factors, which need not necessarily be treated as capital and labor, we can use our results to explain the impact of trade liberalization on a broad variety of economic phenomena, including globalization-driven shifts in the returns on human capital, the structure of wages paid to different kinds of labor, etc.

Finally, the country with the higher capital-population ratio is a net exporter of manufacturing goods, i.e., there is partial specialization in the countries and home market effect is observed.

These findings highlight the importance of variable markups in international trade studies. Possible extensions include partially substitutable labor and capital, general equilibrium settings, and footloose capital.

References

[1] Baldwin, R., Forslid, R., Martin, P., Ottaviano G.I.P., Robert-Nicoud F. 2003. *Economic Geography and Public Policy*. Princeton University Press, Princeton, NJ.

[2] Behrens, K., Murata, Y. 2007. General equilibrium models of monopolistic competition: A new approach. *Journal of Economic Theory* 136(1): 2007, 776-787.

[3] Bernard, A.B., Jensen, J.B., Redding, S., Schott, P.K. 2007. Firms in international trade. CEP Discussion Paper No. 795.

[4] Bertoletti, P., Epiphani, P. 2012. Consumer behavior, monopolistic competition, and international trade. Pavia University, Pavia, Italy, and Bocconi University, Milan, Working Paper.

[5] Brakman, S., Heijdra, B.J. 2004. *The Monopolistic Competition Revolution in Retrospect*. Cambridge University Press, , Cambridge, MA.

[6] Brander, J., Krugman, P.R. 1983. A reciprocal dumping model of international trade. *Journal of International Economics* 15(3): 313–321.

[7] Campbell, J.R., Hopenhayn, H.A. 2005. Market size matters. *Journal of Industrial Economics* 53(1): 1-25

[8] Davis, D.R., Weinstein, D.E. 1999. Economic geography and regional production structure: An Empirical investigation. *European Economic Review* 43(2): 379–407.

[9] Dinopoulos, E., Fujiwara, K., Shimomura, K. 2011. International trade and volume patterns under quasilinear preferences. *Review of Development Economics* 15(1): 154–167.

[10] Dixit, A.K., Stiglitz J.E. 1977. Monopolistic competition and optimum product diversity. *American Economic Review* 67, 1977, 297-308.

[11] Feenstra, R.C. *Advanced international trade: theory and evidence*, Princeton University Press, Princeton, NJ.

[12] Greenhut, M.L., Ohta, H., Sailors, J. 1985. Reverse dumping: A form of spatial price discrimination. *Journal of Industrial Economics* 34(2): 167–181.

[13] Grossman, G., Helpman, E. 1994. Protection for Sale. *American Economic Review* 84: 833–50.

[14] Hallak, J.C. 2006. Product quality and the direction of trade. *Journal of International Economics* 68(1): 238–265.

[15] Hallak, J.C., Schott, P. 2008. Estimating cross-country differences in product quality. NBER Working Paper No. 13807.

[16] Helpman, E., Krugman, P.R. 1987. *Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition, and the International Economy.* MIT Press, Cambridge, MA.

[17] Hummels, D., Klenow, P.J. 2005. The variety and quality of a nation's exports. *American Economic Review* 95(3): 704–723.

[18] Krugman, P. 1979. Increasing returns, monopolistic competition, and international trade. *Journal of international economics* 9(4): 469-479.

[19] Krugman, P. 1980. Scale Economies, Product Differentiation, and the Pattern of Trade. *American Economic Review* 70(5): 950–959.

[20] Krugman, P. 1981. Intraindustry Specialization and the Gains from Trade. *Journal of Political Economy* 89(5): 959-973.

[21] Manova, K., Zhang, Z. 2009. Quality heterogeneity across firms and export destinations (*now entitled* Export prices across firms and destinations). NBER Working Paper No. 15342.

[22] Martin, J. 2009. Spatial price discrimination in international markets. CREST-INSEE mimeograph.

[23] Martin, P., Rogers, C.A. 1995. Industrial location and public infrastructure. *Journal of International Economics* 39(3-4): 335–351.

[24] Mas-Colell, A. 1990. *The Theory of General Economic Equilibrium: A Differentiable Approach.* Econometric Society Monograph Vol. 9. Cambridge University Press, Cambridge, MA.

[25] Melitz, M., Ottaviano, G. 2008. Market Size, Trade, and Productivity. *Review of Economic Studies* 75: 295–316.

[26] Melitz, M. J., Redding, S. J. 2012. Heterogeneous Firms and Trade. NBER Working Paper No. 18652.

[27] Mrazova, M., Neary J.P. 2012. Selection Effects with Heterogeneous Firms. CEP Discussion Paper No 1174.

[28] Ottaviano, G. 2001. Monopolistic competition, trade, and endogenous spatial fluctuations. *Regional Science and Urban Economics* 31(1): 51–77.

[29] Ottaviano, G., Tabuchi, T., Thisse, J.-F. 2002. Agglomeration and trade revisited. *International Economic Review* 43(2): 409–436.

[30] Pfluger, M. 2004. A simple, analytically solvable, Chamberlinian agglomeration model. *Regional Science and Urban Economics* 34(5): 565–573.

[31] Schott, P.K. 2001. Do Rich and Poor Countries Specialize in a Different Mix of Goods? Evidence from Product-Level U.S. Trade Data. NBER Working Paper No. 8492.

[32] Schott, P.K. 2004. Across-product versus within-product specialization in international trade. *The Quarterly Journal of Economics* 119(2): 647–678.

[33] Syverson, C. 2007. Prices, spatial competition and heterogeneous producers: An empirical test. *Journal of Industrial Economics* 55(2): 197–222.

[34] Takatsuka, H., Zeng, D.-Z. 2012. Mobile capital and the home market effect. *Canadian Journal of Economics* 45(3): 1062–1082.

[35] Vives, X. 1999. *Oligopoly Theory*. MIT Press. Cambridge, Mass.

[36] Zhelobodko, E., Kokovin, S., Parenti, M., Thisse, J.-F. 2012. Monopolistic competition: Beyond the constant elasticity of substitution. *Econometrica* 80: 2765–2784.

[37] Zhelobodko, E., Kokovin S., Thisse, J.-F. 2010. Monopolistic competition: beyond the CES. CEPR Discussion Paper No.7947.

5 Appendix.

5.1 Appendix A: producer's program

In all proofs below we need first and second order conditions of the producer's optimization program. This program of Home producer i is

$$(p_k^{HH}(x_k^{HH}, \mu^H) - 1)\lambda L x_k^{HH} + (p_k^{HF}(x_k^{HF}, \mu^F) - \tau)(1-\lambda) L x_k^{HF} - \pi^H \to \max_{x^{HH}, x^{HF}},$$

that could be rewritten as follows

$$(V'(m^H)u'(x_k^{HH}) - 1)\lambda L x_k^{HH} + (V'(m^F)u'(x_k^{HF}) - \tau)(1-\lambda) L x_k^{HF} - \pi^H \to \max_{x^{HH}, x^{HF}}.$$

The first order condition of the producer's program is:

$$\left\{\begin{array}{c} \lambda L V'(m^H)(u''(x_k^{HH})x_k^{HH} + u'(x_k^{HH})) - \lambda L = 0 \\ (1-\lambda)LV'(m^F)(u''(x_k^{HF})x_k^{HF} + u'(x_k^{HF})) - \tau(1-\lambda)L = 0 \end{array}\right. ,$$

or, after simplification,

$$\left\{\begin{array}{c} V'(m^H)u'(x_k^{HH})\left(1 + \frac{u''(x_k^{HH})x_k^{HH}}{u'(x_k^{HH})}\right) = 1 \\ V'(m^F)u'(x_k^{HF})\left(1 + \frac{u''(x_k^{HF})x_k^{HF}}{u'(x_k^{HF})}\right) = \tau \end{array}\right. \Leftrightarrow$$

$$\left\{\begin{array}{c} p(x_k^{HH})\left(1 - r_u(x_k^{HH})\right) = 1 \\ p(x_k^{HF})\left(1 - r_u(x_k^{HF})\right) = \tau \end{array}\right. \Leftrightarrow$$

$$\left\{\begin{array}{c} p(x_k^{HH}) = \frac{1}{1-r_u(x_k^{HH})} \\ p(x_k^{HF}) = \frac{\tau}{1-r_u(x_k^{HF})} \end{array}\right. .$$

The strict second order condition of the producer's program:

$$\left\{\begin{array}{c} \lambda L V'(m^H)(u''(x_k^{HH}) + u'''(x_k^{HH})x_k^{HH} + u''(x_k^{HH})) < 0 \\ (1-\lambda)LV'(m^F)(u''(x_k^{HF}) + u'''(x_k^{HF})x_k^{HF} + u''(x_k^{HF})) < 0 \end{array}\right. \Leftrightarrow$$

$$\left\{\begin{array}{c} u''(x_k^{HH})\left(2 + \frac{u'''(x_k^{HH})x_k^{HH}}{u''(x_k^{HH})}\right) < 0 \\ u''(x_k^{HF})\left(2 + \frac{u'''(x_k^{HF})x_k^{HF}}{u''(x_k^{HF})}\right) < 0 \end{array}\right. \Leftrightarrow$$

$$\left\{\begin{array}{c} 2 - r_{u'}(x_k^{HH}) > 0 \\ 2 - r_{u'}(x_k^{HF}) > 0 \end{array}\right. \Leftrightarrow$$

$$\left\{\begin{array}{c} r_{u'}(x_k^{HH}) < 2 \\ r_{u'}(x_k^{HF}) < 2 \end{array}\right. .$$

5.2 Appendix B: equilibrium equations and consumption

At equilibrium, the inverse demands equal the producer's optimal prices, i.e.,

$$V'\left[\lambda K u(x^{HH}) + (1-\lambda)Ku(x^{FH})\right]u'(x^{HH}) = \frac{1}{1 - r_u(x^{HH})}$$

$$V'\left[\lambda K u(x^{HH}) + (1-\lambda)Ku(x^{FH})\right]u'(x^{FH}) = \frac{\tau}{1 - r_u(x^{FH})}$$

$$V'\left[\lambda K u(x^{HF})+(1-\lambda)K u(x^{FF})\right]u'(x^{FF})=\frac{1}{1-r_u(x^{FF})}$$

$$V'\left[\lambda K u(x^{HF})+(1-\lambda)K u(x^{FF})\right]u'(x^{HF})=\frac{\tau}{1-r_u(x^{HF})}$$

Dividing the equations we get the system described in **Proposition 1**:

$$\frac{u'(x^{HH})(1-r_u(x^{HH}))}{u'(x^{FH})(1-r_u(x^{FH}))}=\frac{1}{\tau},$$

$$V'\left[\lambda K u(x^{HH})+(1-\lambda)K u(x^{FH})\right]u'(x^{HH})(1-r_u(x^{HH}))=1,$$

$$\frac{u'(x^{FF})(1-r_u(x^{FF}))}{u'(x^{HF})(1-r_u(x^{HF}))}=\frac{1}{\tau},$$

$$V'\left[\lambda K u(x^{HF})+(1-\lambda)K u(x^{FF})\right]u'(x^{FF})(1-r_u(x^{FF}))=1.$$

Proof of Proposition 2. Rewrite equation (15) as follows

$$\frac{u'(x^{HH})(1-r_u(x^{HH}))}{u'(x^{FH})(1-r_u(x^{FH}))}=\frac{1}{\tau}.$$

Defining marginal revenue as $MR\equiv u'(x)(1-r_u(x))=u'(x)+xu''(x)=(xu'(x))'=(xp)'$, the left hand side of the above equation is the ratio of marginal revenues. Under the second-order condition, the marginal revenue decreases with output. The right hand side of our equation is less then one ($\frac{1}{\tau}<1$). Thus, in Home country the individual consumption of local variety is bigger than individual consumption of the imported variety ($x^{HH}>x^{FH}$). The same argument for Foreign gives $x^{FF}>x^{HF}$.

Let us define two functions, y and G.

1. Imported consumption $z(x,\,\tau)$ as a function of domestic one is defined from equation

$$\frac{u'(x)}{u'(z(x,\,\tau))}=\frac{1}{\tau}\cdot\frac{1-r_u(z(x,\,\tau))}{1-r_u(x)};$$

   where $x=x^{HH}$, $z=x^{FH}$ or $x=x^{FF}$, $z=x^{HF}$.

2. Function $G(x,\,s,\,K,\,\tau)$ expressing marginal utility is defined as

$$G(x,\,\lambda,\,K,\,\tau)=V'(\lambda K u(x)+(1-\lambda)K u(z(x,\,\tau)))u'(x)(1-r_u(x)).$$

To we study the behavior z as a function of x, we differentiate the equation defining z with respect to x:

$$u''(x)(1-r_u(x)) - u'(x)r_u'(x) = \frac{1}{\tau}\cdot(u''(z)(1-r_u(z)) - u'(z)r_u'(z))\frac{\partial z}{\partial x} \Rightarrow$$

$$u'(x)\left(\frac{xu''(x)}{xu'(x)}(1-r_u(x)) - r_u'(x)\right) = \frac{1}{\tau}\cdot u'(z)\left(\frac{zu''(z)}{zu'(z)}(1-r_u(z)) - r_u'(z)\right)\frac{\partial z}{\partial x} \Rightarrow$$

$$\frac{u'(x)r_u(x)}{x}\left(1-r_u(x)+1+r_u(x)-r_{u'}(x)\right) = \frac{1}{\tau}\cdot\frac{u'(z)r_u(z)}{z}\left(1-r_u(z)+1+r_u(z)-r_{u'}(z)\right)\frac{\partial z}{\partial x}$$

$$\frac{u'(x)r_u(x)}{x}\left(2-r_{u'}(x)\right) = \frac{1}{\tau}\cdot\frac{u'(z)r_u(z)}{z}\left(2-r_{u'}(z)\right)\frac{\partial z}{\partial x} \Rightarrow$$

$$\frac{\partial z}{\partial x} = \tau\cdot\frac{u'(x)r_u(x)(2-r_{u'}(x))z}{u'(z)r_u(z)(2-r_{u'}(z))x} = \tau\cdot\frac{u''(x)(2-r_{u'}(x))}{u''(z)(2-r_{u'}(z))} > 0.$$

Thus, we have obtained positive dependence between the consumptions of local and imported varieties in each country.

Further, the derivative of z with respect to trade cost is found from

$$u'(x)(1-r_u(x)) = (u''(z)(1-r_u(z)) - u'(z)r_u'(z))\frac{\partial z}{\partial \tau} \Rightarrow$$

$$u'(x)(1-r_u(x)) = (u''(z)(1-r_u(z)) - u'(z)\frac{r_u(z)}{z}(1+r_u(z)-r_{u'}(z)))\frac{\partial z}{\partial \tau} \Rightarrow$$

$$u'(x)(1-r_u(x)) = \frac{u'(z)}{z}(-r_u(z)(1-r_u(z)) - r_u(z)(1+r_u(z)-r_{u'}(z)))\frac{\partial z}{\partial \tau} \Rightarrow$$

$$u'(x)(1-r_u(x)) = \frac{u'(z)}{z}(-r_u(z)+(r_u(z))^2 - r_u(z) - (r_u(z))^2 + r_u(z)r_{u'}(z)))\frac{\partial z}{\partial \tau} \Rightarrow$$

$$u'(x)(1-r_u(x)) = \frac{u'(z)}{z}(-2r_u(z) + r_u(z)r_{u'}(z)))\frac{\partial z}{\partial \tau} \Rightarrow$$

$$u'(x)(1-r_u(x)) = \frac{u'(z)r_u(z)}{z}(r_{u'}(z)-2)\frac{\partial z}{\partial \tau} \Rightarrow$$

$$u'(x)(1 - r_u(x)) = u''(z)(2 - r_{u'}(z))\frac{\partial z}{\partial \tau} \Rightarrow$$

$$\frac{\partial z}{\partial \tau} = \frac{u'(x)(1 - r_u(x))}{u''(z)(2 - r_{u'}(z))} < 0.$$

Now we turn to studying function $G(x, \lambda, K, \tau)$. Based on our findings, the argument of function $V'(\lambda K u(x) + (1-\lambda) K u(z(x, \tau)))$ strictly increases with individual consumption of local variety x, Home capital share λ, total capital endowment K. It also strictly decreases with trade cost τ. Then function $V'(\lambda K u(x) + (1-\lambda) K u(z(x, \tau)))$ strictly decreases with x, λ, K and strictly increases with τ. As we have seen, $u'(x)(1 - r_u(x))$ strictly decreases with x. Consequently function $G(x, \lambda, K, \tau)$ strictly decreases with λ, x and K and strictly increases with τ. Since function $G(x, \lambda, K, \tau)$ is strictly monotone with x there is at most one solution to the equation determining Home consumption:

$$G(x^{HH}, \lambda, K, \tau) = 1.$$

Based on the monotonicity of function G, the solution x^{HH} to this equation strictly decreases with λ and K and strictly increases with τ.

So, we find out that: (i) the consumption of domestic variety (x^{HH}, x^{FF}) decreases with country's capital share (λ or $(1-\lambda)$) and K and strictly increases with τ; (ii) the consumption of imported variety (x^{FH}, x^{HF}) decreases with country's capital share, K and τ.

Similarly, the solution of equation

$$G(x^{FF}, 1-\lambda, K, \tau) = 1.$$

determines the individual consumption of varieties in Foreign. Since function $G(x, \lambda, K, \tau)$ strictly decreases with λ, x and K and strictly increases with τ, the consumption of domestic varieties is less in Home country, posessing a bigger share of capital (i.e., $x^{FF} > x^{HH}$). Similar reasoning brings us to expression $x^{HF} > x^{FH}$.

In section 3.2.1 we have proved that $x^{HH} > x^{HF}$. Consequently there is the only sorting individual consumptions:

$$x^{FF} > x^{HH} > x^{HF} > x^{FH},$$

QED.

5.3 Appendix C: comparative statics of closed economy

We derive the comparative statics of Subsection 3.1 step by step, in the same order as its claims.

(i) **Consumption**. Differentiating equation (19) with respect to capital endowment K we get

$$\frac{dx}{dK} = -\frac{V''(Ku(x))u(x)u'(x)(1-r_u(x))}{KV''(Ku(x))\left(u'(x)\right)^2(1-r_u(x)) + V'(Ku(x))u''(x)(2-r_{u'}(x))} =$$

$$= \frac{\frac{r_V}{K}u'(x)(1-r_u(x))}{-\frac{r_V}{u(x)}\left(u'(x)\right)^2(1-r_u(x)) + u''(x)(2-r_{u'}(x))} = \frac{\frac{r_V}{K}(1-r_u(x))}{-\frac{r_V}{u(x)}u'(x)(1-r_u(x)) - \frac{r_{u(x)}}{x}(2-r_{u'}(x))} = -\frac{x}{K}\frac{r_V}{r_V\varepsilon_{u(x)} - \varepsilon_{MR}},$$

where $\varepsilon_{MR} = -\frac{r_u(x)(2-r_{u'}(x))}{1-r_u(x)} < 0$ is the elasticity of the marginal revenue.

(ii) **Price and demand elasticity.** We denote the expenditure on manufactured good as $E_m(K)$:

$$E_m(K) = Kp(x)x = \frac{Kx}{1-r_u(x)}.$$

First we differentiated function $r_u(x)$ with respect to x:

$$r_u'(x) = \left(-\frac{xu''(x)}{u'(x)}\right)' = -\frac{(u''(x)+xu'''(x))u'(x) - x(u''(x))^2}{(u'(x))^2} =$$

$$= \frac{-\frac{xu''(x)}{u'(x)} - \frac{xu'''(x)u''(x)x}{u''(x)u'(x)} + \left(\frac{xu''(x)}{u'(x)}\right)^2}{x} =$$

$$= \frac{r_u(x)}{x}(1 + r_u(x) - r_{u'}(x)).$$

The expenditure elasticity w.r.t. the capital endowment is

$$\varepsilon_{E_m}(K) = \frac{\frac{x}{1-r_u(x)} + \frac{1-r_u(x)+xr_u'(x)}{(1-r_u(x))^2}K}{\frac{x}{1-r_u(x)}} = 1 + K\frac{1-r_u(x)+r_u(x)(1+r_u(x)-r_{u'}(x))}{x(1-r_u(x))}\cdot\frac{dx}{dK} =$$

$$= 1 + K\frac{1+r_u^2(x)-r_u(x)r_{u'}(x)}{x(1-r_u(x))}\cdot\frac{dx}{dK} = 1 + K\frac{(1-r_u(x))^2 + r_u(x)(2-r_{u'}(x))}{x(1-r_u(x))}\cdot\frac{dx}{dK} < 1,$$

since $r_u(x) < 1$, $r_{u'}(x) < 2$, and $\frac{dx}{dK} < 0$.

(iii) **Capital price (interest rate).** Capital price is

$$\pi = (p-1)Lx = L\frac{xr_u(x)}{1-r_u(x)}.$$

We find capital price derivative with respect to capital endowment:

$$\frac{\partial \pi}{\partial K} = \frac{(r_u(x) + xr_u'(x))(1-r_u(x)) + r_u'(x)xr_u(x)}{(1-r_u(x))^2}\frac{\partial x}{\partial K} =$$

$$= \frac{r_u(x) - r_u^2(x) + xr_u'(x)}{(1-r_u(x))^2}\frac{\partial x}{\partial K} =$$

$$= \frac{r_u(x) - r_u^2(x) + r_u(x)(1 + r_u(x) - r_{u'}(x))}{(1-r_u(x))^2}\frac{\partial x}{\partial K} = r_u(x)\frac{2 - r_{u'}(x)}{(1-r_u(x))^2}\frac{\partial x}{\partial K} =$$

$$= \frac{-\varepsilon_{MR}}{1-r_u(x)}\frac{\partial x}{\partial K} < 0,$$

Q.E.D.

5.4 Appendix D: dumping

Proof of Proposition 3. Since $x^{FF} > x^{HF}$ and $x^{HH} > x^{FH}$, from decreasing demand we immediately obtain inequalities $p^{HH} < p^{FH}$and $p^{FF} < p^{HF}$, or, in other words, $p^{ii} < p^{ij}$.

The ordering of individual consumption is

$$x^{FF} > x^{HH} > x^{HF} > x^{FH}.$$

Claim (i). In the IED case

$$p(x) = \frac{1}{1-r_u(x)} > \frac{1}{1-r_u(z)} = p(z),$$

since $x > z$. So,

$$\frac{1}{1-r_u(x^{FF})} > \frac{1}{1-r_u(x^{HH})} > \frac{1}{1-r_u(x^{HF})} > \frac{1}{1-r_u(x^{FH})}.$$

Consequently,

$$p^{FF} > p^{HH} > \frac{p^{HF}}{\tau} > \frac{p^{FH}}{\tau}.$$

Claim (ii). In the DED case

$$p(x) = \frac{1}{1 - r_u(x)} < \frac{1}{1 - r_u(z)} = p(z),$$

that means

$$\frac{1}{1 - r_u(x^{FF})} < \frac{1}{1 - r_u(x^{HH})} < \frac{1}{1 - r_u(x^{HF})} < \frac{1}{1 - r_u(x^{FH})}.$$

Consequently,

$$p^{FF} < p^{HH} < \frac{p^{HF}}{\tau} < \frac{p^{FH}}{\tau}.$$

Claim (iii). Under trade liberalization difference $(p^{HH} - p^{HF}/\tau)$ decreases in the IED case, since p^{HH} decreases, p^{HF}/τ increases, and $p^{HH} > p^{HF}/\tau$. In the DED case difference $(p^{HF}/\tau - p^{HH})$ decreases under trade liberalization, since p^{HH} increases, p^{HF}/τ decreases, and $p^{HH} < p^{HF}/\tau$. Similar is the comparison between the domestic and imported prices of Foreign producer. Consequently, under trade liberalization dumping (reverse dumping) becomes weaker.

Under increasing capital share we have exactly the same behavior of difference in domestic and imported prices for Home (richer in capital) country. But there is the opposite effect for Foreign (poorer in capital) country. So the greater difference in capital endowment—the weaker (reverse) dumping practice by Home firms and the stronger (reverse) dumping practiced by Foreign firms,

QED.

5.5 Appendix E: comparison of profits and trade flows

Proof of Proposition 5.

Claim (i). The capital prices in Home π^H and Foreign π^F are

$$\pi^H = \theta L((p^{HH} - 1)x^{HH} + (p^{HF} - \tau)x^{HF}),$$

$$\pi^F = (1 - \theta)L((p^{FF} - 1)x^{FF} + (p^{FH} - \tau)x^{FH}).$$

Since the countries have the same populations ($\theta = 0.5$) we have

$$\pi^H - \pi^F = \theta L((p^{HH} - 1)x^{HH} + (p^{HF} - \tau)x^{HF} - (p^{FF} - 1)x^{FF} - (p^{FH} - \tau)x^{FH}) =$$

$$= \theta L \left(\frac{x^{HH} r_u(x^{HH})}{1 - r_u(x^{HH})} - \tau \frac{x^{FH} r_u(x^{FH})}{1 - r_u(x^{FH})} - \left(\frac{x^{FF} r_u(x^{FF})}{1 - r_u(x^{FF})} - \tau \frac{x^{HF} r_u(x^{HF})}{1 - r_u(x^{HF})} \right) \right).$$

Consider function $s(x) = \frac{x r_u(x)}{1-r_u(x)} - \tau \frac{z(x,\tau) r_u(z(x,\tau))}{1-r_u(z(x,\tau))}$.

Its derivative with respect to x is

$$\frac{\partial z}{\partial x} = \frac{(r_u(x) + x r'_u(x))(1 - r_u(x)) + r'_u(x) x r_u(x)}{(1 - r_u(x))^2} - \tau \frac{(r_u(z) + z r'_u(z))(1 - r_u(z)) + r'_u(z) z r_u(z)}{(1 - r_u(z))^2} \frac{\partial z}{\partial x} =$$

$$= \frac{r_u(x) - r_u^2(x) + x r'_u(x)}{(1 - r_u(x))^2} - \tau \frac{r_u(z) - r_u^2(z) + z r'_u(z)}{(1 - r_u(z))^2} \frac{\partial z}{\partial x} =$$

$$= \frac{r_u(x) - r_u^2(x) + r_u(x)(1 + r_u(x) - r_{u'}(x))}{(1 - r_u(x))^2} - \tau \frac{r_u(z) - r_u^2(z) + r_u(z)(1 + r_u(z) - r_{u'}(z))}{(1 - r_u(z))^2} \frac{\partial z}{\partial x} =$$

$$= r_u(x) \frac{2 - r_{u'}(x)}{(1 - r_u(x))^2} - \tau r_u(z) \frac{2 - r_{u'}(z)}{(1 - r_u(z))^2} \frac{\partial z}{\partial x} = r_u(x) \frac{2 - r_{u'}(x)}{(1 - r_u(x))^2} - \tau^2 r_u(z) \frac{2 - r_{u'}(z)}{(1 - r_u(z))^2} \cdot \frac{u''(x)(2 - r_{u'}(x))}{u''(z)(2 - r_{u'}(z))} =$$

$$= r_u(x) \frac{2 - r_{u'}(x)}{(1 - r_u(x))^2} - \tau^2 r_u(z) \frac{u''(x)(2 - r_{u'}(x))}{u''(z)(1 - r_u(z))^2} = \frac{2 - r_{u'}(x)}{(1 - r_u(x))^2} \left(r_u(x) - \tau^2 \frac{r_u(z) u''(x)(1 - r_u(x))^2}{u''(z)(1 - r_u(z))^2} \right) =$$

$$= \frac{2 - r_{u'}(x)}{(1 - r_u(x))^2} \left(r_u(x) - \tau^2 \frac{r_u(z) u''(x)}{u''(z)} \left(\frac{u'(z)}{\tau u'(x)} \right)^2 \right) = \frac{2 - r_{u'}(x)}{(1 - r_u(x))^2} \left(r_u(x) - \frac{r_u(z) u''(x)(u'(z))^2}{u''(z)(u'(x))^2} \right) =$$

$$= \frac{2 - r_{u'}(x)}{(1 - r_u(x))^2} \left(r_u(x) - \frac{z r_u(z) r_u(x) u'(z)}{x r_u(z) u'(x)} \right) = \frac{2 - r_{u'}(x)}{(1 - r_u(x))^2} r_u(x) \left(1 - \frac{z u'(z)}{x u'(x)} \right) > 0,$$

since function $xu'(x) = \lambda x p(x)$ strictly increases and $x > z$. Function $s(x)$ strictly increases and $x^{HH} < x^{FF}$, so we have

$$\pi^H - \pi^F = \theta L(s(x^{HH}) - s(x^{FF})) < 0 \Rightarrow \pi^H < \pi^F,$$

So Foreign capital price exceeds Home capital price under same countries populations.

Claim (ii). Since the countries' sizes are equal, the total export volumes for home e^H and foreign e^F country are

$$e^H = \lambda K \cdot \frac{L}{2} \cdot p^{HF} x^{HF},$$

$$e^F = (1-\lambda)K \cdot \frac{L}{2} \cdot p^{FH} x^{FH}.$$

Consider function $v(x) = px$. We study monotonicity of this function:

$$\frac{\partial v(x)}{\partial x} = \frac{\partial}{\partial x}(px) = \frac{\partial}{\partial x}\left(\frac{x}{1-r_u(x)}\right) = \frac{1-r_u(x)+xr_u'(x)}{(1-r_u(x))^2} = \frac{1-r_u(x)+r_u(x)(1+r_u(x)-r_{u'}(x))}{(1-r_u(x))^2} =$$

$$= \frac{1+r_u^2(x)-r_u(x)r_{u'}(x)}{(1-r_u(x))^2} > \frac{1+r_u^2(x)-2r_u(x)}{(1-r_u(x))^2} = \frac{(1-r_u(x))^2}{(1-r_u(x))^2} = 1 > 0.$$

We have found that total country's export strictly increases with x. Since $x^{HF} > x^{FH}$ and $\lambda > 1-\lambda$, total Home export exceeds Foreign one:

$$e^H > e^F.$$

Claim (iii). The gross firm sizes in Home and Foreign are:

$$q^H = x^{HH}\theta L + \tau x^{HF}(1-\theta)L = \theta L(x^{HH} + \tau x^{HF}),$$

$$q^F = \theta L\left(x^{FF} + \tau x^{FH}\right) \Rightarrow$$

$$q^H - q^F = \theta L\left(x^{HH} - \tau x^{FH} - \left(x^{FF} - \tau x^{HF}\right)\right).$$

Let $s_2(x) = x - \tau z(x,\tau)$. We differentiated it with respect to x:

$$\frac{\partial s_2(x)}{\partial x} = 1 - \tau\frac{\partial z}{\partial x} = 1 - \tau \cdot \frac{u''(x)(2-r_{u'}(x))}{u''(z)(2-r_{u'}(z))} = 1 - \frac{u'(z)(1-r_u(z))}{u'(x)(1-r_u(x))} \cdot \frac{u''(x)(2-r_{u'}(x))}{u''(z)(2-r_{u'}(z))} =$$

$$= 1 - \frac{u'(z)(1+\frac{zu''(z)}{u'(z)})}{u'(x)(1+\frac{xu''(x)}{u'(x)})} \cdot \frac{u''(x)(2-r_{u'}(x))}{u''(z)(2-r_{u'}(z))} = 1 - \frac{(u'(z)+zu''(z))}{(u'(x)+xu''(x))} \cdot \frac{u''(x)(2-r_{u'}(x))}{u''(z)(2-r_{u'}(z))} = 1 - \frac{(zu'(z))'}{(xu'(x))'} \cdot \frac{u''(x)(2-r_{u'}(x))}{u''(z)(2-r_{u'}(z))}$$

Since $x > z$, we have $(zu'(z))' > (xu'(x))'$. Consider function $u''(x)(2-r_{u'}(x))$, it increases when

$$(u''(x)(2-r_{u'}(x)))' = u'''(x)(3-r_{u''}(x)) \geq 0.$$

So under assumptions $u'''(x) \geq 0$ and $r_{u''}(x) \leq 3$ we get

$$\frac{\partial s_2(x)}{\partial x} > 0.$$

Consequently,

$$q^H - q^F = \theta L \left(x^{HH} - \tau x^{FH} - \left(x^{FF} - \tau x^{HF}\right)\right) = \theta L(s_2(x^{HH}) - s_2(x^{FF})) < 0 \Leftrightarrow q^H < q^F,$$

since $x^{HH} < x^{FF}$.

It means that the gross firm size in Home is less then in Foreign.

Similarly we get the conclusion about net firm sizes $y^H < y^F$.

Q.E.D.

5.6 Appendix F: gross firm size under trade liberalization

Proof of Proposition 6. Here we study the firm sizes behavior when trade costs are low, i.e., $\tau \approx 1$. It means that all individual consumptions are approximately the same ($x^{HH} = x^{FF} = x^{FH} = x^{HF}$) in the region studied. But their derivatives with respect to trade costs are not the same. Consider equations (15), (16):

$$\tau u'(x^{HH})(1 - r_u(x^{HH})) = u'(x^{FH})(1 - r_u(x^{FH})),$$

$$V'\left[\lambda K u(x^{HH}) + (1-\lambda) K u(x^{FH})\right] u'(x^{HH})(1 - r_u(x^{HH})) = 1.$$

We find elasticities with respect to trade cost:

$$1 + \varepsilon_{u'(x^{HH})(1-r_u(x^{HH}))} \varepsilon_{x^{HH}} = \varepsilon_{u'(x^{FH})(1-r_u(x^{FH}))} \varepsilon_{x^{FH}},$$

$$\varepsilon_{V'}\left(\varepsilon_{u(x^{HH})}\varepsilon_{x^{HH}} \frac{\lambda K u(x^{HH})}{\lambda K u(x^{HH}) + (1-\lambda) K u(x^{FH})} + \varepsilon_{u(x^{FH})}\varepsilon_{x^{FH}} \frac{(1-\lambda) K u(x^{FH})}{\lambda K u(x^{HH}) + (1-\lambda) K u(x^{FH})}\right) +$$

$$+ \varepsilon_{u'(x^{HH})(1-r_u(x^{HH}))} \varepsilon_{x^{HH}} = 0,$$

where $\varepsilon_{u'(x^{ij})(1-r_u(x^{ij}))}$ is the elasticity of marginal revenue $MR = u'(x^{ij})(1 - r_u(x^{ij}))$ with respect to τ. Let $x = x^{HH} = x^{FF} = x^{FH} = x^{HF}$, then all marginal revenue and utility elasticities are the same, and we denote: $\varepsilon_{MR} = \varepsilon_{u'(x)(1-r_u(x))} = \varepsilon_{u'(x^{ij})(1-r_u(x^{ij}))}$, $\varepsilon_{u(x^{ij})} = \varepsilon_{u(x)}$, and $r_V = -\varepsilon_{V'}$. Then

$$1 + \varepsilon_{MR}\varepsilon_{x^{HH}} = \varepsilon_{MR}\varepsilon_{x^{FH}},$$

$$-r_V \left(\lambda\varepsilon_{u(x)}\varepsilon_{x^{HH}} + (1-\lambda)\varepsilon_{u(x)}\varepsilon_{x^{FH}}\right) + \varepsilon_{MR}\varepsilon_{x^{HH}} = 0.$$

Using the first equation

$$\varepsilon_{x^{FH}} = \frac{1}{\varepsilon_{MR}} + \varepsilon_{x^{HH}}$$

we insert it to the second one

$$-r_V\varepsilon_{u(x)} \left(\lambda\varepsilon_{x^{HH}} + (1-\lambda)\left(\frac{1}{\varepsilon_{MR}} + \varepsilon_{x^{HH}}\right)\right) + \varepsilon_{MR}\varepsilon_{x^{HH}} = 0 \Rightarrow$$

$$-r_V\varepsilon_{u(x)} \left(\varepsilon_{x^{HH}} + \frac{1-\lambda}{\varepsilon_{MR}}\right) + \varepsilon_{MR}\varepsilon_{x^{HH}} = 0 \Rightarrow$$

$$-r_V\varepsilon_{u(x)}\varepsilon_{x^{HH}} - \frac{(1-\lambda)r_V\varepsilon_{u(x)}}{\varepsilon_{MR}} + \varepsilon_{MR}\varepsilon_{x^{HH}} = 0 \Rightarrow$$

$$\varepsilon_{x^{HH}} = -\frac{(1-\lambda)r_V\varepsilon_{u(x)}}{\varepsilon_{MR}(r_V\varepsilon_{u(x)} - \varepsilon_{MR})} > 0.$$

Then the elasticity of imported consumption is

$$\varepsilon_{x^{FH}} = \frac{1}{\varepsilon_{MR}} + \varepsilon_{x^{HH}} = \frac{1}{\varepsilon_{MR}} - \frac{(1-\lambda)r_V\varepsilon_{u(x)}}{\varepsilon_{MR}(r_V\varepsilon_{u(x)} - \varepsilon_{MR})} =$$

$$= \frac{1}{\varepsilon_{MR}}\left(1 - \frac{(1-\lambda)r_V\varepsilon_{u(x)}}{r_V\varepsilon_{u(x)} - \varepsilon_{MR}}\right) = \frac{1}{\varepsilon_{MR}}\frac{r_V\varepsilon_{u(x)} - \varepsilon_{MR} - (1-\lambda)r_V\varepsilon_{u(x)}}{r_V\varepsilon_{u(x)} - \varepsilon_{MR}} =$$

$$= \frac{\lambda r_V\varepsilon_{u(x)} - \varepsilon_{MR}}{\varepsilon_{MR}(r_V\varepsilon_{u(x)} - \varepsilon_{MR})} < 0.$$

We have found the elasticities of Home individual consumption

$$\varepsilon_{x^{HH}} = -\frac{(1-\lambda)r_V\varepsilon_{u(x)}}{\varepsilon_{MR}(r_V\varepsilon_{u(x)} - \varepsilon_{MR})} > 0,$$

$$\varepsilon_{x^{FH}} = \frac{\lambda r_V \varepsilon_{u(x)} - \varepsilon_{MR}}{\varepsilon_{MR}(r_V \varepsilon_{u(x)} - \varepsilon_{MR})} < 0.$$

Similarly we use the equilibrium equations for Foreign consumption

$$\tau u'(x^{FF})(1 - r_u(x^{FF})) = u'(x^{HF})(1 - r_u(x^{HF})),$$

$$V'\left[\lambda K u(x^{HF}) + (1-\lambda) K u(x^{FF})\right] u'(x^{FF})(1 - r_u(x^{FF})) = 1$$

and find the elasticities with respect to trade costs

$$1 + \varepsilon_{u'(x^{FF})(1-r_u(x^{FF}))}\varepsilon_{x^{FF}} = \varepsilon_{u'(x^{HF})(1-r_u(x^{HF}))}\varepsilon_{x^{HF}} \Rightarrow$$

$$1 + \varepsilon_{MR}\varepsilon_{x^{FF}} = \varepsilon_{MR}\varepsilon_{x^{HF}} \Rightarrow$$

$$\varepsilon_{x^{HF}} - \varepsilon_{x^{FF}} = \frac{1}{\varepsilon_{MR}}.$$

Similarly we derive another elasticity

$$\varepsilon_{V'}\left(\varepsilon_{u(x^{HF})}\varepsilon_{x^{HF}}\frac{\lambda K u(x^{HF})}{\lambda K u(x^{HF}) + (1-\lambda) K u(x^{FF})} + \varepsilon_{u(x^{FF})}\varepsilon_{x^{FF}}\frac{(1-\lambda) K u(x^{FF})}{\lambda K u(x^{HF}) + (1-\lambda) K u(x^{FF})}\right) +$$

$$+\varepsilon_{u'(x^{FF})(1-r_u(x^{FF}))}\varepsilon_{x^{FF}} = 0 \Rightarrow$$

$$-r_V\left(\lambda \varepsilon_{u(x)}\varepsilon_{x^{HF}} + (1-\lambda)\varepsilon_{u(x)}\varepsilon_{x^{FF}}\right) + \varepsilon_{MR}\varepsilon_{x^{FF}} = 0 \Rightarrow$$

$$-r_V\left(\lambda \varepsilon_{u(x)}(\varepsilon_{x^{HF}} - \varepsilon_{x^{FF}}) + \varepsilon_{u(x)}\varepsilon_{x^{FF}}\right) + \varepsilon_{MR}\varepsilon_{x^{FF}} = 0 \Rightarrow$$

$$-r_V \varepsilon_{u(x)}\left(\lambda \frac{1}{\varepsilon_{MR}} + \varepsilon_{x^{FF}}\right) + \varepsilon_{MR}\varepsilon_{x^{FF}} = 0 \Rightarrow$$

$$-\lambda \frac{r_V \varepsilon_{u(x)}}{\varepsilon_{MR}} + \varepsilon_{x^{FF}}(\varepsilon_{MR} - r_V \varepsilon_{u(x)}) = 0.$$

Thus we have found the elasticities of individual consumption in Foreign

$$\varepsilon_{x^{FF}} = -\frac{\lambda r_V \varepsilon_{u(x)}}{\varepsilon_{MR}(r_V \varepsilon_{u(x)} - \varepsilon_{MR})} > 0,$$

$$\varepsilon_{x^{HF}} = \frac{1}{\varepsilon_{MR}} + \varepsilon_{x^{FF}} = \frac{1}{\varepsilon_{MR}} - \frac{\lambda r_V \varepsilon_{u(x)}}{\varepsilon_{MR}(r_V \varepsilon_{u(x)} - \varepsilon_{MR})} =$$

$$= \frac{(1-\lambda) r_V \varepsilon_{u(x)} - \varepsilon_{MR}}{\varepsilon_{MR}(r_V \varepsilon_{u(x)} - \varepsilon_{MR})} < 0.$$

Claim (i). Now we turn to gross firm size behavior in Home:

$$q^H = \theta L x^{HH} + \tau(1-\theta) L x^{HF}.$$

The derivative of the gross firm size in Home with respect to trade costs is

$$\frac{\partial q^H}{\partial \tau} = \theta L \frac{\partial x^{HH}}{\partial \tau} + (1-\theta) L \left(\tau \frac{\partial x^{HF}}{\partial \tau} + x^{HF} \right) =$$

$$= \theta L \varepsilon_{x^{HH}} \frac{x^{HH}}{\tau} + (1-\theta) L \left(\tau \varepsilon_{x^{HF}} \frac{x^{HF}}{\tau} + x^{HF} \right).$$

Using elasticities for individual consumption we get

$$\frac{\partial q^H}{\partial \tau} = Lx \frac{-\theta(1-\lambda) r_V \varepsilon_{u(x)}}{\varepsilon_{MR}(r_V \varepsilon_{u(x)} - \varepsilon_{MR})} + (1-\theta) L \left(x \frac{(1-\lambda) r_V \varepsilon_{u(x)} - \varepsilon_{MR}}{\varepsilon_{MR}(r_V \varepsilon_{u(x)} - \varepsilon_{MR})} + x \right) =$$

$$= Lx \left[\frac{-\theta(1-\lambda) r_V \varepsilon_{u(x)}}{\varepsilon_{MR}(r_V \varepsilon_{u(x)} - \varepsilon_{MR})} + (1-\theta) \frac{(1-\lambda) r_V \varepsilon_{u(x)} - \varepsilon_{MR} + \varepsilon_{MR}(r_V \varepsilon_{u(x)} - \varepsilon_{MR})}{\varepsilon_{MR}(r_V \varepsilon_{u(x)} - \varepsilon_{MR})} \right] =$$

$$= Lx \frac{-\theta(1-\lambda) r_V \varepsilon_{u(x)} + (1-\theta)(1-\lambda) r_V \varepsilon_{u(x)} - (1-\theta)\varepsilon_{MR} + (1-\theta)\varepsilon_{MR}(r_V \varepsilon_{u(x)} - \varepsilon_{MR})}{\varepsilon_{MR}(r_V \varepsilon_{u(x)} - \varepsilon_{MR})} =$$

$$= Lx \frac{-\theta(1-\lambda) r_V \varepsilon_{u(x)} + (1-\theta)(1-\lambda) r_V \varepsilon_{u(x)} + (1-\theta)\varepsilon_{MR}(r_V \varepsilon_{u(x)} - \varepsilon_{MR} - 1)}{\varepsilon_{MR}(r_V \varepsilon_{u(x)} - \varepsilon_{MR})} =$$

$$= Lx \frac{(1-\theta)(1-\lambda) r_V \varepsilon_{u(x)} + (1-\theta)(1-\lambda) r_V \varepsilon_{u(x)} + (1-\theta)\varepsilon_{MR}(r_V \varepsilon_{u(x)} - \varepsilon_{MR} - 1) - (1-\lambda) r_V \varepsilon_{u(x)}}{\varepsilon_{MR}(r_V \varepsilon_{u(x)} - \varepsilon_{MR})}$$

The firm size in Home increases with trade liberalization when

$$2(1-\theta)(1-\lambda)r_V\varepsilon_{u(x)} + (1-\theta)\varepsilon_{MR}(r_V\varepsilon_{u(x)} - \varepsilon_{MR} - 1) - (1-\lambda)r_V\varepsilon_{u(x)} > 0 \Rightarrow$$

$$(1-\theta)\left[2(1-\lambda)r_V\varepsilon_{u(x)} + \varepsilon_{MR}(r_V\varepsilon_{u(x)} - \varepsilon_{MR} - 1)\right] > (1-\lambda)r_V\varepsilon_{u(x)} \Rightarrow$$

$$\left[\begin{array}{c} \begin{cases} 1-\theta > \frac{(1-\lambda)r_V\varepsilon_{u(x)}}{(1-\lambda)r_V\varepsilon_{u(x)}+(1-\lambda)r_V\varepsilon_{u(x)}+\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR}-1)} \\ 2(1-\lambda)r_V\varepsilon_{u(x)} + \varepsilon_{MR}(r_V\varepsilon_{u(x)} - \varepsilon_{MR} - 1) > 0 \end{cases} \\ \text{or} \\ \begin{cases} 1-\theta < \frac{(1-\lambda)r_V\varepsilon_{u(x)}}{(1-\lambda)r_V\varepsilon_{u(x)}+(1-\lambda)r_V\varepsilon_{u(x)}+\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR}-1)} \\ 2(1-\lambda)r_V\varepsilon_{u(x)} + \varepsilon_{MR}(r_V\varepsilon_{u(x)} - \varepsilon_{MR} - 1) < 0 \end{cases} \end{array}\right. \Rightarrow$$

$$\left[\begin{array}{c} \begin{cases} \theta < 1 - \frac{(1-\lambda)r_V\varepsilon_{u(x)}}{2(1-\lambda)r_V\varepsilon_{u(x)}+\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR}-1)} \\ \lambda < 1 + \frac{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR}-1)}{2r_V\varepsilon_{u(x)}} \end{cases} \\ \text{or} \\ \begin{cases} \theta > 1 - \frac{(1-\lambda)r_V\varepsilon_{u(x)}}{2(1-\lambda)r_V\varepsilon_{u(x)}+\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR}-1)} \\ \lambda > 1 + \frac{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR}-1)}{2r_V\varepsilon_{u(x)}} \end{cases} \end{array}\right. .$$

We denote $\bar{\theta}^H = 1 - \frac{(1-\lambda)r_V\varepsilon_{u(x)}}{2(1-\lambda)r_V\varepsilon_{u(x)}+\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR}-1)}$ and $\bar{\lambda}^H = 1 + \frac{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR}-1)}{2r_V\varepsilon_{u(x)}}$.

We are interested in behavior of the firm's size when $0 < \theta < 1$ and $\frac{1}{2} < \lambda < 1$. Here we have two cases:

(a) If $r_V\varepsilon_{u(x)} - \varepsilon_{MR} - 1 < 0$ then $\bar{\lambda}^H > 1$. It is easy show that in this area the firm size in Home increases when $\theta < \bar{\theta}^H$;

(b) If $r_V\varepsilon_{u(x)} - \varepsilon_{MR} - 1 > 0$ then $\bar{\lambda}^H < 1$. This case brings us to the same result: the firm size in Home increases when $\theta < \bar{\theta}^H$.

Thus we have proved claim (i) of Proposition 6 and turn to proving claim (ii).

Claim (ii). The gross firm size in Foreign is:

$$q^F = (1-\theta)Lx^{FF} + \tau\theta Lx^{FH}.$$

The derivative of the firm size under small trade costs is:

$$\frac{\partial q^F}{\partial \tau} = (1-\theta)L\frac{\partial x^{FF}}{\partial \tau} + \theta L\left(\tau\frac{\partial x^{FH}}{\partial \tau} + x^{FH}\right) =$$

$$= (1-\theta)L\varepsilon_{x^{FF}}\frac{x^{FF}}{\tau} + \theta L\left(\tau\varepsilon_{x^{FH}}\frac{x^{FH}}{\tau} + x^{FH}\right).$$

Inserting here elasticities $\varepsilon_{x^{FF}}$ and $\varepsilon_{x^{FH}}$ we get

$$\frac{\partial q^F}{\partial \tau} = -\lambda(1-\theta)Lx\frac{r_V\varepsilon_{u(x)}}{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})} + \theta L\left(\frac{\lambda r_V\varepsilon_{u(x)}-\varepsilon_{MR}}{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}x + x\right) =$$

$$= Lx\left[\frac{-\lambda(1-\theta)r_V\varepsilon_{u(x)}}{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})} + \theta\frac{\lambda r_V\varepsilon_{u(x)}-\varepsilon_{MR}+\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}\right] =$$

$$= Lx\frac{-\lambda(1-\theta)r_V\varepsilon_{u(x)}+\theta\lambda r_V\varepsilon_{u(x)}-\theta\varepsilon_{MR}+\theta\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})} =$$

$$= Lx\frac{(2\theta-1)\lambda r_V\varepsilon_{u(x)}-\theta\varepsilon_{MR}+\theta\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})} =$$

$$= Lx\frac{(2\theta-1)\lambda r_V\varepsilon_{u(x)}+\theta\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR}-1)}{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}.$$

The gross firm size in Foreign increases with trade liberalization when

$$(2\theta-1)\lambda r_V\varepsilon_{u(x)}+\theta\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR}-1) > 0 \Rightarrow$$

$$2\theta\lambda r_V\varepsilon_{u(x)}+\theta\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR}-1) > \lambda r_V\varepsilon_{u(x)} \Rightarrow$$

$$\theta\left(2\lambda r_V\varepsilon_{u(x)}+\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR}-1)\right) > \lambda r_V\varepsilon_{u(x)} \Rightarrow$$

$$\left[\begin{array}{l}\begin{cases}\theta > \frac{\lambda r_V\varepsilon_{u(x)}}{2\lambda r_V\varepsilon_{u(x)}+\varepsilon_{MR}(r_V\varepsilon_{u(x)}-1-\varepsilon_{MR})} \\ 2\lambda r_V\varepsilon_{u(x)}+\varepsilon_{MR}(r_V\varepsilon_{u(x)}-1-\varepsilon_{MR}) > 0\end{cases} \\ \begin{cases}\theta < \frac{\lambda r_V\varepsilon_{u(x)}}{2\lambda r_V\varepsilon_{u(x)}+\varepsilon_{MR}(r_V\varepsilon_{u(x)}-1-\varepsilon_{MR})} \\ 2\lambda r_V\varepsilon_{u(x)}+\varepsilon_{MR}(r_V\varepsilon_{u(x)}-1-\varepsilon_{MR}) < 0\end{cases}\end{array}\right. \Rightarrow$$

$$\left[\begin{cases} \theta > \frac{\lambda r_V\varepsilon_{u(x)}}{2\lambda r_V\varepsilon_{u(x)}+\varepsilon_{MR}(r_V\varepsilon_{u(x)}-1-\varepsilon_{MR})} \\ \lambda > -\frac{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-1-\varepsilon_{MR})}{2r_V\varepsilon_{u(x)}} \end{cases} \right. \\ \left. \begin{cases} \theta < \frac{\lambda r_V\varepsilon_{u(x)}}{2\lambda r_V\varepsilon_{u(x)}+\varepsilon_{MR}(r_V\varepsilon_{u(x)}-1-\varepsilon_{MR})} \\ \lambda < -\frac{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-1-\varepsilon_{MR})}{2r_V\varepsilon_{u(x)}} \end{cases} \right. .$$

We denote $\bar{\theta}^F = \frac{\lambda r_V\varepsilon_{u(x)}}{2\lambda r_V\varepsilon_{u(x)}+\varepsilon_{MR}(r_V\varepsilon_{u(x)}-1-\varepsilon_{MR})}$ and $\bar{\lambda}^F = -\frac{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-1-\varepsilon_{MR})}{2r_V\varepsilon_{u(x)}}$.

Again we are interested in behavior of the firm's size when $0 < \theta < 1$ and $\frac{1}{2} < \lambda < 1$. We have two cases:

(a) If $r_V\varepsilon_{u(x)} - \varepsilon_{MR} - 1 < 0$ then $\bar{\lambda}^F < 0$. Easy show that in this area the firm size in Foreign increases when $\theta > \bar{\theta}^F$;

(b) If $r_V\varepsilon_{u(x)} - \varepsilon_{MR} - 1 > 0$ then $\bar{\lambda}^F > 0$. This case brings us to the same result: the gross firm size in Foreign increases when $\theta > \bar{\theta}^F$, that completes the proof of claim (ii).

Claim (iii). To explain the difference between Figures 2a, 2b and prove claim (iii), we can compare thresholds $\bar{\theta}^H$ and $\bar{\theta}^F$. Since $\lambda > 1/2$ we are interested in area when $\lambda \in (0.5, 1]$. We do not present the full proof but it is easy shown that:

(a) If $r_V\varepsilon_{u(x)} - \varepsilon_{MR} < 1$ then both thresholds increase and $\bar{\theta}^H > \bar{\theta}^F$. (This case presented at Figure 2a.);

(b) If $r_V\varepsilon_{u(x)} - \varepsilon_{MR} > 1$ then both thresholds decrease and $\bar{\theta}^H < \bar{\theta}^F$. (See Figure 2b.)

Q.E.D.

Additional remark in this section says that the difference in firm sizes is monotone. To prove this, note that the behavior of difference in firm sizes is determined as:

$$\frac{\partial(q^H - q^F)}{\partial\tau} = Lx\frac{r_V\varepsilon_{u(x)}\left[(1-\lambda)(2\theta-1)-(1-\theta)\varepsilon_{MR}\right]+(1-\theta)\varepsilon_{MR}(1+\varepsilon_{MR})}{\varepsilon_{MR}(\varepsilon_{MR}-r_V\varepsilon_{u(x)})} -$$

$$-Lx\frac{-r_V\varepsilon_{u(x)}(\lambda(2\theta-1)+\theta\varepsilon_{MR})+\theta\varepsilon_{MR}(1+\varepsilon_{MR})}{\varepsilon_{MR}(\varepsilon_{MR}-r_V\varepsilon_{u(x)})}$$

$$= Lx\frac{r_V\varepsilon_{u(x)}\left[(1-\lambda)(2\theta-1)-(1-\theta)\varepsilon_{MR}\right]+(1-\theta)\varepsilon_{MR}(1+\varepsilon_{MR})+r_V\varepsilon_{u(x)}(\lambda(2\theta-1)+\theta\varepsilon_{MR})-\theta\varepsilon_{MR}(1+\varepsilon_{MR})}{\varepsilon_{MR}(\varepsilon_{MR}-r_V\varepsilon_{u(x)})} =$$

$$= Lx\frac{r_V\varepsilon_{u(x)}\left[2\theta-1+(2\theta-1)\varepsilon_{MR}\right]-(2\theta-1)\varepsilon_{MR}(1+\varepsilon_{MR})}{\varepsilon_{MR}(\varepsilon_{MR}-r_V\varepsilon_{u(x)})} =$$

$$= Lx(2\theta-1)\frac{r_V\varepsilon_{u(x)}(1+\varepsilon_{MR})-\varepsilon_{MR}(1+\varepsilon_{MR})}{\varepsilon_{MR}(\varepsilon_{MR}-r_V\varepsilon_{u(x)})} =$$

$$= Lx(2\theta-1)\frac{(r_V\varepsilon_{u(x)}-\varepsilon_{MR})(1+\varepsilon_{MR})}{\varepsilon_{MR}(\varepsilon_{MR}-r_V\varepsilon_{u(x)})} =$$

$$= -Lx(2\theta-1)\frac{1+\varepsilon_{MR}}{\varepsilon_{MR}}$$

So,

$$\frac{\partial(q^H-q^F)}{\partial\tau} = -Lx(2\theta-1)\frac{1+\varepsilon_{MR}}{\varepsilon_{MR}}$$

and we have found that the sign of monotonicity of the gross form sizes differential is determined by this expression.

5.7 Appendix G: net firm size under trade liberalization

Proof of Proposition 7. Now we eliminate the transportation sector as a "consumer", and study the impact of globalization on *net* firm size y^H:

$$y^H = \theta Lx^{HH} + (1-\theta)Lx^{HF}.$$

We get the derivative with respect to trade cost:

$$\frac{\partial y^H}{\partial\tau} = \theta L\frac{\partial x^{HH}}{\partial\tau} + (1-\theta)L\frac{\partial x^{HF}}{\partial\tau} =$$

$$= \theta L\varepsilon_{x^{HH}}\frac{x^{HH}}{\tau} + (1-\theta)L\varepsilon_{x^{HF}}\frac{x^{HF}}{\tau}.$$

Now we use the individual consumption elasticities under small trade cost from Appendix F and get:

$$\frac{\partial y^H}{\partial\tau} = \frac{Lx}{\tau}\left(-\theta\frac{(1-\lambda)r_V\varepsilon_{u(x)}}{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})} + (1-\theta)\frac{(1-\lambda)r_V\varepsilon_{u(x)}-\varepsilon_{MR}}{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}\right) =$$

$$= \frac{Lx}{\tau\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}\big((1-\theta)(1-\lambda)r_V\varepsilon_{u(x)} - (1-\theta)\varepsilon_{MR} - \theta(1-\lambda)r_V\varepsilon_{u(x)}\big) =$$

$$= \frac{Lx}{\tau\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}\big((1-2\theta)(1-\lambda)r_V\varepsilon_{u(x)} - (1-\theta)\varepsilon_{MR}\big) =$$

$$= -\frac{Lx}{\tau\varepsilon_{MR}(r_V\varepsilon_{u(x)} - \varepsilon_{MR})}\left((2\theta-1)(1-\lambda)r_V\varepsilon_{u(x)} + (1-\theta)\varepsilon_{MR}\right).$$

The net Firm size in Home increases with trade liberalization when

$$(2\theta-1)(1-\lambda)r_V\varepsilon_{u(x)} + (1-\theta)\varepsilon_{MR} < 0 \Rightarrow$$

$$2\theta(1-\lambda)r_V\varepsilon_{u(x)} - (1-\lambda)r_V\varepsilon_{u(x)} + \varepsilon_{MR} - \theta\varepsilon_{MR} < 0 \Rightarrow$$

$$\theta(2(1-\lambda)r_V\varepsilon_{u(x)} - \varepsilon_{MR}) < (1-\lambda)r_V\varepsilon_{u(x)} - \varepsilon_{MR} \Rightarrow$$

$$\theta < \frac{(1-\lambda)r_V\varepsilon_{u(x)} - \varepsilon_{MR}}{2(1-\lambda)r_V\varepsilon_{u(x)} - \varepsilon_{MR}}.$$

We denote $\tilde{\theta}^H = \frac{(1-\lambda)r_V\varepsilon_{u(x)} - \varepsilon_{MR}}{2(1-\lambda)r_V\varepsilon_{u(x)} - \varepsilon_{MR}}$ and find similar derivative for net firm size in Foreign:

$$y^F = \theta L x^{FH} + (1-\theta)Lx^{FF}$$

$$\frac{\partial y^F}{\partial\tau} = \theta L\frac{\partial x^{FH}}{\partial\tau} + (1-\theta)L\frac{\partial x^{FF}}{\partial\tau} =$$

$$= \theta L\varepsilon_{x^{FH}}\frac{x^{FH}}{\tau} + (1-\theta)L\varepsilon_{x^{FF}}\frac{x^{FF}}{\tau}.$$

Using equations for individual consumption we get

$$\frac{\partial y^F}{\partial\tau} = \frac{Lx}{\tau}\left(\theta\frac{\lambda r_V\varepsilon_{u(x)} - \varepsilon_{MR}}{\varepsilon_{MR}(r_V\varepsilon_{u(x)} - \varepsilon_{MR})} - (1-\theta)\frac{\lambda r_V\varepsilon_{u(x)}}{\varepsilon_{MR}(r_V\varepsilon_{u(x)} - \varepsilon_{MR})}\right) =$$

$$= \frac{Lx}{\tau\varepsilon_{MR}(r_V\varepsilon_{u(x)} - \varepsilon_{MR})}\left(\theta(\lambda r_V\varepsilon_{u(x)} - \varepsilon_{MR}) - (1-\theta)\lambda r_V\varepsilon_{u(x)}\right) =$$

$$= \frac{Lx}{\tau\varepsilon_{MR}(r_V\varepsilon_{u(x)} - \varepsilon_{MR})}\left(\lambda\theta r_V\varepsilon_{u(x)} - \theta\varepsilon_{MR} - \lambda r_V\varepsilon_{u(x)} + \theta\lambda r_V\varepsilon_{u(x)}\right) =$$

$$= \frac{Lx}{\tau\varepsilon_{MR}(r_V\varepsilon_{u(x)} - \varepsilon_{MR})}\left(\theta(2\lambda r_V\varepsilon_{u(x)} - \varepsilon_{MR}) - \lambda r_V\varepsilon_{u(x)}\right).$$

Thus, the net firm size in Foreign increases with trade liberalization when

$$\theta(2\lambda r_V \varepsilon_{u(x)} - \varepsilon_{MR}) - \lambda r_V \varepsilon_{u(x)} > 0 \Rightarrow$$

$$\theta > \frac{\lambda r_V \varepsilon_{u(x)}}{2\lambda r_V \varepsilon_{u(x)} - \varepsilon_{MR}}.$$

We denote $\tilde{\theta}^F = \frac{\lambda r_V \varepsilon_{u(x)}}{2\lambda r_V \varepsilon_{u(x)} - \varepsilon_{MR}}$ and conclude that if $\theta < \tilde{\theta^H}$, then y^H increases, and if $\theta > \tilde{\theta^F}$, then y^F increases, **Q.E.D.**

5.8 Appendix H: simulations confirming robust globalization effects on firm size

We are going to show an example of monotone behavior of firm sizes for wide range of levels of trade cost ($1 \leq \tau \leq 1.25$), shown in Figure 5 below. The idea is to confirm that Proposition 6 is likely to be extended from $\tau \approx 1$ to a wide range of trade cost. We take low-tier utility function $u(x) = \sqrt{x} - \frac{2}{5}x$. and upper-tier utility $V(m) = \ln(m)$ for Figure .

Using $K = 1$ and $L = 10$, Figure 5 presents pattern of changes in q^H, q^F when trade freeness $1/\tau$ increases from 0.8 to 1. In particular, diagrams (a)-(c) display the case $\bar{\theta}^H(\lambda) > \bar{\theta}^F(\lambda)$ related to Figure 2a under upper-tier utility $V(m) = \sqrt{m}$. Namely, Figure 5a relates to lower area of Figure 2a under $\theta = 0.3 < \bar{\theta}^F$. Figure 5b displays the middle case under $\bar{\theta}^F < \theta = 0.55 < \bar{\theta}^H$. Finally, Figure 5c shows the upper case under $\theta = 0.8 > \bar{\theta}^H$.

By contrast, Figure 2b is transformed into Fig. 5 (d)-(e), which presents pattern for case $\bar{\theta}^H(\lambda) > \bar{\theta}^F(\lambda)$ under upper-tier utility $V(m) = \ln(m)$. Figure 5d corresponds to the case of $\lambda = 0.6$ and $\theta = 0.3 < \bar{\theta}^H$; Figure 5e related to middle case of Figure 2b when countries a fairly similar in populations under $\bar{\theta}^H = \theta = 0.51 < \bar{\theta}^F$ and Figure 5f displays case of $\theta = 0.6 > \bar{\theta}^H$.

Commenting these simulations, we observe that in the range $1/\tau > 0.8$, i.e., $\tau < 1.25$ the conclusions of Proposition 6 hold true.

5.9 Appendix I: capital price under trade liberalization

Proof of Proposition 8. Here we study behavior of capital prices π^H and π^F under small trade cost. Firstly, we get an auxiliary result:

$$\varepsilon_{MR} = \varepsilon_{u'(x)(1-r_u(x))} = \frac{(u'(x) + xu''(x))'}{u'(x) + xu''(x)}x = \frac{2u''(x) + xu'''(x)}{u'(x) + xu''(x)}x = \frac{-2r_u(x) + r_u(x)r_{u'}(x)}{1 - r_u(x)} = -r_u(x)\frac{2 - r_{u'}(x)}{1 - r_u(x)}.$$

Claim (i). We study the derivative of capital price with respect to trade cost:

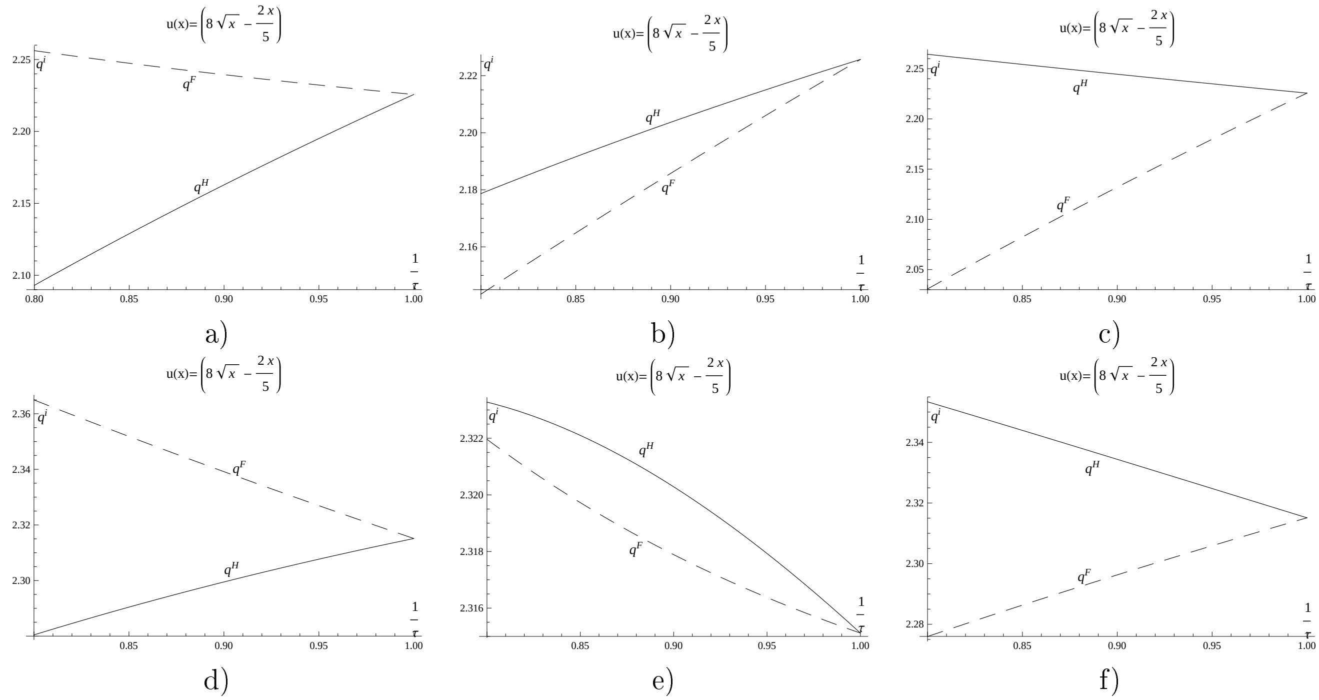


Figure 5: Firm size behavior under small trade cost.

$$\pi^H = \theta L(p^{HH} - 1)x^{HH} + (1-\theta)L(p^{HF} - \tau)x^{HF}) =$$

$$= \theta L \frac{x^{HH} r_u(x^{HH})}{1 - r_u(x^{HH})} + \tau(1-\theta)L\frac{x^{HF} r_u(x^{HF})}{1 - r_u(x^{HF})},$$

$$\frac{\partial \pi^H}{\partial \tau} = \theta L r_u(x^{HH}) \frac{2 - r_{u'}(x^{HH})}{(1 - r_u(x^{HH}))^2} \frac{\partial x^{HH}}{\partial \tau} + (1-\theta)L\left(\tau r_u(x^{HF}) \frac{2 - r_{u'}(x^{HF})}{(1 - r_u(x^{HF}))^2} \frac{\partial x^{HF}}{\partial \tau} + \frac{x^{HF} r_u(x^{HF})}{1 - r_u(x^{HF})}\right) =$$

$$= \theta L r_u(x) \frac{2 - r_{u'}(x)}{(1 - r_u(x))^2} \frac{\partial x^{HH}}{\partial \tau} + (1-\theta)L\left(r_u(x) \frac{2 - r_{u'}(x)}{(1 - r_u(x))^2} \frac{\partial x^{HF}}{\partial \tau} + \frac{x r_u(x)}{1 - r_u(x)}\right) =$$

$$= L\frac{r_u(x)}{1 - r_u(x)}\left(\theta \frac{2 - r_{u'}(x)}{1 - r_u(x)} \frac{\partial x^{HH}}{\partial \tau} + (1-\theta)\left(\frac{2 - r_{u'}(x)}{1 - r_u(x)} \frac{\partial x^{HF}}{\partial \tau} + x\right)\right) =$$

$$= L\frac{r_u(x)}{1 - r_u(x)}\left(\theta \frac{2 - r_{u'}(x)}{1 - r_u(x)} \varepsilon_{x^{HH}} x + (1-\theta)\left(\frac{2 - r_{u'}(x)}{1 - r_u(x)} \varepsilon_{x^{HF}} x + x\right)\right) =$$

$$= L\frac{xr_u(x)}{1-r_u(x)}\left(-\theta\frac{2-r_{u'}(x)}{1-r_u(x)}\frac{(1-\lambda)r_V\varepsilon_{u(x)}}{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}+(1-\theta)\left(\frac{2-r_{u'}(x)}{1-r_u(x)}\frac{(1-\lambda)r_V\varepsilon_{u(x)}-\varepsilon_{MR}}{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}+1\right)\right)=$$

$$= L\frac{xr_u(x)}{1-r_u(x)}\left(-\theta\frac{2-r_{u'}(x)}{1-r_u(x)}\frac{(1-\lambda)r_V\varepsilon_{u(x)}}{-r_u(x)\frac{2-r_{u'}(x)}{1-r_u(x)}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}+(1-\theta)\left(\frac{2-r_{u'}(x)}{1-r_u(x)}\frac{(1-\lambda)r_V\varepsilon_{u(x)}-\varepsilon_{MR}}{-r_u(x)\frac{2-r_{u'}(x)}{1-r_u(x)}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}+1\right)\right)$$

$$= L\frac{x}{1-r_u(x)}\left(\theta\frac{(1-\lambda)r_V\varepsilon_{u(x)}}{(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}+(1-\theta)\left(-\frac{(1-\lambda)r_V\varepsilon_{u(x)}-\varepsilon_{MR}}{(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}+r_u(x)\right)\right)=$$

$$= L\frac{x}{1-r_u(x)}\cdot\frac{\theta(1-\lambda)r_V\varepsilon_{u(x)}-(1-\theta)(1-\lambda)r_V\varepsilon_{u(x)}+(1-\theta)\varepsilon_{MR}+(1-\theta)r_u(x)(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}{r_V\varepsilon_{u(x)}-\varepsilon_{MR}}=$$

$$= L\frac{x}{1-r_u(x)}\cdot\frac{-(1-\theta)(1-\lambda)r_V\varepsilon_{u(x)}-(1-\theta)(1-\lambda)r_V\varepsilon_{u(x)}+(1-\theta)\varepsilon_{MR}+(1-\theta)r_u(x)(r_V\varepsilon_{u(x)}-\varepsilon_{MR})+(1-\lambda)r_V\varepsilon_{u(x}}{r_V\varepsilon_{u(x)}-\varepsilon_{MR}}$$

$$= L\frac{x}{1-r_u(x)}\cdot\frac{(1-\theta)(-2(1-\lambda)r_V\varepsilon_{u(x)}+\varepsilon_{MR}+r_u(x)(r_V\varepsilon_{u(x)}-\varepsilon_{MR}))+(1-\lambda)r_V\varepsilon_{u(x)}}{r_V\varepsilon_{u(x)}-\varepsilon_{MR}}$$

Observing this expression we conclude that capital price in Home increases under trade liberalization when

$$(1-\theta)(-2(1-\lambda)r_V\varepsilon_{u(x)}+\varepsilon_{MR}+r_u(x)(r_V\varepsilon_{u(x)}-\varepsilon_{MR}))+(1-\lambda)r_V\varepsilon_{u(x)}<0\Rightarrow$$

$$(1-\theta)(-2r_V\varepsilon_{u(x)}+2\lambda r_V\varepsilon_{u(x)}+\varepsilon_{MR}+r_u(x)r_V\varepsilon_{u(x)}-r_u(x)\varepsilon_{MR})<-(1-\lambda)r_V\varepsilon_{u(x)}\Rightarrow$$

$$(1-\theta)(2\lambda r_V\varepsilon_{u(x)}-r_V\varepsilon_{u(x)}+r_V\varepsilon_{u(x)}(r_u(x)-1)+\varepsilon_{MR}(1-r_u(x))<-(1-\lambda)r_V\varepsilon_{u(x)}\Rightarrow$$

$$(1-\theta)((2\lambda-1)r_V\varepsilon_{u(x)}-(r_V\varepsilon_{u(x)}-\varepsilon_{MR})(1-r_u(x))<-(1-\lambda)r_V\varepsilon_{u(x)}\Rightarrow$$

$$\left[\begin{array}{l}\begin{cases}1-\theta < -\frac{(1-\lambda)r_V\varepsilon_{u(x)}}{(2\lambda-1)r_V\varepsilon_{u(x)}-(r_V\varepsilon_{u(x)}-\varepsilon_{MR})(1-r_u(x))}\\(2\lambda-1)r_V\varepsilon_{u(x)}-(r_V\varepsilon_{u(x)}-\varepsilon_{MR})(1-r_u(x))>0\end{cases}\\\begin{cases}1-\theta > -\frac{(1-\lambda)r_V\varepsilon_{u(x)}}{(2\lambda-1)r_V\varepsilon_{u(x)}-(r_V\varepsilon_{u(x)}-\varepsilon_{MR})(1-r_u(x))}\\(2\lambda-1)r_V\varepsilon_{u(x)}-(r_V\varepsilon_{u(x)}-\varepsilon_{MR})(1-r_u(x))<0\end{cases}\end{array}\right. \Rightarrow$$

$$\left[\begin{array}{l}\begin{cases}\theta > 1+\frac{(1-\lambda)r_V\varepsilon_{u(x)}}{(2\lambda-1)r_V\varepsilon_{u(x)}-(r_V\varepsilon_{u(x)}-\varepsilon_{MR})(1-r_u(x))}\\\lambda > \frac{1}{2}+\frac{(r_V\varepsilon_{u(x)}-\varepsilon_{MR})(1-r_u(x))}{2r_V\varepsilon_{u(x)}}\end{cases}\\\begin{cases}\theta < 1+\frac{(1-\lambda)r_V\varepsilon_{u(x)}}{(2\lambda-1)r_V\varepsilon_{u(x)}-(r_V\varepsilon_{u(x)}-\varepsilon_{MR})(1-r_u(x))}\\\lambda < \frac{1}{2}+\frac{(r_V\varepsilon_{u(x)}-\varepsilon_{MR})(1-r_u(x))}{2r_V\varepsilon_{u(x)}}\end{cases}\end{array}\right. .$$

We denote $\hat{\theta}^H = 1+\frac{(1-\lambda)r_V\varepsilon_{u(x)}}{(2\lambda-1)r_V\varepsilon_{u(x)}-(r_V\varepsilon_{u(x)}-\varepsilon_{MR})(1-r_u(x))}$ and $\hat{\lambda}^H = \frac{1}{2}+\frac{(r_V\varepsilon_{u(x)}-\varepsilon_{MR})(1-r_u(x))}{2r_V\varepsilon_{u(x)}}$. The first case in the above expression is impossible since $\hat{\theta}^H > 1$ when $\lambda > \hat{\lambda}^H$. Thus, the second case takes place, that means that capital price in Home increases when

$$\begin{cases}\theta < \hat{\theta}^H\\\lambda < \hat{\lambda}^H\end{cases} .$$

Claim (ii). The derivative for Foreign capital price is:

$$\frac{\partial\pi^F}{\partial\tau} = (1-\theta)Lr_u(x^{FF})\frac{2-r_{u'}(x^{FF})}{(1-r_u(x^{FF}))^2}\frac{\partial x^{HH}}{\partial\tau} + \theta L\left(\tau r_u(x^{FH})\frac{2-r_{u'}(x^{FH})}{(1-r_u(x^{FH}))^2}\frac{\partial x^{FH}}{\partial\tau} + \frac{x^{FH}r_u(x^{FH})}{1-r_u(x^{FH})}\right) =$$

$$= (1-\theta)Lr_u(x)\frac{2-r_{u'}(x)}{(1-r_u(x))^2}\frac{\partial x^{FF}}{\partial\tau} + \theta L\left(r_u(x)\frac{2-r_{u'}(x)}{(1-r_u(x))^2}\frac{\partial x^{FH}}{\partial\tau} + \frac{xr_u(x)}{1-r_u(x)}\right) =$$

$$= L\frac{r_u(x)}{1-r_u(x)}\left((1-\theta)\frac{2-r_{u'}(x)}{1-r_u(x)}\frac{\partial x^{FF}}{\partial\tau} + \theta\left(\frac{2-r_{u'}(x)}{1-r_u(x)}\frac{\partial x^{FH}}{\partial\tau} + x\right)\right) =$$

$$= L\frac{r_u(x)}{1-r_u(x)}\left((1-\theta)\frac{2-r_{u'}(x)}{1-r_u(x)}\varepsilon_{x^{FF}}x + \theta\left(\frac{2-r_{u'}(x)}{1-r_u(x)}\varepsilon_{x^{FH}}x + x\right)\right) =$$

$$= L\frac{xr_u(x)}{1-r_u(x)}\left(-(1-\theta)\frac{2-r_{u'}(x)}{1-r_u(x)}\frac{\lambda r_V\varepsilon_{u(x)}}{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})} + \theta\left(\frac{2-r_{u'}(x)}{1-r_u(x)}\frac{\lambda r_V\varepsilon_{u(x)}-\varepsilon_{MR}}{\varepsilon_{MR}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})} + 1\right)\right) =$$

$$= L\frac{x r_u(x)}{1-r_u(x)}\left(-(1-\theta)\frac{2-r_{u'}(x)}{1-r_u(x)}\frac{\lambda r_V \varepsilon_{u(x)}}{-r_u(x)\frac{2-r_{u'}(x)}{1-r_u(x)}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}+\theta\left(\frac{2-r_{u'}(x)}{1-r_u(x)}\frac{\lambda r_V\varepsilon_{u(x)}-\varepsilon_{MR}}{-r_u(x)\frac{2-r_{u'}(x)}{1-r_u(x)}(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}+1\right)\right)$$

$$= L\frac{x}{1-r_u(x)}\left((1-\theta)\frac{\lambda r_V\varepsilon_{u(x)}}{(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}+\theta\left(-\frac{\lambda r_V\varepsilon_{u(x)}-\varepsilon_{MR}}{(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}+r_u(x)\right)\right)=$$

$$= L\frac{x}{1-r_u(x)}\cdot\frac{(1-\theta)\lambda r_V\varepsilon_{u(x)}-\theta\lambda r_V\varepsilon_{u(x)}+\theta\varepsilon_{MR}+\theta r_u(x)(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}{r_V\varepsilon_{u(x)}-\varepsilon_{MR}}=$$

$$= L\frac{x}{1-r_u(x)}\cdot\frac{\lambda r_V\varepsilon_{u(x)}-\theta\lambda r_V\varepsilon_{u(x)}-\theta\lambda r_V\varepsilon_{u(x)}+\theta\varepsilon_{MR}+\theta r_u(x)(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}{r_V\varepsilon_{u(x)}-\varepsilon_{MR}}.$$

The capital price in Foreign increases with trade liberalization when

$$\lambda r_V\varepsilon_{u(x)}-\theta(2\lambda r_V\varepsilon_{u(x)}-\varepsilon_{MR}-r_u(x)(r_V\varepsilon_{u(x)}-\varepsilon_{MR}))<0\Rightarrow$$

$$\theta((2\lambda-1)r_V\varepsilon_{u(x)}+(1-r_u(x))(r_V\varepsilon_{u(x)}-\varepsilon_{MR}))>\lambda r_V\varepsilon_{u(x)}\Rightarrow$$

$$\left[\begin{array}{l}\begin{cases}\theta>\frac{\lambda r_V\varepsilon_{u(x)}}{(2\lambda-1)r_V\varepsilon_{u(x)}+(1-r_u(x))(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}\\(2\lambda-1)r_V\varepsilon_{u(x)}+(1-r_u(x))(r_V\varepsilon_{u(x)}-\varepsilon_{MR})>0\end{cases}\\\begin{cases}\theta<\frac{\lambda r_V\varepsilon_{u(x)}}{(2\lambda-1)r_V\varepsilon_{u(x)}+(1-r_u(x))(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}\\(2\lambda-1)r_V\varepsilon_{u(x)}+(1-r_u(x))(r_V\varepsilon_{u(x)}-\varepsilon_{MR})<0\end{cases}\end{array}\right.\Rightarrow$$

$$\left[\begin{array}{l}\begin{cases}\theta>\frac{\lambda r_V\varepsilon_{u(x)}}{(2\lambda-1)r_V\varepsilon_{u(x)}+(1-r_u(x))(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}\\\lambda>\frac{1}{2}-\frac{(1-r_u(x))(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}{2r_V\varepsilon_{u(x)}}\end{cases}\\\begin{cases}\theta<\frac{\lambda r_V\varepsilon_{u(x)}}{(2\lambda-1)r_V\varepsilon_{u(x)}+(1-r_u(x))(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}\\\lambda<\frac{1}{2}-\frac{(1-r_u(x))(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}{2r_V\varepsilon_{u(x)}}\end{cases}\end{array}\right..$$

We denote $\hat{\theta}^F=\frac{\lambda r_V\varepsilon_{u(x)}}{(2\lambda-1)r_V\varepsilon_{u(x)}+(1-r_u(x))(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}$ and $\hat{\lambda}^F=\frac{1}{2}-\frac{(1-r_u(x))(r_V\varepsilon_{u(x)}-\varepsilon_{MR})}{2r_V\varepsilon_{u(x)}}$. The second case among the above inequalities is impossible since $\lambda>\frac{1}{2}$ and $\hat{\lambda}^F<\frac{1}{2}$. It means that the capital price in Foreign increases when

$$\theta>\hat{\theta}^F.$$

Claim (iii). Here we compare thresholds $\hat{\theta}^H$ and $\hat{\theta}^F$.

Again we are only interesting in area when $\lambda \in (0.5, 1]$. Easy is to show that:

(a) If $(r_V \varepsilon_{u(x)} - \varepsilon_{MR})(1 - r_u(x)) < r_V \varepsilon_{u(x)}$ then both thresholds decrease and $\hat{\theta}^H < \hat{\theta}^F$. (This case is presented at Figure 4a.)

(b) If $(r_V \varepsilon_{u(x)} - \varepsilon_{MR})(1 - r_u(x)) > r_V \varepsilon_{u(x)}$ then both thresholds increase and $\hat{\theta}^H > \hat{\theta}^F$. (See Figure 4b.)

Q.E.D.

Additional remark from this section studies the difference in capital prices:

$$\frac{\partial(\pi^H - \pi^F)}{\partial \tau} = L\frac{x}{1 - r_u(x)}\left((1-\theta)r_u(x) - \frac{(2\theta-1)(1-\lambda)r_V\varepsilon_{u(x)} + (1-\theta)\varepsilon_{MR}}{\varepsilon_{MR} - r_V\varepsilon_{u(x)}}\right) -$$

$$-L\frac{x}{1 - r_u(x)}\left(\frac{(2\theta-1)\lambda r_V\varepsilon_{u(x)} - \theta\varepsilon_{MR}}{\varepsilon_{MR} - r_V\varepsilon_{u(x)}} + \theta r_u(x)\right) =$$

$$= L\frac{x}{1 - r_u(x)}\left((1-\theta)r_u(x) - \frac{(2\theta-1)(1-\lambda)r_V\varepsilon_{u(x)} + (1-\theta)\varepsilon_{MR}}{\varepsilon_{MR} - r_V\varepsilon_{u(x)}} - \frac{(2\theta-1)\lambda r_V\varepsilon_{u(x)} - \theta\varepsilon_{MR}}{\varepsilon_{MR} - r_V\varepsilon_{u(x)}} - \theta r_u(x)\right) =$$

$$= L\frac{x}{1 - r_u(x)}\left((1-\theta)r_u(x) - \theta r_u(x) - \frac{(2\theta-1)(1-\lambda)r_V\varepsilon_{u(x)} + (1-\theta)\varepsilon_{MR} + (2\theta-1)\lambda r_V\varepsilon_{u(x)} - \theta\varepsilon_{MR}}{\varepsilon_{MR} - r_V\varepsilon_{u(x)}}\right) =$$

$$= L\frac{x}{1 - r_u(x)}\left(-(2\theta-1)r_u(x) - \frac{(2\theta-1)r_V\varepsilon_{u(x)} - (2\theta-1)\varepsilon_{MR}}{\varepsilon_{MR} - r_V\varepsilon_{u(x)}}\right) =$$

$$= L\frac{x}{1 - r_u(x)}(2\theta-1)\left(-r_u(x) + 1\right) = (2\theta-1)Lx$$

So, we conclude that the difference in capital prices

$$\frac{\partial(\pi^H - \pi^F)}{\partial \tau} = (2\theta-1)Lx$$

doesn't depend on capital asymmetry and always increases under $\theta > \frac{1}{2}$.

5.10 Appendix K: simulations confirming robust globalization effects on capital price

We are going to show an example of monotone behavior of firm sizes for wide range of levels of trade cost ($1 \leq \tau \leq 1.25$), shown in Figure 6 below. The idea is to confirm that Proposition 8 is likely to be extended from $\tau \approx 0$ to a wide range of trade cost. We take low-tier utility function $u(x) = \sqrt{x} \pm \frac{2}{5}x$ and upper-tier utility $V(m) = \ln(m)$ (similar results

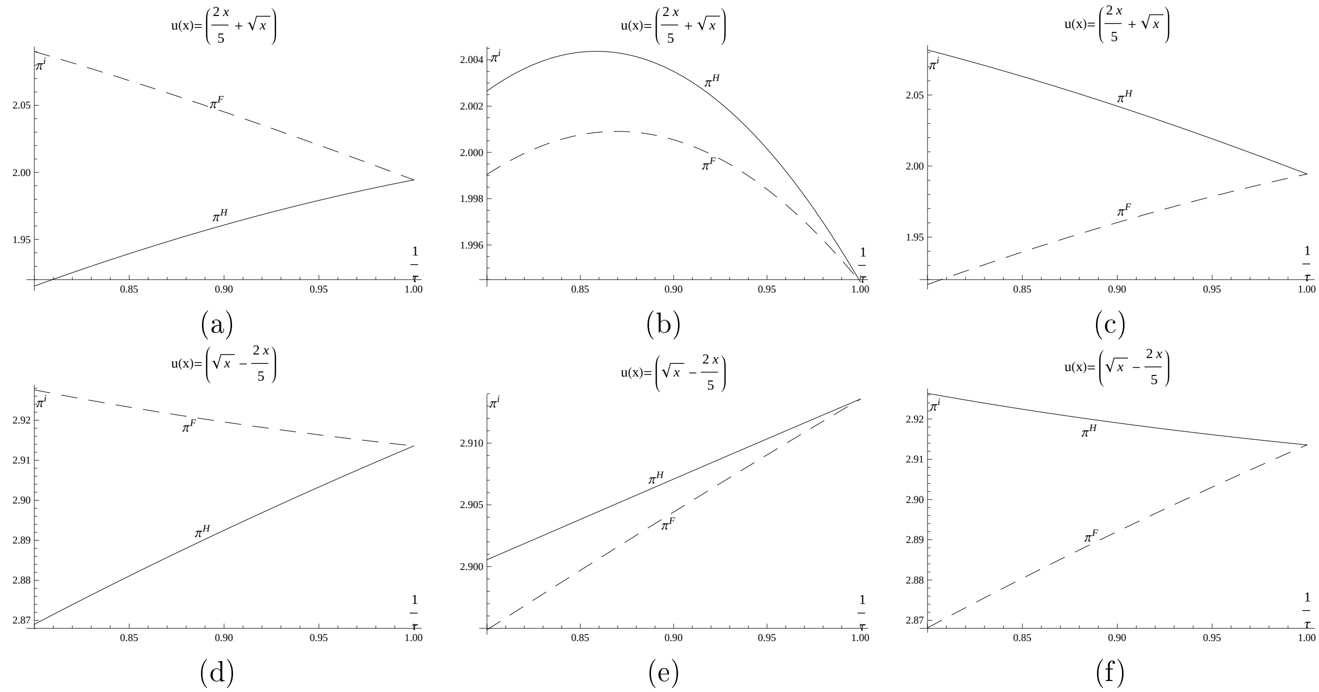


Figure 6: Capital prices behavior under trade liberalization.

we obtained in several other examples).

Using $K = 1$ and $L = 10$, Fig. 6 presents pattern of changes in π^H, π^F when trade freeness $1/\tau$ increases from 0.8 to 1. In particular, pictures (a)-(c) display the case $\hat{\theta}^H(\lambda) < \hat{\theta}^F(\lambda)$ related to Figure 3a (left panel) with DED-class utility $u(x) = \sqrt{x} + \frac{2}{5}x$ (we did not find such a pattern under IED utilities). Everywhere we observe monotonicity on $1 \leq \tau \leq 1.25$ except case (b) where monotone behavior holds only for trade cost $1 \leq \tau \leq 1.10$. Namely, (a) shows $\lambda = 0.51 < \hat{\lambda}^H = 0.972$ and $\theta = 0.4 < \hat{\theta}^H = 0.470$; (b) shows $\hat{\theta}^H = 0.470 < \theta = 0.505 < \hat{\theta}^F = 0.530$ and $\lambda = 0.51$; (c) shows $\theta = 0.6 > \hat{\theta}^F = 0.530$ and $\lambda = 0.51$.

By contrast, Fig. 6 pictures (d)-(f) relate to Figure 3b (right panel) and $u(x) = \sqrt{x} - \frac{2}{5}x$: the pattern for case $\hat{\theta}^H(\lambda) > \hat{\theta}^F(\lambda)$. Here case (d) shows $\lambda = 0.51$ and $\theta = 0.4 < \hat{\theta}^F = 0.439$; (e) shows $\hat{\theta}^F = 0.439 < \theta = 0.51 < \hat{\theta}^H = 0.561$ and $\lambda = 0.51$; (f) shows $\theta = 0.6 > \hat{\theta}^H = 0.561$ and $\lambda = 0.51$.

Commenting these simulations, we observe that in the range $1/\tau > 0.9$, i.e., $\tau < 1.1$ the conclusions of Proposition 8 hold true.