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на тему: **Pricing options on stocks with non-homogenous jump dynamics: The case of litigation jumps**

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Аннотация

Данная работа посвящена анализу и оценке опционов на акции компаний, часто становящихся участниками судебных разбирательств, претензий регуляторных или антимонопольных органов. Выбор именно этой тематики обусловлен приобретенной в последние годы «популярностью» судов как формы конкуренции компаний и направления их существенных расходов. В первую очередь настолько значительной статьей расходов стали патентные споры среди IT-компаний, таких как Apple Inc., Samsung и т.п.

Рассматриваются публичные компании, акции которых могут быть подвержены неожданным скачкам на фоне новостей, что нарушает предпосылку о непрерывности динамики финансовых инструментов, необходимую для применения базовый формул их оценки. Автор ставит себе цель не только пополнить «банк моделей» опционов новым подходом к расчету влияния скачков, но и оценить скрытый рыночный параметр – подверженность компании резким изменения в рыночной стоимости из-за новостей определенного типа. Этот показатель можно рассматривать как рыночную оценку рискованности участия компании в определенных регуляторных спорах. Он может быть использован на практике в принятии решений касательно юридической политики фирм, а также при взвешивании перспектив и вероятного исхода судебных тяжб, в которых участвует компания, инвесторами и аналитиками.

В работе рассматриваются несколько основных типов судебных процессов. При моделировании процесса появления новостей о судебных процессах на рынке используется новый подход: вместо того, чтобы оценивать динамику этого процесса на основе данных каждого деривативa в отдельности, предлагается проанализировать статистику появления ключевых слов, относящихся к данному итпу судебных процессов, в запросах и новостях системы Bloomberg – основной базы данных, используемой в работе могими профессиональными участниками рынка. На каждый из рассматриваемых типов разбирательств подобраны ключевые слова, позволяющие построить и оценить процесс появления новостей на финансовом рынке. На основе этих процессов оценивается динамический параметр интенсивности прихода новостей, становящихся причинами скачков в ценах акций компаний.

Волатильности непрерывных и скачкообразных компонентов движения акций оцениваются на основе модели, решение которой получено схожим образом с работой Merton (1976), но с учетом динамки параметра интенсивности скачков. В качестве базы для эмпирического анализа выбраны компании, наиболее часто становившиеся участниками тяжб каждого из рассматриваемых типов за 2010-2013 годы. Оценка интересующих нас параметров по срезу опционов каждой из компании подтверждает, что в некоторых случаях предложенная в данной работе модель более адекватна рыночным данным, чем ранее разработанные стандарты оценки. Более того, компании, которые оказалось предпочтительнее оценивать с помощью нашей модели, находятся в категории судебных тяжб с наибольшим количеством публичных компаний с ликвидными деривативами, что позволяет предположить, что эти категории разбирательств действительно вызывают наиболее сильный отклик на рынке.

Contents

Introduction

The mathematics of financial markets have greatly evolved over the past decades, allowing market players to price complex securities and derivative contracts based on various contingencies. However, in the real world an infinite range of extraordinary events may occurr which still remain to be accounted for in quantative models.

Stock options are one of the basic kinds of actively traded derivative contracts. The baseline approach to their pricing, which was developed in the 1970s by Black, Scholes and Merton, assumes a continuous process governing share price movements. However, it was soon discovered that this assumption is not always true in practice. For instance, the largest drop if the Dow Jones Industrial Average (DJIA) occurred in the midst of the global 2008-2009 financial crisis, on November 15th, 2008, when the index fell 7.87%[1]. By contrast, in November 2012 the NYSE and NASDAQ experienced a rally, with the DJIA jumping 1.7%, S&P500 increasing 2%, and Apple Inc stock (NASDAQ: AAPL) rocketing 7.2% on November 19th [2].

These examples indicate that stocks and equity indices cannot be modeled as continuous diffusions, as their most drastic changes often happen instantaneously upon news arrival. The latter fact has spurred quite a large and diverse stream of research on incorporating jumps in security price processes.

While certain authors have developed option pricing methods without taking into account the nature of events causing jumps, others have distinguished between events governing the discontinuities and built models for certain categories. The first type of events to come to mind, and consequently one of the most "popular" ones to model, is earnings announcement. Financial statement publications are regular and can easily be tracked for all publicly traded companies; besides, their dates are usually known in advance. However, it is slightly harder to prove the importance of singling out and designing new models for other kinds if news, which may be infrequent, require tedious data collection or appear to be significant only for certain industries.

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[1] S&P Dow Jones Indices website (http://www.djindexes.com/)

[2] The Wall Street Journal Blogs, MarketBeat, 19.11.2012 (http://blogs.wsj.com/marketbeat/2012/11/19/apple-rally-third-biggest-jump-since-financial-crisis/)

This work aims to contribute to jump process modeling in finance by considering a specific type of jump-inducing news: litigations, antimonopoly and other regulatory proceedings.

Such a choice was motivated by the apparent increasing influence on equity stock prices legal conflicts have had in recent years. Perhaps the most prominent evidence of their effect are company expenses, as well as market value gains and losses caused by patent wars mostly involving IT companies. According to *the New York Times*, as at October 2012 IT giants such as Apple Inc and Google's patent litigation legal costs exceeded their R&D expenses[3]. The latter only concerns direct costs; the most severe consequences are stock market responses. For example, after Apple Inc's August 2012 $1.05 bln court victory over Samsung Corporation in a legal dispute over software and design copyright not only the dispute's direct parties, but their market competitors and other firms engaged in large-scale patent disputes were affected. On the news announcement day, Samsung's stock fell 7.5% on the South Korean stock market, while Apple increased 1.88% on the NYSE. Moreover, Microsoft (NYSE:MSFT) shares rose 0.4% and Nokia (HE:NOK1V) increased 7.7%, as these companies manufactured devices based on an alternative operating system, Windows Mobile, rather than the "loser" Android. Another Android proponent, Google, fell 1.4% on the NYSE[4].

The aforementioned example demonstrates that sometimes the companies most affected by litigation-related news are not the ones directly involved in it, but other firms sensitive to this type of legal conflicts. Indeed, any court verdict may become precedent for future legal disputes, and thus greatly influence the outcomes of other lawsuits in the same legal field. Therefore this thesis analyzes news related to specific litigation categories, and their impact on public companies engaged in a large number of such court proceedings. Consequently, the news-generating process in this paper is not company-specific but based on market news arrival data. This differs from the approach undertaken by most researchers, who assume the news arrival parameters are unique for each entity, which is likely to be true only for a narrow sample of firm-specific events not linked to the remainder of the market in any way.

Nevertheless, our model still contains a news-related parameter estimated for each company: the jump volatility due to litigation news of a certain kind. This is another novel feature of our research, which may be used not only in option pricing but in corporate policy decisions: it may be helpful in

[3] The New York Times website, Technology, 7.10.2012 (http://www.nytimes.com/2012/10/08/technology/patent-wars-among-tech-giants-can-stifle-competition.html?pagewanted=all&_r=0)
[4] BBC News website, Technology, 27.08.2012 (http://www.bbc.co.uk/news/technology-19389732)

the estimation of market risks associated with filing a lawsuit or continuing a legal conflict instead of trying to reach a settlement.

It is also important to point out that this paper uses data which, to our knowledge, has never before been used in stock price modeling and option pricing: news keyword search count provided by the Bloomberg Terminal system. We choose keywords which are able to highlight news related to the litigation areas of interest. Later, the search count is fitted to a distribution, which is incorporated in the option pricing model and used for further estimations. In our view, this approach is an adequate way to model news flow, as it captures the instantaneous intensity of topic appearance in the headlines, and uses Bloomberg, one of the main information systems used by financial market professionals.

The remainder of this thesis is organized as follows. First, we provide an overview of existing academic research, both in the field of stock market discontinuities in option pricing and the impact of litigation events on the stock market. A description of the proposed mathematical model and the derivation of an option price formula follow, accompanied with the interpretation of model parameters which may help to characterize the litigation process itself, not only serve for claim pricing. We then outline the estimation and model evaluation procedures, the main characteristics the company sample chosen for testing and the data used. Finally, we present the estimation results and conclusions derived from the comparison of our model with benchmarks developed by other researchers.

Chapter I: Background

I. Option pricing with jump diffusions in the stock price process

The baseline approach to pricing European stock options (options which may only be exercised upon maturity), the Black-Scholes-Merton model, was first published in 1973. The evolution of a security price in this paper was modeled with the following stochastic differential equation:

$$dS_t = \alpha S_t dt + \sigma S_t dW_t$$

In this model, the stock price is governed by a Wiener process (W_t) – an almost surely continuous stochastic process with independent, normally distributed increments. One of the first extensions accounting for discontinuous movements was developed by one of the above mentioned researchers, Merton, who included jumps randomly occurring at unexpected times (Merton, 1976). As an analogue of the continuous-time Wiener process, a Poisson jump process which would govern abnormal stock price variations was introduced. The result of this paper constituted a new way to price European options, which, however, did not imply a closed-form solution.

The subsequent stream of research was mostly dedicated to generalizing models of security price dynamics so that they could encompass jump dynamics but still allowed more or less concise option price formulas. Apart from simply evaluating unexpected market events, jump dynamics appeared to be able to solve certain empirical puzzles, such as the "volatility smile": the fact that implied volatility tends to decrease with the increase of moneyness (ratio of the current share price to the strike price), reach a minimal value for at-the-money options and afterward start to rise again (Derman, Kani, 1994). Papers capturing "smile" effects without incorporating jump diffusions used such approaches as dynamic variance with constant elasticity, which allowed to extend the base model without overcomplication (Cox and Ross, 1976); stochastic volatility with stochastic interest rates (Amin and Ng, 1993; Bakshi and Chen, 1997). Discontinuous modifications aimed to solve the aforementioned empirical puzzle include stochastic volatility in the presence of jump diffusions (Bates, 1996) as well as a unification of the several approached given above: models with stochastic volatility, stochastic interest rates and jump processes (Bakshi, Cao, Chen, 1997). Some researchers further complicated their works by modeling variance as a Markov-switching process, thus assuming that volatility is governed by an unobservable process and is dependent on a number of possible "states of the world" (Rubinstein, 1994; Satoyoshi, Mitsui, 2010).

A popular choice for the theoretical representation of the jump diffusion is a Poisson process. This is due to the fact that a Poisson jump diffusion is a well-known Levy process which, among other things, allows the application of an analogue of the Ito lemma necessary for option prices. The intensity parameter (λ) of the Poisson distribution in most early models is assumed to be constant for a given stock. This assumption leads to the relative simplicity of the models, but is effectively groundless from an empirical point of view.

Nevertheless, numerous papers incorporate dynamic jump intensity; however, such models typically adopt more complicated mathematical and econometric methodologies than the ones used in the papers mentioned above. Certain works introduce both stochastic volatility and stochastic jump intensity; however, the solution of their models requires the rather complex application of Fourier transforms (Kangro et al., 2000; Yan, Hanson, 2006). Dynamic jump intensities are also analyzed in several papers deriving option prices in a portfolio optimization framework, postulating individuals' wealth and intertemporal utility function properties with a CAPM methodology (Kurmann, 2009; Asea, Ncube, 1997). To generalize the analysis for American options (those that can be exercise at any time till maturity) on moderate and high dividend yield stocks, certain papers (Hilliard, Schwartz, 2005) use a bivariate tree approach, the first grid representing the changes in the smooth volatility component while the second one consisting of Poisson jumps of log-normal size. This approach's advantage is the absence of complicating analytical derivations and straightforward computerized implementation.

Other approaches to include dynamic intensity and cope with analytical complications include steps away from the standard combination "lognormal size of jumps – Poisson distribution of jumps arrival process". These include modeling jumps as a double exponential diffusion with asymmetric probabilities of upward and downward jumps (Kou, 2002; Cai, Ning, Kou, 2011) and as a log-negative-binomial process (Heston, 1993) – the latter model allows the resulting option price formula to be independent of the jump probability.

Non-financial literature sometimes uses another type of distribution: mixed Poisson distribution with the intensity parameter following a Gamma process. This event arrival modeling approach has been used to describe processes ranging from hospital admissions and disease incidence (Savani, Zhigljavsky, Zeger, 1988; Bräannäas, Johansson, 1994) to consumer purchase behavior (Savani, Zhigljavsky) and daily number of price change durations (Heinen, 2003). This distribution is relatively easy to incorporate because it maps into a negative binomial one. It is mostly used to model "fat-tailed" news arrivals: when large waiting times between occurrences of a process are more likely than in the standard, fixed-intensity Poisson process. This paper models news arrival with this distribution, which allows to extend the Poisson news arrival model but still results in an analytical formula for the option price. The empirical rational for using this distribution is provided further on.

Developing an option pricing formula need not be the single objective pursued. One of the corollaries of analytical option pricing is the estimation of a stock's implied variance (IV). Johannes, Dubinsky (2005) consider jumps caused by prescheduled earnings announcements and use the resulting IV as a measure of uncertainty of the firm's financial results to be published. The authors use several estimators for IV: a time-series (ex-post) estimator and another one based on options with different maturity expiring after an earnings announcement (ex-ante). This thesis also uses implied model parameters as indications of a company's unobserved characteristics.

As stated before, existing literature on option pricing with jump diffusions usually concentrates on unexpected jump processes in general, not elaborating on the nature of events causing the jumps. However, some papers focused on analyzing specific types of events. Most frequently presented in related works are earnings announcements – investigated, for instance, in the aforementioned Johannes, Dubinsky (2005). Still, there are other types of events that have not been neglected in existing literature. These include mergers & acquisitions (Subramanian, 2004) and FDA responses for health care companies (Linnel, 2012). The former analyzes shares of merging firms to develop a way to incorporate a non-continuous jump in their dynamics. This model differs from most of the others by its relevance to the issue under consideration: the author considers the evolution of M&A parties' stock prices based on the probability of a merger succeeding and proves that their ratio should be deterministic so that it can reach a pre-defined value in case the merger goes through. Further on, Subramanian points out that in case the merger is called off, the share dynamics should change to certain base price processes, which may be derived from related stocks not involved in the agreement. This model also leads to the estimation of an implied parameter relevant for fields outside option pricing: the implied probability of a merger's success.

Linnel (2012) looks into quite an important area of event research: FDA enquiries and recommendations issued on medical and pharmaceutical companies' products. These events are highly relevant for Biotech industries, as a single FDA decision may kill a firm's entire line of business. Furthermore, these decisions are hard to predict for non-pharma professionals, so any working financial analysis tool would be useful. This paper aims to construct a similar kind of tool in the legal domain.

II. Stock market effects of litigations

A number of researchers have already investigated the abnormal returns and peculiarities of stock price dynamics associated with litigation news. Bhagat et al. (1998) find not only that abnormal returns for publicly traded plaintiffs and defendants are significant, but their variation may also be explained by such factors as the subject of the legal dispute, type of opponent (e.g. government or regulatory body, another legal entity, a private citizen), a firm's market capitalization and parameters indicating high risk of insolvency. Bizjak and Coles (1995) limited their event study to antitrust litigation and find significant value losses among defendants, which grow with new lawsuit filings. They also stress that the main threat that explains these wealth losses is not the simple prospect of legal expenses, but the potential prohibition of engagement in certain business practices. These two works refer to an article which focuses on a litigation between Texaco and Pennzoil over the takeover of Getty Oil Company and states that a massive value loss the parties underwent during the litigation (over $3 bln. in total) was only partially regained afterwards, and the loss of value of the defendant did not match the gain in value of the plaintiff (Cutler and Summers, 1988). The case described in this paper is strong evidence of dead-weight losses associated with legal conflicts.

Other papers further categorize lawsuits in interfirm and non-interfirm ones, pointing out that the stock market reacts significantly only to corporate (interfirm) legal conflicts. However, this reaction is non-symmetric in terms of gain and loss balance: the losing side often bears higher overall expenses that the winning side gains as a result of the dispute. Consequently, there is evidence of dead-weight litigation costs (Koku et al., 2000).

Certain papers highlight the need for direct market estimation of court risks. Some of these works are devoted to litigation participation securities, which represent shares of the income resulting from a litigation (awarded by a court as claim recovery or else), which lead to positive excess returns, being indicative of a company's confidence in its case. These securities' market prices provide estimates of the market's belief that the firm will prevail in a dispute (Esty, 2001). The latter paper further argues that precedents are highly important for further litigations of a given type, so such litigation-linked securities trading may provide market reaction characteristics that can influence further related cases. Indeed, the existence of such a security on the market would solve many problems related to projected lawsuit impact estimation; however, this is rarely the case with most litigations.

The following conclusions can be derived from this brief overview of litigation effect research. First of all, litigation news do generate abnormal returns, and they often have an impact not only on the litigants themselves, but on participants of other court disputes for which the case may serve as precedent. Secondly, lawsuits result in dead-weight costs, thus being an important concern both for investors and corporate legal departments considering involvement in court proceedings. Finally, the "marketization" of legal risks (the introduction of a security linked to gains or losses ensuing from a lawsuit) gives a valid idea of market perception of the case and is significant for further similar litigations. This thesis attempts to provide implicit estimates of a company's vulnerability to certain lawsuits, which may be helpful when a security of the aforementioned type does not exist.

Chapter 2: The model

I. Valuation formula

Following the general stream of related research, we model the stock price process as a geometric Brownian motion with a jump component:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t + dj_t$$

$$dj_t = \begin{cases} Y_t - 1 \; if \; a \; jump \; occurs \\ 0 \; otherwise \end{cases} \tag{1}$$

$$Y_t \sim i.i.d.N(1; \sigma_j)$$

In terms of the probability space illustrating the stock market at t – $(\Omega, \Im_t, P)$ – the stock price evolution process consists of an $\Im_t$-adapted continuous diffusion and an also $\Im_t$-adapted jump process. The news arrival intensity and jump size ($Y_t - 1$) are independent.

We assume that news-related jump size has a zero mean, which is due to the fact that pricing is done under the equivalent martingale measure $\mathbb{Q}$. Under this measure, the stock price process is a martingale, therefore $E^{\mathbb{Q}}(S_t|\Im_{t-}) = S_{t-}$ - the expected change in value, whether due to the continuous or jump diffusion components, is zero. The news arrival process has a Poisson distribution, as in Merton (1976); however, its intensity λ is also a random rather than constant variable. The jump diffusion has the following characteristics:

$$f_t(n, \lambda_t) = \frac{{\lambda_t}^n e^{-\lambda_t}}{n!}$$

$$\lambda_t \sim \Gamma(m, \theta): f_\Gamma(x, m, \theta) = \frac{1}{\theta^m} \cdot \frac{1}{\Gamma(m)} \cdot x^{m-1} e^{-\frac{x}{\theta}} \tag{2}$$

$$\Gamma(m) = (m-1)!\ for\ m \in \mathbb{Z}^+$$

$$\Gamma(m) = \int_0^\infty e^{-t} t^{m-1} dt\ \ for\ m \notin \mathbb{Z}^-$$

Appendix I provides the proof of the fact that this distribution mixture maps into a negative binomial distribution. This results in the following probability of a fixed number of jumps occurring over an option's life:

$$P(X = n) = \frac{(1-p)^m p^n (n+m-1)!}{n!\,(m-1)!} \tag{3}$$

For small degrees of freedom, the density of the negative binomial distribution is concentrated near zero and has a "fat" tail, which is precisely what characterizes most influential lawsuits: a lot of news arrive around the dates of certain court sessions stating the parties' reactions etc.; however, major litigation stages are concluded with lengthy time gaps.

In terms of modeling news flow as a market or industry-specific process, a possible methodology would be introducing Markov processes and modeling hidden states of the world (litigation intensity). However, this paper assumes that court dispute intensity is not actually hidden, but can be estimated based on the data which will be presented below. Even more importantly, Markov models do not allow to construct a relatively simple analytical solution to the model, which is given below.

We proceed with a European call option price derivation, a full version of which is given in Appendix 2. The calculations basically follow the Merton (1976) approach, which results in a formula of the following type (we denote the number of jumps occurring by *X*):

$$C(S_t) = \sum_{n=0}^{\infty} Prob\{X = n\} \cdot BS(S, K, T, r, \sigma_{X=n}) \tag{4}$$

In our case, the probability of the number of jumps equaling n is based on the negative binomial probability distribution function. Below is the resulting formula which will be used for further estimations:

$$C_0 = \sum_{n=0}^{\infty} \frac{(1-p)^m p^n (m+n-1)!}{n!\,(m-1)!} BS(S, T, K, \sigma_n^{2\prime}, r) \tag{5}$$

Where $\sigma_n^{2\prime} = \sigma^2 + \frac{s}{T}\sigma_j^2$ is a volatility measure resulting from both kinds of stochastic evolutions (continuous and non-continuous).

II. Comparative statics

The results of theoretical estimation of this paper's proposed model, accompanied with the constant intensity Poisson jump model and simple Black-Scholes with no jumps, are given below. In each case we have chosen Gamma distribution parameters to equal the Poisson fixed intensity in mean for them to be comparable.

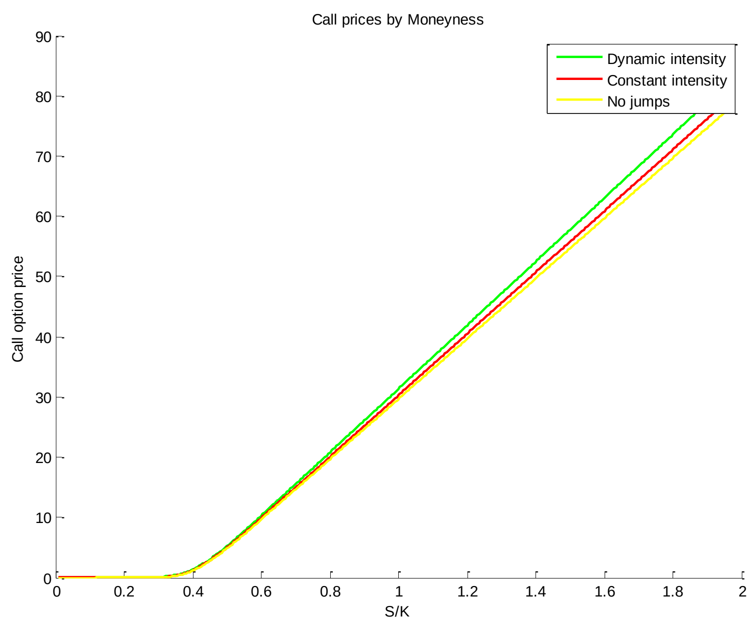


Figure 1. Call option prices for T=20; K=50; r=0.03; y=0; σ=0.03; σ_j=0.25; λ=0.4;k=3; $\theta = {}^{\lambda}/_{k}$

We can see that, when the expected number of news arriving on a given day is low, the three models provide almost equal option prices.

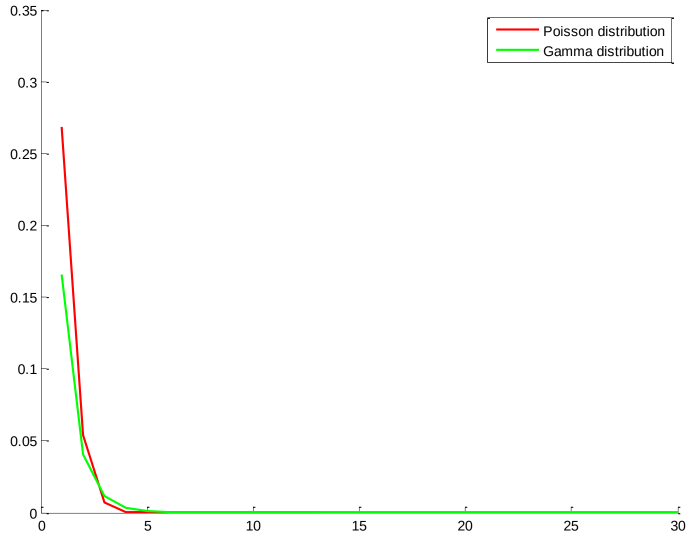


Figure 2. Poisson and Gamma probability distribution functions evaluated along integers for λ*=0.4;k=3;* $\theta = {}^{\lambda}/_{k}$

Indeed, distributions are quite similar for this combination of parameters.

For high times to maturity, considerable jump volatilities compared to the continuous diffusion volatility and a high number of jumps for which the infinite sum in jump process pricing formulas is truncated, both jump diffusion models yield high jump premiums and similar prices:

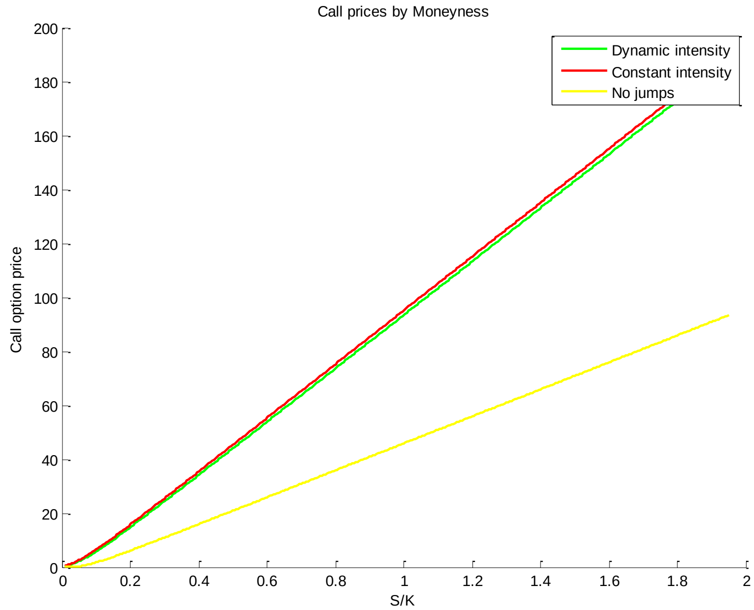


Figure 3. Call option prices for T=50; K=50; r=0.03; y=0; σ=0.1; σ_j=1; λ=0.3;k=0.2; $\theta = {}^{\lambda}/_{k}$

The corresponding distributions are also quite similar:

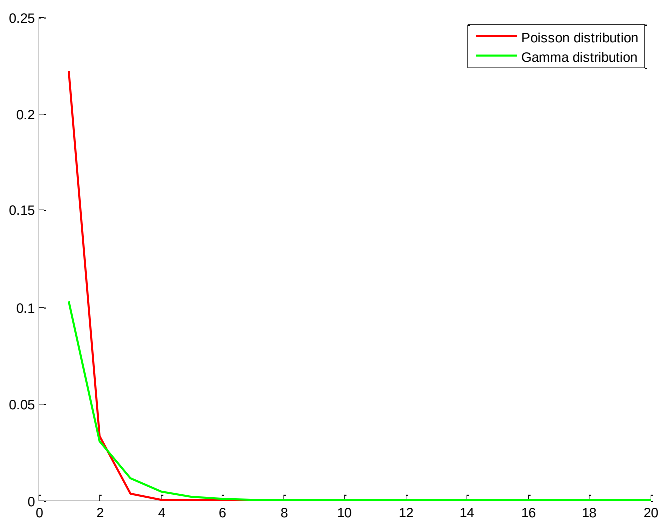


Figure 4. Poisson and Gamma probability distribution functions evaluated along integers for λ=0.3;k=0.2; $\theta = {}^{\lambda}/_{k}$

At lower maturity times, the jump premium from the constant intensity model exceeds the one yielded by the dynamic intensity one. This is due to the fact that the Gamma distribution of jump intensity has “fat tails”, making extreme numbers of jumps over a short period of time more probable; however, a short maturity horizon limits the possibilities of extreme event arrival numbers.

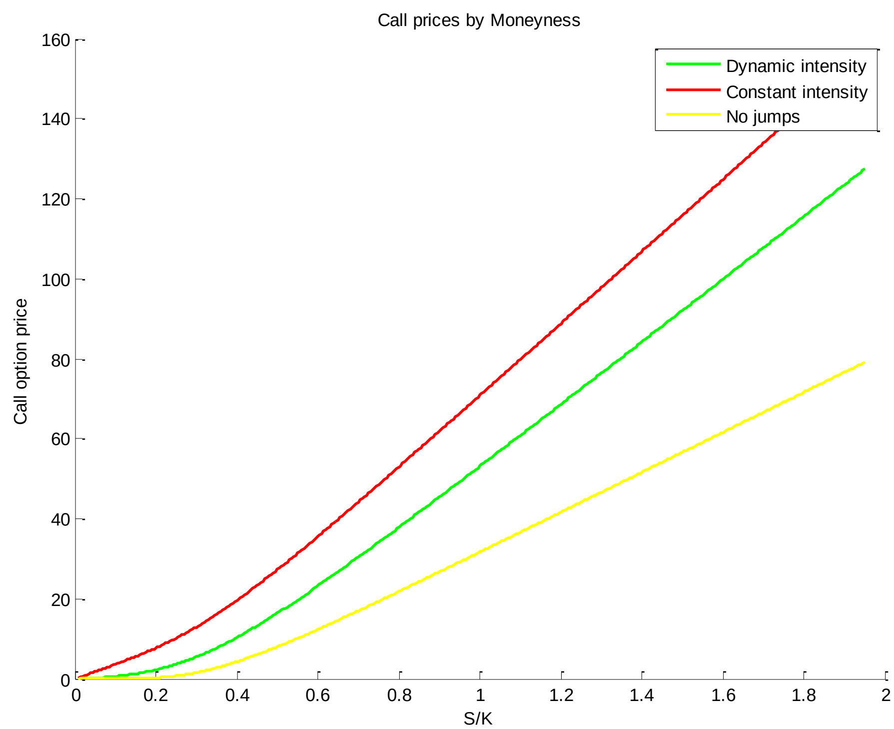


Figure 5. Call option prices for T=20; K=20; r=0.05; y=0; σ=0.1; σ_j=1; λ=0.8;k=0.2; $\theta = {}^{\lambda}/_{k}$

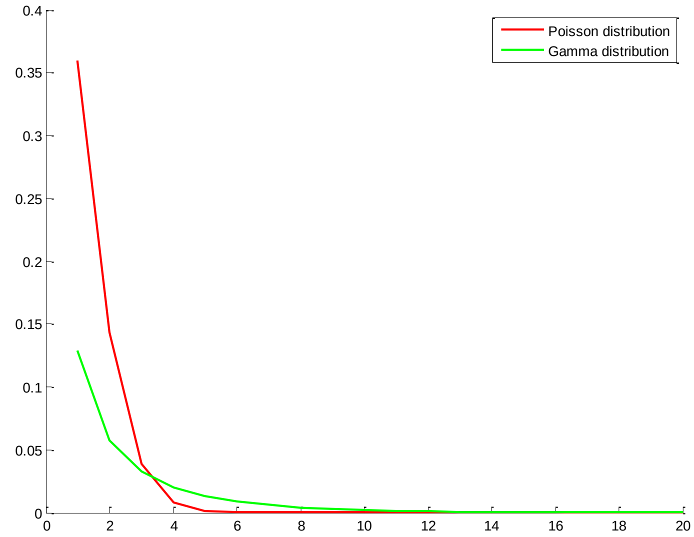


Figure 6. Poisson and Gamma probability distribution functions evaluated along integers for λ*=0.8;k=0.2;* $\theta = {}^{\lambda}/_{k}$

However, for high scale parameters of the Gamma distribution at short option maturities the litigation jump risk premium may attain high level compared to other models:

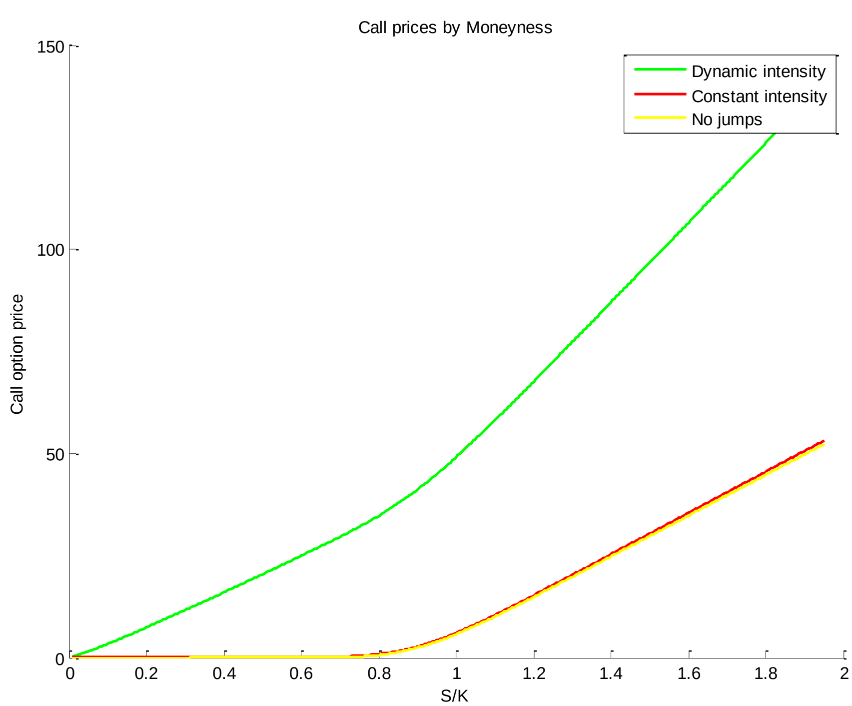


Figure 7. Call option prices for T=2; K=4; r=0.05; y=0; σ=0.1; σ_j=1; λ=3;k=4; $\theta = {}^{\lambda}/_{k}$

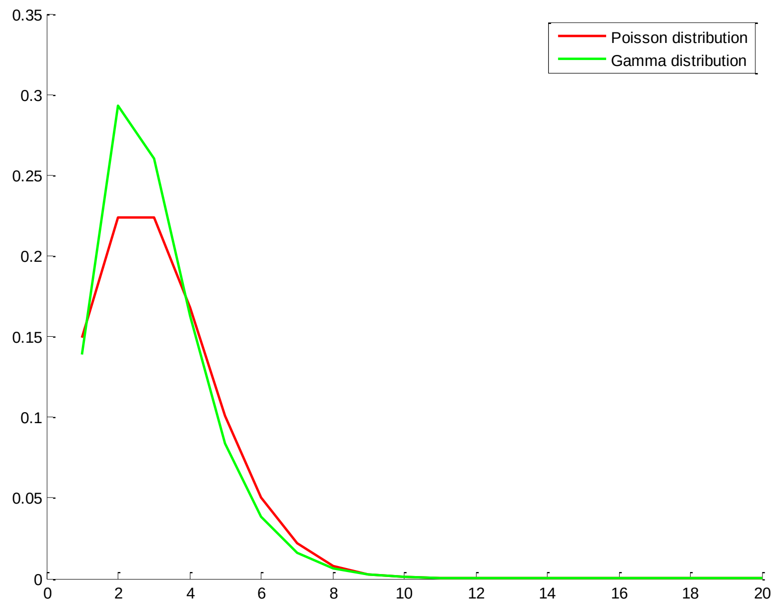


Figure 8. Poisson and Gamma probability distribution functions evaluated along integers for λ=3;k=4; $\theta = {}^{\lambda}/_{k}$

With such distribution characteristics, the dynamic intensity model leads to higher probabilities of a relatively small number of jumps occurring; given the short interval till maturity, this results in higher expected volatility induced by numerous jumps.

Evidently, depending on the time to maturity of a given option and the distribution characteristics of a given jump process, the three different models may yield similar or greatly varying results. One would expect the jump diffusions produce similar results on long horizons to maturity when their respective jump intensities are low. The comparison result will depend on the shape and scale of the Gamma distribution on short maturities with high jump intensities.

The next section will provide the empirical rationale for choosing a dynamic Gamma-distributed new arrival intensity parameter for the case of litigation news.

Chapter 3: Methodology

I. News arrival process

As previously stated, this model does not consider the news to be purely company-specific, but rather deals with news on a stream of litigation which numerous public companies are involved in and therefore sensitive to.

In litigations, it is often impossible to single out two or even three distinct parties of a dispute. Moreover, new related to a certain legal conflict can affect other similar disputes, as it acts as a precedent and changes investors' beliefs concerning other proceedings' outcomes.

The Bloomberg Terminal provides a wide range of news-related research tools. One of them is the possibility of extracting the story count for certain keywords appearing in news. This option is provided in the News & Research Menu of the terminal (TREN<GO>). In the opinion of the author of this work, this story count is a good proxy of the intensity of litigation news arrival – it represents the actual number of news stories involving the word combinations of interest at a given point in time. Below is a comparison of graphs illustrating the story count for a patent lawsuit-related query and Apple Inc. call option implied volatility calculated by Bloomberg.

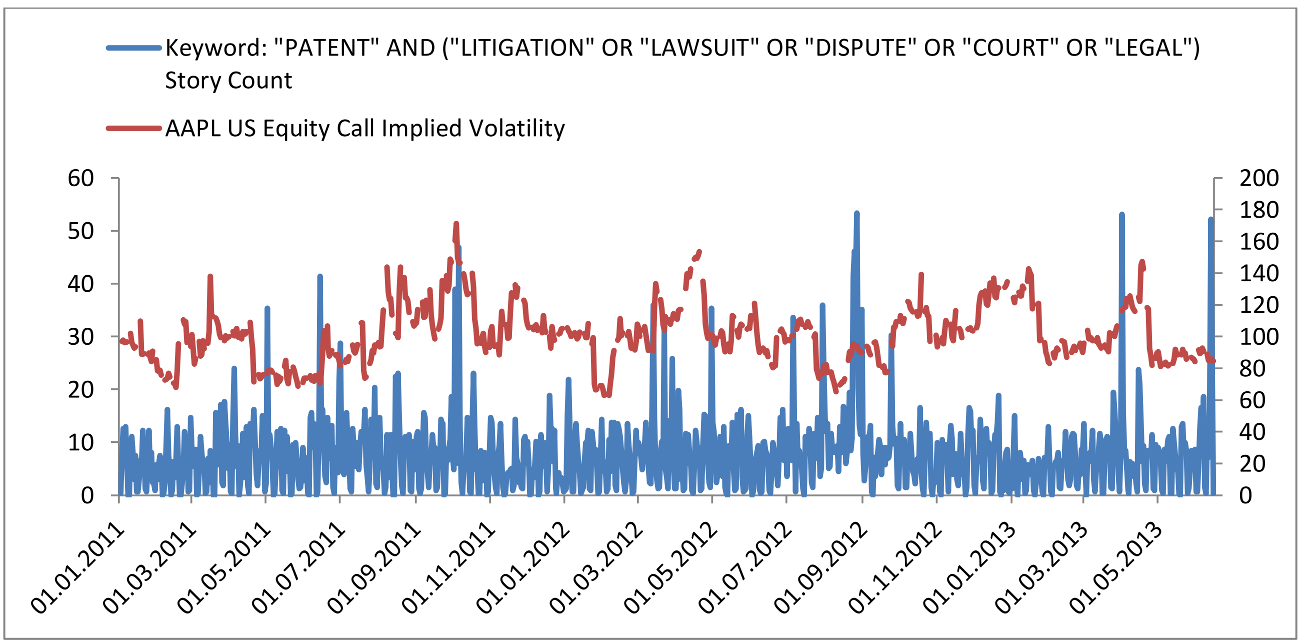


Figure 9. Bloomberg News& Research trend analysis, story count for patent litigation query, compared to AAPL call option implied volatility, Jan 2011 – Jun 2013

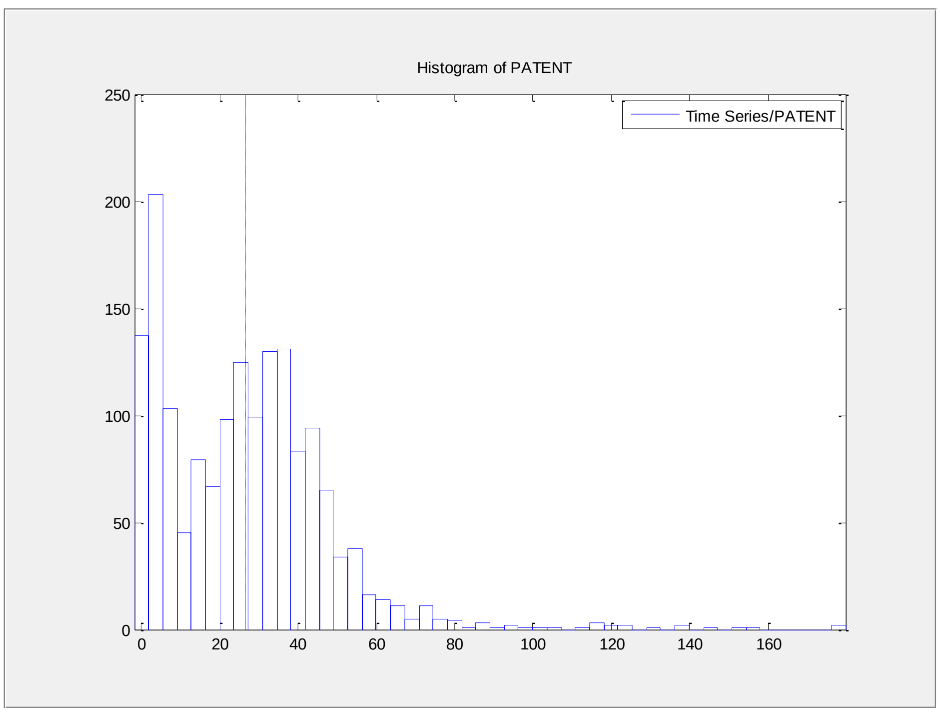
Apparently some news arrival intensity peaks coincide with unusually high levels of implied volatility, which may indicate that a jump component related to these peak intensity levels is present in the option's dynamics.

We have constructed several keyword queries to capture the news on the main types of litigations which may be of interest and also involve a large number of publicly traded companies with liquid options trading on the market:

"PATENT" AND ("LITIGATION" OR "LAWSUIT" OR "DISPUTE" OR "COURT" OR "LEGAL")
"FRAUD" AND ("LITIGATION" OR "LAWSUIT" OR "DISPUTE" OR "COURT" OR "LEGAL")
("REAL*" AND "PROPERTY") AND ("LITIGATION" OR "LAWSUIT" OR "DISPUTE" OR "COURT" OR "LEGAL")
"INJURY" AND ("LITIGATION" OR "LAWSUIT" OR "DISPUTE" OR "COURT" OR "LEGAL")
"ANTITRUST" AND ("LITIGATION" OR "LAWSUIT" OR "DISPUTE" OR "COURT" OR "LEGAL")
"BANKING" AND ("LITIGATION" OR "LAWSUIT" OR "DISPUTE" OR "COURT" OR "LEGAL")

Figure 10. Keyword search queries constructed to illustrate the main types of lawsuits considered

We continue by looking at the empirical distributions of these intensities. Below are the histograms of news story count for the main litigation types:



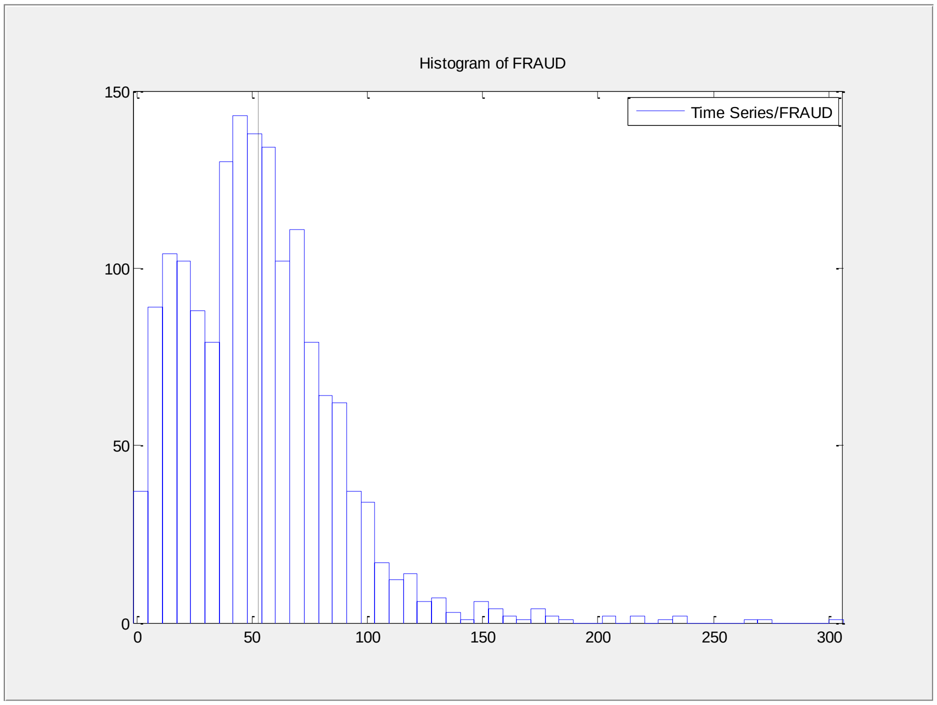


Figure 11. Histograms of story count for the main types of litigation queries considered

From these illustrations, it is clear that the information arrival intensity can hardly be viewed as constant. Also, it seems reasonable to model it with a gamma distribution given its theoretical form.

Having identified the type of news of interest, we continue with the following steps. First, for each type of news flow, we fit the empirical distribution to a gamma one (choosing a lookback interval for distribution parameter calibration). Each following day, the parameters are updated. Having obtained this distribution's historical properties, we incorporate it as a jump-generating process into the formula (5) we developed. We calibrate the models for each stream of litigation for a sample of companies with the highest number of docket filings for this type of legal proceedings. Jump volatilities for each stock, which we estimate in the course of model calibration as the average over its options, are interpreted as sensitivity levels to the given legal dispute types. We predict that these sensitivity parameters will be related to the percentage of cases of a certain type a company is involved in. We evaluate this hypothesis based on actual results later on.

We then compare this model to a basic version of the Black-Scholes-Merton formula with lognormal jumps estimated for the same sample of companies and to the simple Black and Scholes formula with no jumps. The Black-Scholes-Merton formula assumes company jump intensity estimation along with other parameters in the course of model calibration; it has one optimization variable more than the model we aim to estimate.

We calibrate model parameters by solving the following constrained optimization problem:

$$\min_{\sigma,\sigma_j} \sum_{t=0}^{T} \left(\frac{\sum_{n=0}^{MaxJumps} Prob\{n\ jumps\} BS\left(S, T, K, \sigma_n^{2'}, r\right) - M_t}{M_t} \right)^2 \tag{6}$$

$$s.t. A \leq \begin{pmatrix} \sigma \\ \sigma_j \end{pmatrix} \leq B$$

Where $Prob\{n\ jumps\}$ is calculated according to the previously derived formula (5). The default constraints used for all volatility parameters were $\{0\}$ as the lower bound for all volatilities and $\{2\}$ as their upper bound. This is a minimal restriction to the problem, which will allow us to capture the "natural" diffusion parameters.

II. Company sample and data

Our choice of companies for analysis was based on their presence among top 100 US public companies in terms of docket filings number as provided by Bloomberg Docket analytics (DCKT<GO>). Thus, for each legal news stream we choose some of the most active litigants. Naturally, the company selection was also based on the presence of actively traded options on these companies' stock. We also chose only low-dividend companies to eliminate early exercise effects in order to be able to use our formula for American options, not only European ones. Daily closing price and trailing dividend yields are obtained from Bloomberg.

Table 1. Sample of most active publicly traded litigants with liquid option contracts by litigation category

TICKER	NAME	GICS_SECTOR_NAME	GICS_INDUSTRY_NAME	SHARE OF DOCKET FILINGS AMONG TOP 100			
				2013	2012	2011	2010
Patent							
AAPL US Equity	APPLE INC	Information Technology	Computers & Peripherals	4,20%	1,65%	1,96%	2,43%
DELL US Equity	DELL INC	Information Technology	Computers & Peripherals	1,77%	0,70%	0,83%	1,02%
GOOG US Equity	GOOGLE INC-CL A	Information Technology	Internet Software & Services	1,59%	0,62%	0,74%	0,92%
MYL US Equity	MYLAN INC	Health Care	Pharmaceuticals	0,19%	0,07%	0,09%	0,11%
CSCO US Equity	CISCO SYSTEMS INC	Information Technology	Communications Equipment	1,12%	0,44%	0,52%	0,65%
Fraud							
BAC US Equity	BANK OF AMERICA CORP	Financials	Diversified Financial Services	2,05%	2,47%	2,22%	3,62%
TECUA US Equity	TECUMSEH PRODUCTS CO-CLASS A	Industrials	Machinery	0,00%	0,00%	0,00%	0,41%
CECO US Equity	CAREER EDUCATION CORP	Consumer Discretionary	Diversified Consumer Services	0,68%	0,35%	2,33%	1,65%
SHLD US Equity	SEARS HOLDINGS CORP	Consumer Discretionary	Multiline Retail	0,00%	0,71%	0,00%	0,21%
Real property							
CAM US Equity	CAMERON INTERNATIONAL CORP	Energy	Energy Equipment & Services	0,12%	0,01%	0,05%	1,31%
RIG US Equity	TRANSOCEAN LTD	Energy	Energy Equipment & Services	0,78%	0,01%	0,06%	0,94%
Personal injury							
OI US Equity	OWENS-ILLINOIS INC	Materials	Containers & Packaging	0,17%	0,28%	1,43%	2,11%
FWLT US Equity	FOSTER WHEELER AG	Industrials	Construction & Engineering	0,02%	0,06%	0,14%	1,01%
OC US Equity	OWENS CORNING	Industrials	Building Products	0,00%	0,01%	1,75%	0,85%
Antitrust							
NFLX US Equity	NETFLIX INC	Consumer Discretionary	Internet & Catalog Retail	0,00%	0,00%	0,15%	0,22%
Banking							
C US Equity	CITIGROUP INC	Financials	Diversified Financial Services	1,82%	1,57%	0,35%	0,17%

Collectively these companies cover a diverse range of industries, which makes them an interesting sample for analysis. The share of docket filings over the recent years represents a measure of each company's "contribution" to its legal conflict category. As mentioned above, one could suggest that the jump volatility parameter estimates we will get will be the implied analog of these sensitivities. Within litigation types covering the largest numbers of companies in the sample (patent, fraud) it will be possible to compare estimated jump volatilities and see if there ratios correspond to those of docket filing shares in any way.

Option price data and contract specifics are obtained from OptionMetrics, which provides information on daily bid and offer prices, strikes, types of contracts, maturity and so forth. The currently available data encompasses a period of time up to January 2013. We choose to analyze a two-year period (2011-2012) to incorporate the most up-to-date data available and cover a period of upsurge in certain legal activity, such as "patent wars".

When selecting option contracts for analysis, we eliminate certain extremes which may bias our calculations. We consider only contracts with implied volatility over 0.05 and less than 2. We also do not consider options which have not been traded for longer than 1 day to control for contract liquidity. We exclude options if they are a "special settlement", if the midpoint of the bid/ask price is below intrinsic value and if the underlying price is not available for that day. The levels of moneyness (the ratio of current stock price to the strike price) for options in our sample range from 0.5 to 2. We only considered option contracts for which prices are available for a minimum of 40 days, to exclude extremely close-to-maturity contracts which may result in unrealistic jump premiums based on our models.

Unfortunately, due to the significant amount of machine time and memory required the author was unable to estimate and average results over all option contracts satisfying the above mentioned conditions. Among the contracts satisfying our criteria, for each company we chose a minimum of twenty option contracts trading over the previously specified time period. These contracts were further used to estimate model parameters according to minimization problem (6). The evaluation of our model over different maturities, as done theoretically, and for various jump volatility constraints remains a topic for further research.

Results

The results of model parameters' estimation with the optimal pricing algorithm chosen for each stock are presented below:

Table 2. Results of chosen model's estimation on the sample of most active litigants: implied continuous diffusion volatility and jump volatility

Patent	Average share of docket filings	Gamma implied volatilty	Gamma implied jump volatility	Preferable model
AAPL US Equity	2,56%	0,275	0,191	Poisson Constant jumps
DELL US Equity	1,08%	0,356	1,118	Litigation Dynamic jumps
GOOG US Equity	0,97%	0,111	1,641	Litigation Dynamic jumps
MYL US Equity	0,11%	0,000	0,110	Poisson Constant jumps
CSCO US Equity	0,68%	0,223	0,350	Constant Poisson jups
Fraud				
BAC US Equity	2,59%	0,100	0,100	Ordinary BS
TECUA US Equity	0,10%	0,541	0,100	Ordinary BS
CECO US Equity	1,26%	0,105	1,753	Litigation Dynamic jumps
SHLD US Equity	0,23%	0,139	2,000	Litigation Dynamic jumps
Real property				
CAM US Equity	0,37%	0,331	0,100	Ordinary BS
RIG US Equity	0,45%	0,101	0,100	Ordinary BS
Personal injury				
OI US Equity	1,00%	0,359	0,148	Ordinary BS
FWLT US Equity	0,31%	0,286	0,712	Poisson Constant jumps
OC US Equity	0,65%	0,221	1,130	Poisson Constant jumps
Antitrust				
NFLX US Equity	0,09%	0,381	0,100	Ordinary BS
Banking				
C US Equity	0,98%	0,317	0,236	Poisson Constant jumps

The model proposed in this paper has been chosen as the one yielding the smallest mean square deviation from market option prices in four cases out of sixteen. These do not always correspond to the segments' leaders in terms of litigation volume (for the two absolute leaders in litigation share – Apple Inc and Bank of America Corporation – constant intensity Poisson jumps and the baseline BS model have been chosen, respectively. However, the firms which it appeared to be preferable to analyzed with our model can be considered "runners-up" in terms of litigation volume share; moreover, an important result is that the model has been chosen within the sector with the largest number of public companies with liquid options. These sectors may actually be the ones with the most sensitive market responses to litigation news; therefore, within these groups our results appear to be more trustworthy.

As expected, for the companies singled out by our model the implied Gamma jump volatility is quite high, meaning that a large proportion of these stocks' dynamics was due to litigation jumps.

Several companies – Bank of America Corporation, Mylan Inc and Transocean Ltd – appear to have been inadequately priced by our model: their resulting parameter values did not diverge much from initial values used in optimization. This may be due to the fact that their business activity is highly linked to other news-generating events, which may be more helpful in their pricing. For instance, Mylan Inc's legal conflicts constitute only a minor share within the patent dispute category.

The analysis performed is evidently only a first step toward the research if litigation news impact on options. The author would like to expand the estimation sample and conduct other tests (such as the distinction between long- and short-maturity options and imposing additional constraints on volatilities) in the future.

Conclusion

This thesis presents a model of option price dynamics involving jumps caused by legal disputes, regulatory proceedings and such. The jumps incorporated in the proposed model arrive governed by a Poisson process with a dynamic intensity rate. Assuming this rate to be gamma-distributed, we obtain an analytical solution to the model in a Merton (1976) framework. The rationale for choosing a gamma distribution for the intensity parameter is explained by actual data characteristics. A novel feature of this research is the estimation of jump characteristics based on actual news flow data, which is obtained from business news analysis data provided by Bloomberg Terminal.

Having modeled our version of stock price dynamics theoretically, we show that the Gamma-distributed intensity model is a very flexible one. In comparison with the fixed-intensity Poisson model and the baseline Black-Scholes formula, our model may yield very similar results on long horizons with low distribution scale parameter; an excessively high jump premium in case of short time to maturity and a high scale; and low premium relative to fixed-intensity jumps in case of short time to maturity and a moderate scale values.

Our empirical testing sample consists of the most prominent litigants within 6 legal dispute categories: patent, fraud, real property, personal injury, antitrust and banking. We have chosen public companies with actively traded option contracts and low dividend yields. As is often the case in similar research, a complete model empirical testing course would take up a lot of machine time and resources. We have limited our scope to option most appropriate for analytical evaluation (eliminated evident arbitrage or non-liquid contracts), and then chose numerous random option contracts to parametrise our models.

The model developed in this paper proved to be most adequate in the analysis of several companies from the litigation categories comprising the highest number of publicly traded companies: patent and fraud litigations. The estimates of Gamma implied jump volatility for these entities is quite high, meaning that indeed a large proportion of these companies' volatility may be attributed to litigation news.

However, twelve other companies from our sample have proved to be more concisely estimated by other models. Ordinary Black and Scholes apparently still remains an important pricing benchmark; nevertheless, for large companies often highlighted in the news such as Apple Inc, jump diffusion still seem to be more appropriate.

The research conducted ion this thesis may be expanded in a great variety of way, including, but not limited to, a broadening an the range of empirically estimated companies, separating option sample with different maturity and moneyness levels and modifying keyword queries in order to get a more precise proxy for litigation news flow.

As a potential field of further research, the author of this paper would like to propose using another type of Bloomberg news-related data: the company heat news story flow parameter calculated by

Bloomberg. This indicator is calculated based on the number of key news stories published related to a company on each given day. As this paper places special emphasis on actual news arrival as a source for empirical parametrization of news arrival processes, this indicator would be the perfect candidate for a jump-generating process generalized over all kinds of company-related news. However, its employment in analysis and comparison with other models, such as the one presented in this paper, remains a subject for further research.

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27. Bloomberg Terminal data

Appendix

I. Poisson mixture of independent Gamma distributions

The probability of n jumps occurring in a Poisson process with a given intensity λ is:

$$P(X=n\,|\,\lambda)=\frac{\lambda^n e^{-\lambda}}{n!}$$

Thus, the unconditional probability of observing n jumps in a mixed Poisson process specified in this paper will be given by:

$$P(X=n)=\int P(X=n\,|\,\lambda)\,f(\lambda)d\lambda=\int_0^\infty \frac{\lambda^n e^{-\lambda}}{n!}\cdot\frac{1}{\theta^m}\cdot\frac{1}{\Gamma(m)}\cdot\lambda^{m-1}e^{-\lambda/\theta}d\lambda$$

Let $\theta=\dfrac{p}{1-p}$. Then:

$$P(X=n)=\int_0^\infty \frac{\lambda^n e^{-\lambda}}{n!}\cdot\frac{(1-p)^m}{p^m}\cdot\frac{1}{\Gamma(m)}\cdot\lambda^{m-1}e^{-\lambda(1-p)/p}d\lambda=\frac{(1-p)^m}{n!\,p^m\Gamma(m)}\cdot\int_0^\infty \lambda^{n+m-1}e^{-\lambda/p}d\lambda=\frac{(1-p)^m}{n!\,p^m\Gamma(m)}\cdot\Gamma(n+m)\cdot p^{n+m}=$$

$$=\frac{(1-p)^m p^n}{n!}\cdot\frac{\Gamma(n+m)}{\Gamma(m)}$$

Recalling that $\Gamma(a) = (a-1)!$ for an integer *a*, we get:

$P(X = n) = \frac{(1-p)^m p^n \cdot (n+m-1)!}{n!(m-1)!}$ – the probability mass function of a negative binomial distribution, where *m* is the number of failures of an experiment (in our case, time periods without jumps) which occurred before *n* successes were observed.

II. A derivation of the European call option price formula

Recall the dynamics of the stock price given in Equation (1). According to financial theory, under the risk neutral probability measure the stock price must be a martingale. Thus:

$$E\big(d(e^{-rt}S_t|\Im_t)\big) = E(e^{-rt}(-rS_t + dS_t)|\Im_t) = e^{-rt} \cdot E(\mu S_t dt + \sigma S_t dW_t + dj_t - rS_t|\Im_t)$$
$$= e^{-rt} \cdot [E((\mu - r)S_t dt|\Im_t) + E(S_t dj_t|\Im_t)] = 0$$

Therefore, under the risk-neutral measure, the stock's drift becomes ($\Im_t$ further omitted from the expected value formulas for simplicity):

$$\mu^* = r - E(dj_t) = r - E\big(\lambda_t(Y-1)\big) = r - E(\lambda)E(Y-1) - cov(\lambda, Y) = r - \varphi$$

$$\varphi = E(\lambda)E(Y-1)$$

$cov(\lambda, Y) = 0$ by assumption of independence of jump intensity and size.

According to the Girsanov theorem, under this risk-neutral measure, the Brownian motion governing stock price dynamics will be:

$$dW_t^* = dW_t - X_t dt$$

$$X_t = -\frac{\mu - rS_t + \varphi}{\sigma}$$

As well as in the case of Poisson jump with constant intensity, an analogue of the Ito lemma can be now applied. It states that: for any scalar function $f(t, X) \in C^2$:

$$df(t, S_t) = f_S\big((r - \varphi)S_t dt + \sigma S_t dW_t\big) + \frac{1}{2} f_{SS}\sigma^2 S_t^2 dt + f_t dt + f dq_c$$

Where:

$$dq_c = \begin{cases} \dfrac{f(S_tY,t)}{f(S_t,t)} - 1 \; if \; a \; jump \; of \; size \; Y \; occurs \\ 0 \; otherwise \end{cases}$$

Thus, for $f(S_t,t) = lnS_t$ we obtain:

$$d(\ln S_t) = \left(r - \varphi - \frac{\sigma^2}{2}\right)dt + \sigma dW_t + \sum_{s \le t} \ln Y_s$$

where *s* is the number of jumps that occurred before time *t*.

Therefore, the actual stock price is:

$$S_t = S_0 \exp\left(\left(r - \varphi - \frac{\sigma^2}{2}\right)t + \sigma W_t\right)\prod_{s \le t} Y_s = S_0 \exp\left(\left(r - \varphi - \frac{\sigma^2}{2}\right)t + \sigma W_t + \sum_{s \le t} J_s\right)$$

Where $J_s = \ln Y_s, J_s \sim i.i.d.\, N(0; \sigma_j)$

We now carry on with a derivation of a European call option price conditional of s – the number of jumps occurring before the option's maturity:

$$C_0^s = e^{-rT}\left(E\left(S_T \cdot 1_{S_T \ge K}\right) - K \cdot E\left(1_{S_T \ge K}\right)\right) = e^{-rT}(P_1 - P_2)$$

Denoting $W_T = \sigma\sqrt{T}z$, $z \sim N(0;1)$, we proceed:

$$P_2 = K \cdot E\left(1_{\sigma\sqrt{T}z + \sum_s J_s \ge \ln K/_{S_0} - \left(r - \varphi - \sigma^2/_2\right)T}\right) = K\Phi\left(d_2^j\right)$$

Where $-d_2^j = \frac{\ln K/_{S_0} - \left(r - \varphi - \sigma^2/_2\right)}{\sqrt{T\sigma^2 + s\sigma_j^2}}$

$$P_1 = S_0 \cdot E\left(\exp\left(\left(r-\varphi-\frac{\sigma^2}{2}\right)T+\sigma W_T+\sum J_s\right)\cdot 1_{\vartheta\geq -d_2^j}\right)$$

$$= S_0 \cdot \int_{-d_2^j}^{\infty} \exp\left(\left(r-\varphi-\frac{\sigma^2}{2}\right)T+\sigma W_T+\sum J_s\right)\cdot\frac{1}{\sqrt{2\pi}}\exp\left(\frac{-\vartheta^2}{2}\right)d\vartheta$$

$$= S_0 \cdot \exp\left((r-\varphi)T+\frac{s\sigma_j^2}{2}\right)\cdot\int_{-d_2^j}^{\infty}\frac{1}{2\pi}\exp\left(-\frac{\left(\vartheta-\sqrt{T\sigma^2+s\sigma_j^2}\right)^2}{2}\right)d\vartheta$$

$$= S_0 \exp\left((r-\varphi)T+\frac{s\sigma_j^2}{2}\right)\Phi\left(d_1^j\right)$$

where $d_1^j = d_2^j + \sqrt{T\sigma^2 + s\sigma_j^2}$

We have obtained the formula for a call option price conditional on s jumps occurring during the life of the option:

$$C_0^s = S_0 \exp\left(-\varphi T+\frac{s\sigma_j^2}{2}\right)\Phi\left(d_1^j\right) - Ke^{-rT}\Phi\left(d_2^j\right)$$

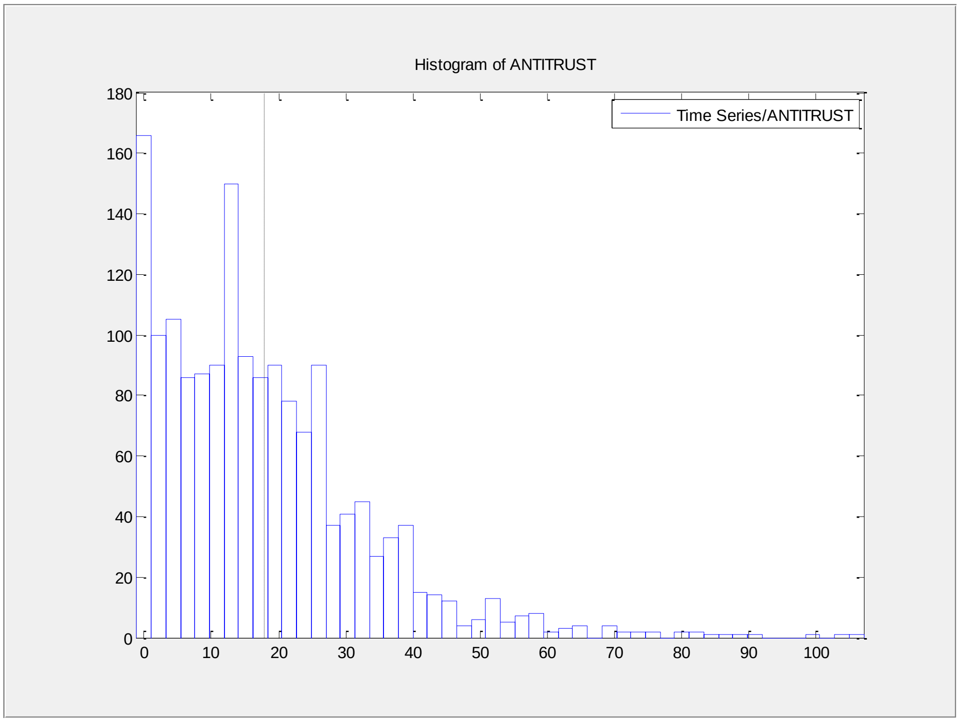
Only the term $\frac{s\sigma_j^2}{2}$ appears to be different from the standard Black-Scholes (BS) formula. In line with Merton (1976), we denote $\sigma_s^{2\prime} = \sigma^2 + \frac{s}{T}\sigma_j^2$ and express the formula for s jumps as:

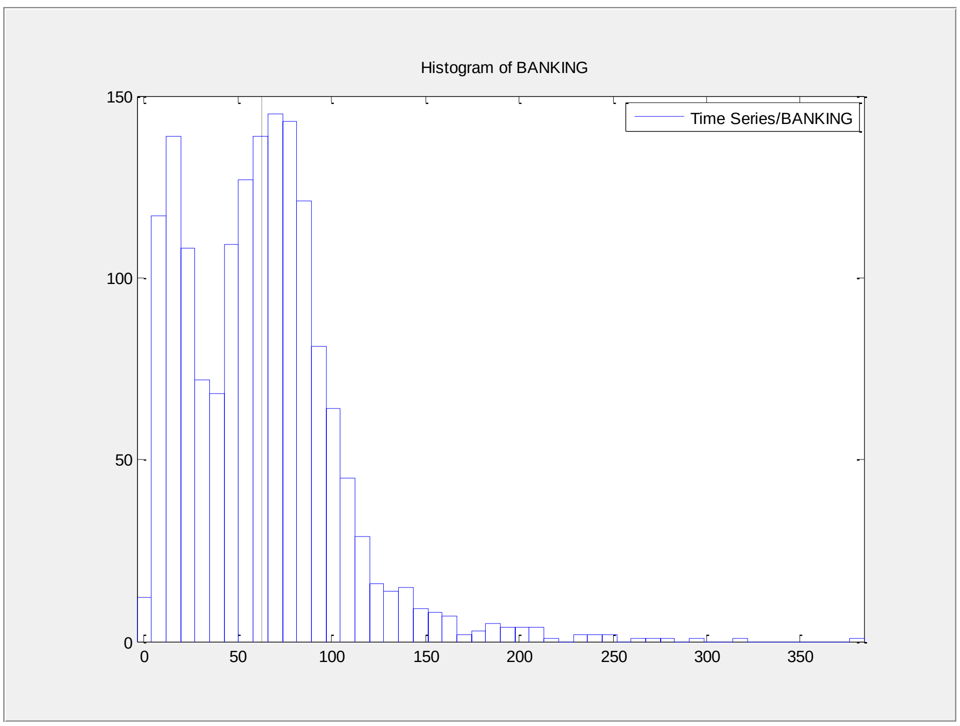
$C_0^s = BS(S,T,K,\sigma_s^{2\prime},r)$ – standard BS formula with a new volatility term.

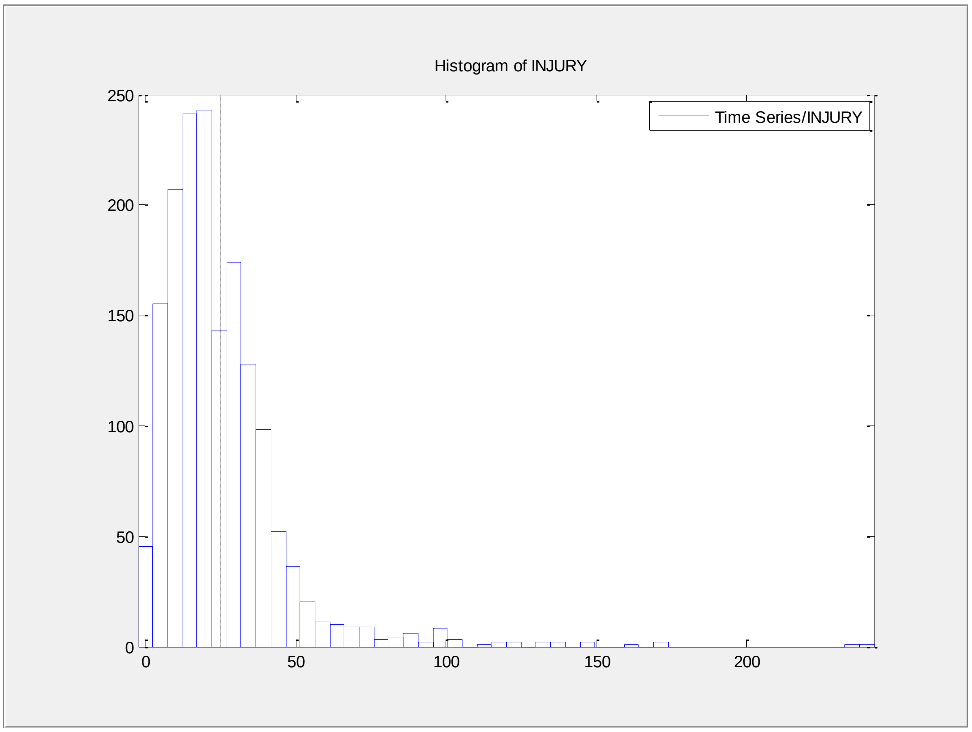
To arrive to the final option price expression, we need to aggregate the expected value over all possible numbers of jumps:

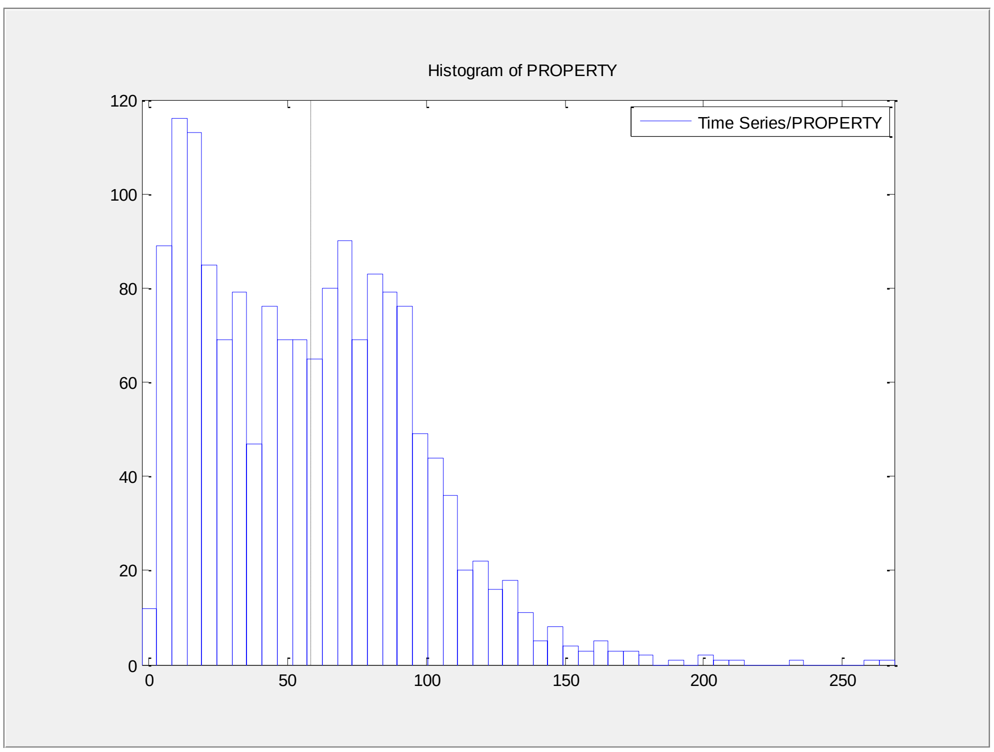
$$C_0 = \sum_{s=0}^{\infty}\frac{(1-p)^m p^s (m+s-1)!}{s!\,(m-1)!}BS(S,T,K,\sigma_s^{2\prime},r)$$

III. Empirical distributions of other news arrival processes considered






Histogram of INJURY
Time Series/INJURY
250
200
150
100
50
0
0
50
100
150
200


Histogram of PROPERTY
Time Series/PROPERTY
120
100
80
60
40
20
0
0
50
100
150
200
250