Национальный исследовательский университет -
Высшая школа экономики

Международный Институт Экономики и Финансов

ВЫПУСКНАЯ КВАЛИФИКАЦИОННАЯ РАБОТА

на тему:

Прыжки волатильности в дни запланированных корпоративных событий.

Студент 4 курса 2 группы
Николенко Иван Андреевич

Научный руководитель
к.э.н., д.ф., доцент, Гельман Сергей Викторович

МОСКВА, 2013 год

Краткое содержание

В данной работе приводится новая модель оценки опционов на основе случайных прыжков в цене и волатильности акции в связи с заранее запланированными корпоративными событиями (например, в связи с публикацией квартальной отчетности компаний). Утверждается, что информационные шоки в такие моменты оказывают влияние на волатильность цены акции. Модель описывается как в рыночной, так и в риск-нейтральной вероятностной мере и включает в себя элементы стохастической волатильности и прыжков в заранее определенные даты. Для нахождения формулы оценки опционов в замкнутой форме применяется анализ дисконтированной характеристической функции логарифма цены акции в момент исполнения опциона. Несмотря на то, что краткое эмпирическое исследование модели не позволяет нам утверждать, что модель значимо лучше предсказывает цены опционов, мы полагаем, что данная модель обладает большим потенциалом для будущих исследований.

National Research University -
Higher School of Economics

International College of Economics and Finance

Graduation Thesis

Jumps in volatility due to scheduled corporate events.

Author:
Ivan Nikolenko

Scientific Advisor:
Dr. Sergey Gelman.

Moscow, 2013

Abstract

In this paper we provide a new option pricing model which takes into account random jumps in both stock price and stock volatility due to scheduled events, such as earnings announcements. We claim that announcement surprises can have potential upward pressure on stock volatility. The model is specified in both the market and the risk-neutral probability measures and incorporates stochastic volatility and jumps at a pre-determined date. We apply transform analysis to derive a closed-form solution for the price of a European call option under the specification of our model. Although the empirical study does not provide sufficient evidence in favor of our model, we think that the model has potential fruitful future applications.

Keywords: option pricing, continuous time, scheduled events, stochastic volatility, volatility jumps.

Acknowledgements

I would like to thank Dr. Sergey Gelman, Dr. Leonid Timochouk, Dmitry Storcheus and Lana Zakharova for their invaluable advice and guidance.

Contents

List of notations...4

Introduction. 5

The basics of BS model...9

The option pricing model (SVJJC) 10

Expected future variance. 16

Comparative Statics. 17

Empirical study. 19

Conclusion 24

References..25

Appendices 27

List of notations

$(\Omega, \mathcal{F}, P)$ – probability space (a space of elementary outcomes Ω supplied by a σ-algebra $\mathcal{F}$ and probability measure P)

$\mathbb{P}$ - market probability measure

$\mathbb{Q}$ - risk-neutral probability measure

S_t – value of a stock

V_t – spot variance of returns

W_t – Wiener process

C – value of a call option

T – option expiration date

K – strike price

r – risk-free interest rate

SV – stochastic volatility

SVJ – stochastic volatility with scheduled jumps in S_t

SVJJ – stochastic volatility with scheduled uncorrelated co-jumps in S_t and V_t

SVJJC - stochastic volatility with scheduled correlated co-jumps in S_t and V_t

Introduction.

Since the major work of Fisher Black and Myron Scholes ([2]) in 1973 option pricing theory has made several significant steps forward. However, the model introduced in their paper is still very popular among investors due to its tractability and easy implementation. Finally, almost all alternative pricing models stem from the famous Black-Scholes model.

The drawbacks of the Black-Scholes model inspired many researchers to develop their option pricing models for various underlying assets. Namely, the assumption of constant volatility is the most crucial limitation of the model, since it does not allow for the well-known phenomenon of a volatility smile (the plot of implied volatility as a function of an option's strike price). In case of equity options, empirical data showed that the implied volatility for an option with a low strike price (or deep-in-the-money call) is significantly higher than for an option with a high strike price (or deep-out-of-the-money call) – [10]. Volatility smile is an indicator of the non-normality of the distribution of stock returns (with negative skewness and positive excess kurtosis, and hence, with fatter tails – especially the left one - [10]).

Further empirical evidence against the validity of Black-Scholes model is dependence of implied volatilities on options' maturities. When short-dated historical volatility is low, traders expect it to rise and volatility seems to be an increasing function of maturity (and vice versa). (Bakshi, Cao, Chen, 1997)

Several approaches were supposed to solve the problem of the deviations of stock returns distribution from the normal one. In 1975 Merton ([15]) introduced the model in which the process of the stock price incorporated not only a diffusion component but also a jump component, thus relaxing the assumption of almost surely continuous process in the Black-Scholes model. The intuition behind such specification is that under the hypothesis of semi-strong market efficiency at least, i.e. when unexpected events happen and information about them is public, market prices adjust to the incoming public information rapidly. Introduction of random jumps (where jumps arrival follows Poisson process) made extreme events more likely and was consistent with empirical data.

Another major improvement was made by the class of stochastic volatility models, in which volatility is no longer constant, but is itself a stochastic process. We will use one of the most famous stochastic volatility model introduced by Heston ([8]) in our analysis. Heston was able to provide a closed-form solution of his model.

Duffie, Pan and Singleton ([4]) generalized and formalized these two branches (stochastic volatility and random jumps), proceeding to the class of multidimensional affine jump-diffusion processes. Transform analysis was used in their paper to derive option pricing formulae. Despite the generality of the models of this class, most of them imply complicated pricing approaches, and closed-form solutions cannot be constructed sometimes. Another disadvantage is decreased parsimony due to the large number of parameters. However, such general approach to pricing options in affine models provided a solid ground for our model.

In this paper we will focus on a specific class of option pricing models: models of scheduled events. Whereas jump-diffusion models assume that jumps arrive at random times, models of scheduled events consider jumps arriving at the dates which are known in advance. The intuition behind this is as follows: some major information affecting stock prices, such as corporate earnings announcements or macroeconomic news (Fed minutes, non-farm payrolls, etc.) can have a significant impact on stocks, indices, currencies or bonds. At the same time, investors typically know in advance when this information is going to appear. Thus, it is natural to try to model this situations separately from randomly arriving jumps. Once again, a certain degree of market efficiency should be assumed in order to justify this kind of models, namely, informational efficiency in at least semi-strong form.

Dubinsky and Johannes ([3]) concentrate on earnings announcements, which significantly influence stock returns on the day of an announcement, i.e. they lead to significant price jumps. Exploring this jumps (namely, their variance) allowed the authors to estimate uncertainty embedded in an individual firm's earnings announcement ("fundamental uncertainty"). This means that investors are uncertain about the performance of the company before the announcement. At the announcement, this uncertainty is removed via a jump in stock price, which adjusts to the new information.

In this paper we extend and generalize the model of Dubinsky and Johannes. We still concentrate on scheduled events, but we claim that sharp changes in stock prices due to informational content of announcements can affect not only the returns, but also the volatility of the underlying asset. The intuition is as follows: although the new information eliminates the uncertainty related to the announcement, it might also unveil new sources of risk for the company. We claim that it is natural to suppose that significant negative return on the day of announcement will likely have an upward pressure on the volatility. If a company's fundamentals appeared to be significantly worse than expected (which was followed by a jump in the price), investors could become less certain about future fundamentals. This effect is augmented by the leverage effect, i.e. when equity depreciates, leverage increases, which raises the riskiness of the firm. Extreme positive returns might also force investors to re-assess the riskiness of the firm, but this effect is smaller due to the proposition that investors tend to be more sensitive to adverse shocks. Furthermore, in case of positive returns, the leverage effect has the opposite direction, i.e. it has a downward pressure on volatility.

We use this logic to model the asymmetric response of volatility to a scheduled announcement by an almost surely positive increase in the variance of returns. In order to avoid complications in derivation and to guarantee that variance is almost surely positive, we assume that variance cannot jump downwards. Instead, the negative correlation between the jumps in returns and in variance generates the desired asymmetry and leverage effect. This means that given sufficient level of negative correlation, the effect of extreme positive return on variance is almost eliminated on average.

Thus, this paper is devoted to a new option pricing model which accounts for potential jumps in volatility at the days of scheduled announcements. We focus our attention of pricing a European call on a stock without dividend yield, since this is the simplest derivative security. To achieve high level of goodness of fit, we include stochastic volatility in the form used by Heston ([8]). Finally, we include jumps in returns themselves, since earnings announcements affect returns in the first place, and this effect was proved to be significant by Dubinsky and Johannes ([3]).

Having specified the model in the market probability measure $\mathbb{P}$ (the "real-world" probability measure), we move to a risk-neutral probability measure $\mathbb{Q}$ and build the stock process in this measure. Risk-neutral valuation is one of the main ways to price an asset, and it is frequently used. It involves changing the probability measure P to $\mathbb{Q}$, which is called a risk-neutral or equivalent martingale measure (two measures on the same probability space are said to be equivalent if they have the same measure-zero sets, i.e. if these measures agree on which events can happen with probability zero). Intuitively, this means that the real world's probability measure is distorted in such a way that all investors, regardless of their risk-preferences, expect returns at a rate equal to the risk-free rate.

By definition, under a risk-neutral measure all discounted asset prices are martingales. Thus, risk-neutral valuation allows us to obtain derivative instrument's value as the conditional expectation of the discounted payoff:

$$V(S_t, t) = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}(f(S_T)|\mathcal{F}_t)$$

A risk-neutral measure is unique if and only if the model we consider is a complete-market model, i.e. in which every instrument is hedgeable and can be replicated by a portfolio consisting of other assets. One of the problems with such complicated models as Stochastic Volatility models or Jump-Diffusion models is in the non-uniquieness of the risk-neutral measure. The class of Stochastic Volatility models has additional state variable (variance), which is not traded in general and is not observable. However, this issue can be addressed in several ways, including local volatility models where the additional state variable is eliminated), assumptions about the market price of risk (e.g. Heston assumed that it is a constant parameter, which was then eliminated from the model) or adding other tradable assets related to the same volatility (e.g. VIX options). Unfortunately, it is impossible to achieve uniqueness of the risk-neutral measure in case of scheduled jumps, since the risk on the day of the announcement cannot be hedged. In this paper we provide a justification for our pricing formula under the impossibility of a perfect hedge.

Due to complexity of the process, we do not construct a risk-neutral measure explicitly. Instead, we show that it exists in our case.

Using the specification of the model in a risk-neutral measure we move to transform analysis equivalent to the one in [4]. This technique allows us to derive the expression for the discounted characteristic function of the logarithm of the stock price. Having obtained that, we proceed to the option-pricing formula in the closed form by a form of Fourier inversion which was used by [3].

The rest of the paper is organized as follows. In Section 2 we provide the basics of the Black-Scholes model as a ground for further modeling. In Section 3 we provide our model specification and solution in a closed form. Namely, we specify the model in the market measure first. Secondly, we transfer our model to the risk-neutral measure preserving the functional form of the underlying stochastic processes. Having obtained the risk-neutral specification, we prove that there exists a corresponding Radon-Nikodym derivative that allows us to construct the risk-neutral measure. To obtain a closed-form solution, we use transform analysis and derive the corresponding discounted characteristic function of logarithm of the stock price at maturity. In Section 4 we investigate how the introduction of a variance jump affects the estimator of expected future variance. We do this to address the complexity of SV-type models' calibration procedure, where spot variance is included as an additional parameter. In Section 5 we examine the comparative statics of the nested models under our consideration. In the empirical part of the paper, we provide the description of model calibration and its results which does not give us enough evidence that introducing variance jumps improves the goodness of fit. In conclusion, we advise to continue the research on this model.

The basics of BS model.

The Black-Scholes model is based on the fundamental assumption that the stock price is distributed lognormally. Under the market measure the stock price follows Geometric Brownian Motion:

$$\frac{dS_t}{S_t} = \mu\, dt + \sigma\, dW_t$$

Under the risk-neutral measure (which is unique in this model, i.e. the market is complete) the firm-specific parameter μ (expected return) is eliminated, and the drift term becomes rdt:

$$\frac{dS_t}{S_t} = r\, dt + \sigma\, dW_t$$

Let us find the discounted stock price process $\{X_t\}_{t\geq 0} = \{\exp(-rt)\, S_t\}_{t\geq 0}$ by applying the Ito lemma:

$$\begin{aligned} dX_t = d(\exp(-rt)\, S_t) = {} & d(\exp(-rt))S_t + \exp(-rt)\, dS_t \\ & = \exp(-rt)\, (-rdt)S_t + \exp(-rt)\, S_t rdt + \exp(-rt)\, S_t\, \sigma\, dW_t \\ & = -rX_t dt + rX_t dt + \sigma X_t dW_t = \sigma X_t dW_t \end{aligned}$$

Hence, under the risk-neutral measure, the discounted stock process is a martingale.

The following assumptions must also be satisfied:

a. The stock price follows the process specified above. μ and σ are constant parameters.

b. Short selling is permitted.

c. No transaction costs or taxes.

d. All financial instruments are perfectly divisible.

e. There are no dividends.

f. There must be no sustainable regular arbitrage opportunities.

g. Security trading is continuous.

h. The risk-free interest rate, r, is constant.

We will now use the intuition of this basic model in building our SVSJJC model.

The option pricing model (SVJJC)

Market measure

For our model we have all the assumptions from the Black-Scholes model, except from (a).

Under $\mathbb{P}$ the stock price process is given by:

$$dS_t = \mu\, S_t dt + \sqrt{V_t}\, S_t\, dW_t^S + S_{t_0-}\left[e^{Z^S} - 1\right] dN_t$$

$$dV_t = k(\theta - V_t)dt + \sigma^V \sqrt{V_t}\, dW_t^V + Z^V dN_t \qquad (3.1.1)$$

$$dW_t^S\, dW_t^V = \rho\, dt$$

Here the time of jump is t_0, $N_t \colon \mathbb{R}^+ \to \mathbb{R}$ is a deterministic process (function of t) such that $N_t = \begin{cases} 0, t < t_0 \\ 1, t \geq t_0 \end{cases}$. We assume that Feller condition holds, i.e. $k\theta > (\sigma^V)^2/2$. This condition is required for variance to be almost surely positive.

Let us look at the model specification more closely. First of all, we include Heston's specification of Stochastic volatility. Here, $\{V_t\}_{t\geq 0}$ is instantaneous variance (squared volatility) - a square root mean reverting process with long-run mean θ, and rate of reversion k, σ^v is referred to as the volatility of volatility; dW_t^S and dW_t^V are correlated with coefficient of correlation ρ; the risk-free rate *r* is constant.

Some features of the model can be seen already at this stage. It involves two Wiener processes and thus, in order to replicate the derivative security by a portfolio, there should exist an asset whose price is perfectly correlated with the variance process, since volatility itself cannot be traded.

Furthermore, the variance process is mean-reverting in the following sense: greater deviations from the long-run mean θ lead to increase of the absolute value of $k(\theta - V_t)dt$ term which then "drags" diffusion back to θ.

Another feature is that as $V_t \to 0$, $\sqrt{V_t} \to 0$ and $\sigma^V \sqrt{V_t}\, dW_t^V \to 0$, which ensures that the variance never becomes negative.

Intuitively, this model provides greater flexibility than BS and can be regarded as an elegant solution to some major problems. Negative correlation ρ between variance and stock return can generate heavier left tail of returns p.d.f. than BS's lognormal distribution and lighter right tail, affecting the skewness of the distribution in a way consistent with volatility smiles. Parameter σ^v, "volatility of volatility", affects the kurtosis of the distribution, again making the distribution more sensible with respect to the leverage effect ([1]).

The model incorporates one scheduled jump in both stock price and volatility. We consider only one scheduled event to reduce the complexity of the model. Thus, we consider only the closest announcement date given that the option matures before the next announcement.

Following [4], we assume that the jumps have a joint distribution with the following characteristic function:

$$\mathbb{E}\left[e^{c_1 Z^S + c_2 Z^V}\right] = \frac{\exp\left(\mu c_1 + \frac{1}{2}(\sigma^Z)^2 c_1^2\right)}{1 - \nu c_2 - \rho_j \nu c_1}$$

where $\mathbb{E}\left[e^{c_1 Z^S + c_2 Z^V}\right] = \int_{\mathbb{R}^2} e^{c_1 Z^S + c_2 Z^V} d\mathbb{P}$

and $c1$ and $c2$ are complex-valued arguments. Furthermore, we require that correlation $\rho_j < 0$, which is needed to model the leverage effect.

We can see that the characteristic function of marginal distributions are, respectively

$$\mathbb{E}\left[e^{c_1 Z^S}\right] = \mathbb{E}\left[e^{c_1 Z^S + 0 Z^V}\right] = \frac{\exp\left(\mu c_1 + \frac{1}{2}(\sigma^Z)^2 c_1^2\right)}{1 - \rho_j \nu c_1}$$

$$\mathbb{E}\left[e^{c_2 Z^V}\right] = \mathbb{E}\left[e^{0 Z^S + c_2 Z^V}\right] = \frac{1}{1 - \nu c_2}$$

Thus, Z^V is distributed exponentially with mean ν, whereas Z^S has some continuous distribution. In [4] it is also shown that for this function Z^S is conditionally normally distributed. At the same time, the unconditional expectation of e^{Z^S} can be easily derived from the characteristic function:

$$\mathbb{E}\left[e^{c_1 Z^S}\right] = \int_{-\infty}^{+\infty} e^{c_1 Z^S} d\mathbb{P}$$

$$\mathbb{E}\left[e^{Z^S}\right] = \int_{-\infty}^{+\infty} e^{Z^S} d\mathbb{P}$$

Thus, $\mathbb{E}\left[e^{Z^S}\right] = \mathbb{E}\left[e^{c_1 Z^S}\right]$ at $c_1 = 1$.

$$\mathbb{E}\left[e^{Z^S}\right] = \frac{\exp\left(\mu + \frac{1}{2}(\sigma^Z)^2\right)}{1 - \rho_j \nu}$$

Since we consider scheduled announcements which cause instantaneous jumps in spot price. Suppose that $\frac{S_{t_0}}{S_{t_0-}} = e^{Z^S}$, which is the jump size, has mean different from one, i.e. $\mathbb{E}\left[S_{t_0} | \mathcal{F}_{t_0-}\right] \neq S_{t_0-}$. Then there would exist a trading strategy which would bring abnormal returns. Any difference between $\mathbb{E}\left[S_{t_0} | \mathcal{F}_{t_0-}\right]$ and S_{t_0-} would constitute a positive abnormal expected return, since there are no interest accruals from t_{0-} to t_0. Thus, efficient markets would imply $\mathbb{E}[S_{t_0}|\mathcal{F}_{t_0-}] = S_{t_0-}$, or equivalently, $\mathbb{E}[S_{t_0}/S_{t_0-}|\mathcal{F}_{t_0-}] = \mathbb{E}[e^{Z^S}|\mathcal{F}_{t_0-}] = 1$. Finally, this implies $\mathbb{E}\left[\mathbb{E}[e^{Z^S}|\mathcal{F}_{t_0-}]\right] = \mathbb{E}[e^{Z^S}] = 1$

Hence,

$$\frac{\exp\left(\mu + \frac{1}{2}(\sigma^Z)^2\right)}{1-\rho_j\nu} = 1$$

and

$$\mu = \ln(1-\rho_j\nu) - \frac{1}{2}(\sigma^Z)^2$$

Note that we consider equity options, and, therefore, an investor cannot trade volatility directly. So, the mean of variance jump Z^V can be positive.

We will now transfer our process into a risk-neutral measure.

Risk-neutral measure.

Let $\mathbb{Q}$ be a probability measure. The stock process under $\mathbb{Q}$ is assumed to follow:

$$dS_t = r\, S_t dt + \sqrt{V_t}\, S_t\, dW_t^S(\mathbb{Q}) + S_{t_0-}[e^{Z^S} - 1]dN_t$$

$$dV_t = k^{\mathbb{Q}}(\theta^{\mathbb{Q}} - V_t)dt + \sigma^{V(\mathbb{Q})}\sqrt{V_t}\, dW_t^V(\mathbb{Q}) + Z^V dN_t \qquad (3.2.1)$$

$$dW_t^S(\mathbb{Q})\, dW_t^V(\mathbb{Q}) = \rho^{\mathbb{Q}}\, dt$$

$$\theta(c_1, c_2) = \mathbb{E}[e^{c_1 Z^S + c_2 Z^V}] = \frac{\exp\left(\mu^{\mathbb{Q}} c_1 + \frac{1}{2}\left(\sigma^{Z(\mathbb{Q})}\right)^2 c_1^2\right)}{1 - \nu^{\mathbb{Q}} c_2 - \rho_j{}^{\mathbb{Q}} \nu^{\mathbb{Q}} c_1}$$

$$\rho_j{}^{\mathbb{Q}} < 0$$

Assume Feller condition holds, i.e. $k^{\mathbb{Q}}\theta^{\mathbb{Q}} > \left(\sigma^{V(\mathbb{Q})}\right)^2/2$.

First of all we need to show that $\mathbb{Q}$ and $\mathbb{P}$ are equivalent. Given the specification of the process under $\mathbb{Q}$, we can claim that the two measures are equivalent (it is guaranteed by the fact that the jump occurs at the same time t_0), which means that if some set of paths $A \subseteq \mathcal{F}$ has non-zero probability under $\mathbb{P}$, it will have non-zero probability under $\mathbb{Q}$, and vice versa. Namely, for any $t \neq t_0$ both stochastic processes have almost surely continuous paths. At $t = t_0$ jumps in stock price and variance occur almost surely under both measures. Z^S is contained in any interval $(a, b) \subseteq \mathbb{R}$ with positive probability under both measures. Z^V is contained in any interval $(a, b) \subseteq \mathbb{R}_+$ with positive probability under both measures, and Z^V

is almost surely positive under both measures ($Z^V \sim_{\mathbb{P}} Exponential(\nu)$ and $Z^V \sim_{\mathbb{Q}} Exponential(\nu^{\mathbb{Q}})$). Thus, we can claim that the two measures are equivalent.

By Radon-Nikodym theorem ([18]) if $\mathbb{P}$ and $\mathbb{Q}$ are equivalent, then there exists an almost surely positive random variable Z such that for any set $A \subseteq \mathcal{F}$ $\mathbb{Q}(A) = \int_A Z(\omega)\, d\,\mathbb{P}(\omega)$. I.e. we can be sure that Radon-Nikodym derivative, which can help us to construct $\mathbb{Q}$ explicitly, exists. From now on we will consider stock price process under $\mathbb{Q}$.

We can now also claim that under $\mathbb{Q}$, the discounted stock process is a martingale. Consider $X_t = \exp(-rt)S_t$. Applying Ito lemma, we get:

$$dX_t = d(\exp(-rt)S_t) = d(\exp(-rt)) \cdot S_t + \exp(-rt)\, dS_t =$$
$$= -r\exp(-rt)\, S_t dt + \exp(-rt)\left(r\, S_t dt + \sqrt{V_t}\, S_t\, dW_t^S(\mathbb{Q}) + S_{t_0-}\left[e^{Z^S(\mathbb{Q})} - 1\right]dN_t\right) =$$
$$= \sqrt{V_t}\, X_t\, dW_t^S(\mathbb{Q}) + X_{t_0-}\left[e^{Z^S(\mathbb{Q})} - 1\right]dN_t$$

Before and after the jump $\exp(-rt)S_t = S_0 + \int_0^t \exp(-rs)\sqrt{V_s}\, S_s\, dW_s^S(\mathbb{Q})$. Since Ito integral is a martingale, $\exp(-rt)S_t$ is a martingale before and after the jump. For $\exp(-rt)S_t$ to be a martingale at t_0, the pre-jump expected value of S_{t_0} should be equal to S_{t_0-}. This reduces to requirement $\mathbb{E}^{\mathbb{Q}}\left[e^{Z^S(\mathbb{Q})} - 1|\mathcal{F}_{t_0}\right] = 0$, which was already discussed in the section on the market measure. Hence, under $\mathbb{Q}$ we also have the requirement

$$\mu^{\mathbb{Q}} = \ln\left(1 - \rho_j^{\mathbb{Q}}\nu^{\mathbb{Q}}\right) - \frac{1}{2}\left(\sigma^{Z(\mathbb{Q})}\right)^2$$

Under this condition, the discounted stock process is martingale under $\mathbb{Q}$. Hence, $\mathbb{Q}$ is an equivalent martingale (risk-neutral) measure. To price a derivative security contingent on S_t we now turn to the Transform analysis.

Transform analysis.

Let us slightly adjust the stock price process (according to Ito's formula):

$$d\log(S_t) = \left(r - \frac{1}{2}V_t\right)dt + \sqrt{V_t}\;\, dW_t^S(\mathbb{Q}) + Z^S(\mathbb{Q})dN_t$$
$$dV_t = k^{\mathbb{Q}}(\theta^{\mathbb{Q}} - V_t)dt + \sigma^{V(\mathbb{Q})}\sqrt{V_t}\, dW_t^V(\mathbb{Q}) + Z^V(\mathbb{Q})dN_t \qquad (3.3.1)$$

To find the expected value $C_t = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}[f(S_T)|\mathcal{F}_t]$ we will apply analysis based on direct and inverse Fourier transforms. This technique for the topic we consider was developed in [4]. This method allows deriving closed-form solutions even when one cannot derive probability distributions.

Suppose we are given the following Fourier Transform of $\log(S_T)$:

$$\psi(u, S_t, V_t, t, T) = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}\left[e^{u\log(S_T)}|\mathcal{F}_t\right] \qquad (3.3.2)$$

We cannot use the inverse transform directly, since we need integrability condition to be satisfied. Thus, we will use the results developed by [3]. Introduction of a dampened call allows us to get the following option pricing formula:

$$C(k) = \frac{\exp(-\alpha k)}{\pi}\int_0^{\infty} Re[\exp(-i\alpha k)\,\psi_c(s)]ds \qquad (3.3.3)$$

$$\psi_c(s) = \frac{\psi(s-i(\alpha+1), S_t, V_t, t, T, r)}{\alpha^2+\alpha-s^2+i(2\alpha+1)s} \qquad (3.3.4)$$

Where k stands for log-strike and α is assumed to be 2 which is sufficient for integrability according to [3].

Hence, the only thing we need is to find $\psi(u, S_t, V_t, t, T)$. First of all, following the affine jump-diffusion studies in [4] we assume the certain form of ψ:

$$\psi(u, S_t, V_t, t, T) = \exp[\alpha(t) + u\log(S_t) + \beta(t)V_t] \qquad (3.3.5)$$

Here, $\alpha(t) = \alpha_0(t) + \alpha^j(t)$, $\alpha_0(t)$ is continuous, $\alpha^j(t)$ is not.

We will now apply two-dimensional Ito's formula for jump-diffusions to ψ (please refer to A1) and get the following ODEs:

$$\dot{\alpha_0}(t) - r(1-u) + k^{\mathbb{Q}}\theta^{\mathbb{Q}}\beta(t) = 0 \qquad (A1.1)$$

$$\dot{\beta}(t) - \frac{u}{2} - k^{\mathbb{Q}}\beta(t) + \frac{u^2}{2} + \frac{\beta^2(t)}{2}\sigma^{V(\mathbb{Q})^2} + u\beta(t)\sigma^{V(\mathbb{Q})}\rho^{\mathbb{Q}} = 0 \qquad (A1.2)$$

The equation we obtained for $\dot{\beta}(t)$ is the same as in [4] in 4. Thus, we can use the solution provided by Duffie *et al*.

$$b = \sigma^{V(\mathbb{Q})}\rho^{\mathbb{Q}}u - k^{\mathbb{Q}}, \qquad a = u(1-u), \qquad \gamma = \sqrt{b^2 + a(\sigma^{V(\mathbb{Q})})^2}$$ [1]

$$\beta(\tau) = -\frac{a(1-e^{-\gamma\tau})}{2\gamma-(\gamma+b)(1-e^{-\gamma\tau})} \qquad (3.3.6)$$

$$\alpha(\tau) = \alpha_0(\tau) + \ln\left(\theta\big(u, \beta(t_0)\big)\right)(1-N_t) \qquad (3.3.7)$$

Where $\alpha_0(\tau) = -r\tau(1-u) - \frac{k^{\mathbb{Q}}\theta^{\mathbb{Q}}}{\left(\sigma^{V(\mathbb{Q})}\right)^2}\left((\gamma+b)\tau + 2\ln\left[1-\frac{\gamma+b}{2\gamma}(1-e^{-\gamma\tau})\right]\right)$ [2]

Hence, we obtained a solution which has a simpler form than in [4]. Not surprisingly, our solution embeds the solution in [3], namely, for one jump in stock price distributed normally $\ln\left(\theta\big(u, \beta(t_0)\big)\right)(1-N_t) = \begin{cases} -\frac{u}{2}\left(\sigma^{Z(\mathbb{Q})}\right)^2 + \frac{u^2}{2}\left(\sigma^{Z(\mathbb{Q})}\right)^2, t < t_0 \\ 0, t \geq t_0 \end{cases}$

---

[1] More precisely: $\gamma = |\gamma^2|^{1/2}\exp\left(\frac{i\arg(\gamma^2)}{2}\right)$, where $\gamma^2 = b^2 + a\left(\sigma^{V(\mathbb{Q})}\right)^2$

[2] $\forall z \in \mathbb{C}\;\; \ln z = \ln|z| + i\arg(z)$

Finally, using the restricted version of transform $\theta(c_1, c_2)$ for our model, we get:

$$\theta(c_1, c_2) = \frac{\exp\left(\left(\ln\left(1-\rho_j^{\mathbb{Q}}\nu^{\mathbb{Q}}\right)-\frac{1}{2}\left(\sigma^{Z(\mathbb{Q})}\right)^2\right)c_1+\frac{1}{2}\left(\sigma^{Z(\mathbb{Q})}\right)^2 c_1^2\right)}{1-\nu^{\mathbb{Q}}c_2-\rho_j{}^{\mathbb{Q}}\nu^{\mathbb{Q}}c_1} \tag{3.3.8}$$

Finally, we need to prove that $\exp[\alpha(t) + u\log(S_t) + \beta(t)V_t]$ indeed represents the discounted characteristic function of $\log(S_T)$. For the proof, please refer to A2.

Thus, combining (3.3.3), (3.3.4), (3.3.5), (3.3.6), (3.3.7), (3.3.8) we have obtained a closed-form solution for the price of European call option.

Impossibility of perfect hedge.

We should be very careful here, since due to the impossibility of perfect hedge the risk-neutral measure is not unique, i.e. we took only one risk-neutral measure ($\mathbb{Q}$) from the continuum of risk-neutral measures. Perfect hedge is impossible due to stochastic volatility and scheduled jumps.

The problem due to stochastic volatility can be eliminated by assuming that there exists market price of volatility risk, as defined in Heston ([8]). However, it is impossible to hedge the risk of stock and variance movements at the scheduled announcements, primarily because there are no other assets which are prone to the same source of risk. In the absence of perfect hedge risk-averse investors may demand additional premium for this risk.

However, we still avoid adding additional terms to the option pricing formula (as was done in [19], for example). We motivate this decision as follows. First of all [19] reports that similar premium is insignificant most of the time. Secondly, [3] claims that the difference between the real-market jump parameter σ^Z and the risk neutral parameter $\sigma^{Z(\mathbb{Q})}$ is almost negligible, which also motivates low risk premium. Finally, our model is very flexible, i.e. it has 7 embedded parameters at minimum, which means that we can calibrate out model to the market prices without introducing additional premia and not losing the explanatory power of the model. Calibration issues are further discussed in the related section.

Therefore, we advise not to include additional parameters related to the risk premium, since it would have an adverse effect on the parsimony of the model.

Expected future variance.

We will now briefly discuss how the new parameters in our model affect the logic in [3]. In the paper, two types of fundamental uncertainty ($\sigma^{Z(\mathbb{Q})}$) were introduced. The estimators themselves are not in the scope of this paper. However, it is important to see what our model implies for the expected future variance. Under mild conditions on stochastic volatility the stochastic volatility option price is the expectation of the Black-Scholes price where the Black-Scholes implied variance is expected risk-neutral variance. The error between the implied variance and this estimator is getting smaller with the decrease of absolute value of correlation between returns and variance increments. Generally, this is not true for our model (since we have ρ and ρ_j), but for ATM options it is usually very difficult to obtain significant values for the correlations. Furthermore, the errors even in presence of correlation is low (about 1%), as reported by [3].

Thus, in this case we will deal with ATM options and stipulate that $\rho = \rho_j = 0$. This means that our model adds only an independent jump in variance to the model considered by Dubinsky and Johannes. We will now find out what this type of jumps adds to the expected future variance estimator.

According to [3], we can approximate the value of expected future variance by implied Black-Scholes volatility, i.e. we can assume that Black-Scholes implied variance is an accurate proxy for expected risk neutral variance.

$$\sigma_{t,T}^{BS^2} \approx \sigma_{t,T}^2 = (T-t)^{-1}\mathbb{E}^{\mathbb{Q}}\left[\int_t^T V_s\,ds|\mathcal{F}_t\right] + (T-t)^{-1}\left(\sigma^{Z(\mathbb{Q})}\right)^2$$

We derive the value of $\mathbb{E}^{\mathbb{Q}}\left[\int_t^T V_s\,ds|\mathcal{F}_t\right]$ in the A3.

$$\mathbb{E}^{\mathbb{Q}}\left[\int_t^T V_s\,ds|\mathcal{F}_t\right] = \frac{V_t-\theta^{\mathbb{Q}}}{k^{\mathbb{Q}}}\left(1-e^{-k^{\mathbb{Q}}(T-t)}\right) + \frac{\nu^{\mathbb{Q}}}{k^{\mathbb{Q}}}\left(1-e^{-k^{\mathbb{Q}}(T-t_0)}\right) + \theta^{\mathbb{Q}}(T-t)$$

$$\sigma_{t,T}^2 = \frac{V_t-\theta^{\mathbb{Q}}}{k^{\mathbb{Q}}(T-t)}\left(1-e^{-k^{\mathbb{Q}}(T-t)}\right) + \frac{\nu^{\mathbb{Q}}}{k^{\mathbb{Q}}(T-t)}\left(1-e^{-k^{\mathbb{Q}}(T-t_0)}\right) + \theta^{\mathbb{Q}} + \frac{\left(\sigma^{Z(\mathbb{Q})}\right)^2}{T-t} \quad (4.1)$$

This formula is valid only for $t < t_0 < T$.

We can see that our model adds an additional term $\frac{\nu^{\mathbb{Q}}}{k^{\mathbb{Q}}(T-t)}\left(1-e^{-k^{\mathbb{Q}}(T-t_0)}\right)$ which is an expected future variance from the volatility jump. This formula is consistent with the intuition of Dubinsky and Johannes about the movements of implied volatility before the announcement: the new term increases prior to the announcement. After the announcement, this term is effectively eliminated from the expected future variance formula.

Comparative Statics.

Our model is related to several nested models which might be of particular interest when it comes to model comparison. First of all, as $\sigma^{V(\mathbb{Q})} \to 0$, $V_t = \theta^{\mathbb{Q}}$ and $\sigma^{Z(\mathbb{Q})} = \nu^{\mathbb{Q}} = 0$ we obtain a solution to the Black-Scholes model. We are more interested in the comparison between the SV ($\sigma^{Z(\mathbb{Q})} = \nu^{\mathbb{Q}} = 0$), SVJ($\nu^{\mathbb{Q}} = 0$), SVJJ (${\rho_j}^{\mathbb{Q}} = 0$) and SVJJC models. We can see from the graphs that SVJ, SVJJ and SVJJC models are almost equivalent, although each model adds a certain degree of flexibility. Each additional feature increases the value of a call option: positive $\sigma^{Z(\mathbb{Q})}$ increases total expected volatility prior to the announcement, which means that an option is more valuable; positive $\nu^{\mathbb{Q}}$ acts in a similar way; negative ${\rho_j}^{\mathbb{Q}}$ increases the probability of a negative price movement due to Z^S, which means that the probability that a call will be in the money at maturity increases. We can also mention the fact that transition from SV to SVJ significantly alters the price of an option due to embedded discontinuity. The greater is $\sigma^{Z(\mathbb{Q})}$, the greater is the difference. Finally, all option prices tend to $S_t - Ke^{-rt}$ as moneyness increases. This is justified by the put-call parity, which states that $call_t + Ke^{-rt} = S_t + put_t$. Since the value of any option is nonnegative, $put \geq 0$ and $call \geq S_t - Ke^{-rt}$. As moneyness increases, a put is deeper out of the money, which means that its price tends to zero. It is also important to consider the distributions of $\log(S_T)$ which can be derived from the characteristic function by Fourier inversion. Note that these are risk-neutral distributions, not market-implied distributions. First of all, $\mathbb{E}^{\mathbb{Q}}[\log(S_T)\,|\mathcal{F}_t] = \log(S_t) + r(T-t)$ is the mean of each distribution, which is held constant for all of the distributions in the figure. We can see that introduction of stochastic volatility, jumps and negative correlation between them makes the tails of the distribution fatter, especially the left one.

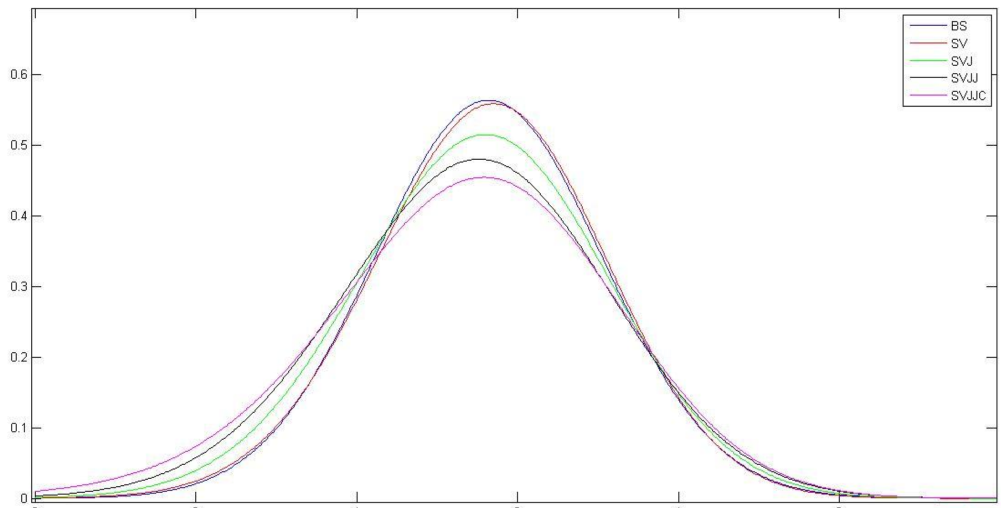

Figure 1: Predicted risk-neutral distributions of $log(S_T)$ conditional on $\mathcal{F}_t$; source: own calculations

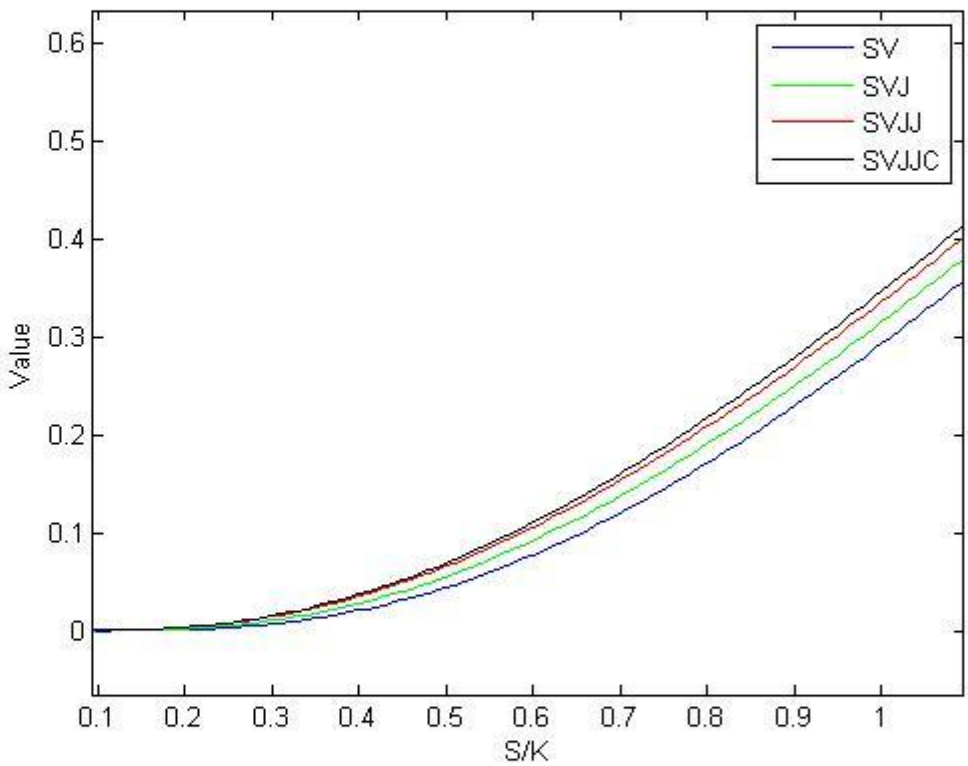

Figure 2: Predicted call prices for $r = 0.05$ $\tau = 1$ $\tau_0 = 0.5$ $k^{\mathbb{Q}} = 2$ $\theta^{\mathbb{Q}} = 0.5$ $\sigma^{V(\mathbb{Q})} = 0.1$ $\rho^{\mathbb{Q}} = -0.7$ $\sigma^{Z(\mathbb{Q})} = 0.3$ $\nu^{\mathbb{Q}} = 0.3$ ${\rho_j}^{\mathbb{Q}} = -1$ $V_t = 0.5$; source: own calculations

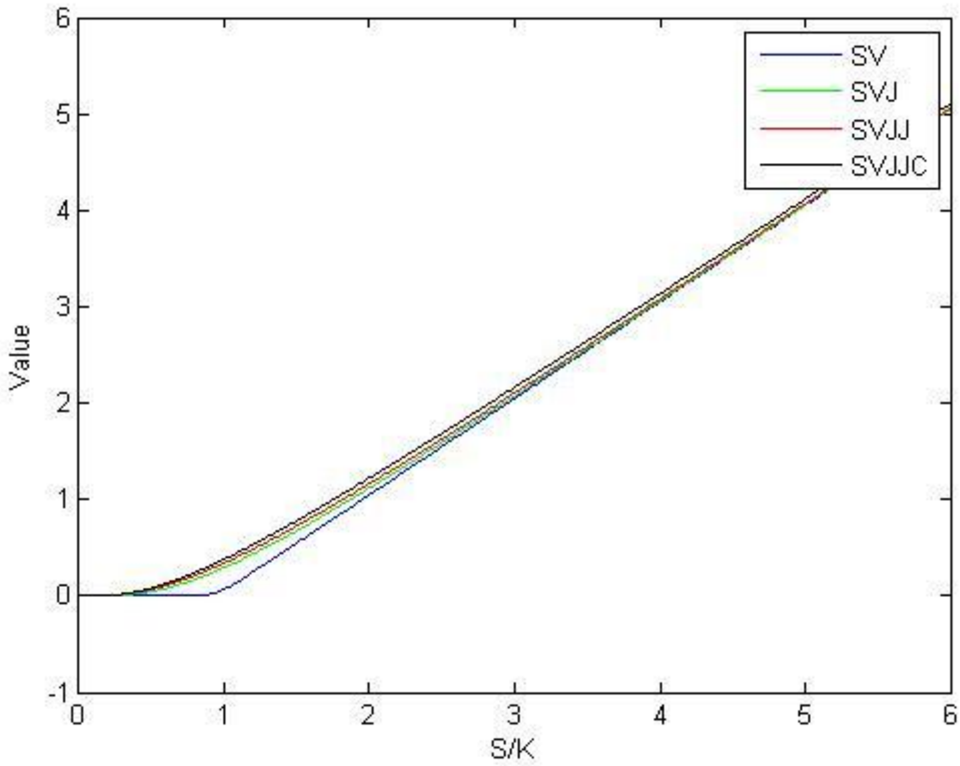

Figure 3: Predicted call prices for $r = 0.05$ $\tau = 1$ $\tau_0 = 0.5$ $k^{\mathbb{Q}} = 2$ $\theta^{\mathbb{Q}} = 0.01$ $\sigma^{V(\mathbb{Q})} = 0.01$ $\rho^{\mathbb{Q}} = -0.7$ $\sigma^{Z(\mathbb{Q})} = 0.7$ $\nu^{\mathbb{Q}} = 0.7$ ${\rho_j}^{\mathbb{Q}} = -1$ $V_t = 0.01$; evident convergence as $\frac{S}{K} \to +\infty$; source: own calculations

Empirical study.

Data description.

To assess the empirical performance of our model, we will perform calibrations to market option data. We take 3 samples of option prices (Apple Computer (AAPL), years 2002, 2004 and 2006) and concentrate our analysis primarily on OTM-options (we take options with $moneyness = \frac{S_t}{K} < 1$), since pricing errors of OTM-options has always been of particular interest for researchers and practitioners. Traditional pricing models (e.g. BS) fail to price OTM-options correctly, whereas they provide fair fit for ATM- and ITM-options.

Apple Computer was also considered in [3] and [19]. We chose this stock because it was actively traded on NASDAQ and because it has been a low-dividend stock (hence traded American calls are equivalent to European calls). Taking an actively traded stock allows us to assume sufficient degree of semi-strong market efficiency, which is necessary for the model of scheduled announcements. Option prices, maturities, strikes, risk-free rates and stock prices were collected from OptionMetrics.

We choose earnings announcements as the scheduled events of interest. Thus, we gathered the dates of earnings announcements from www.fulldisclosure.com.

We adjust our data in the following way (based on [19]): we eliminate all options with 1) maturity in time span of three days around the date of earnings announcement; 2) quote given in time span of three days around the date of earnings announcement; (1 and 2 are needed to avoid significant price disturbances which arise at the date of EA); 3) more than 1 earnings announcement during their life (since our model covers only one jump); 4) prices violating theoretical bounds given by put-call parity ($Call < S_t$ and $Call > S_t - \exp(-r(T-t))K$); 5) implied volatility more than 100%.

Moneyness	**Time to expiration**	**(T-t) < 0.1 years**	**(T-t) >= 0.1 years**		**Subtotal N**
1<=S/K<0.9	price	1.7848	2.2791		
	N	42	355		397
0.9<=S/K<0.7	price	0.3759	0.6084		
	N	79	604		683
S/K>=0.7	price	0.0656	0.0894		
	N	98	522		620
Subtotal N		219	1481		2101

Table 1: Descriptive statistics; source: own calculations

Only 3 samples were considered mainly due to extreme computational complexity of the calibration formulae. Fitting each model took a considerable amount of machine time. Therefore, this calibration analysis does not possess strict statistical power. Instead, it is used to present a general view on the goodness of fit of the model comparing with other models.

Estimation procedure.

We use the approach in [19] to compare three nested models: Heston SV model (). Parameter estimates $\hat{\Theta} = \left[k^{\mathbb{Q}}\ \theta^{\mathbb{Q}}\ \sigma^{V(\mathbb{Q})}\ \rho^{\mathbb{Q}}\ \ \sigma^{Z(\mathbb{Q})}\ \nu^{\mathbb{Q}}\ {\rho_j}^{\mathbb{Q}}\right]$ are obtained as follows:

$$\hat{\Theta} = \underset{\hat{\Theta}, V_t}{argmin} \left[\sum_i \left(Call_i^{market} - Call\left(\hat{\Theta}, V_t\right)_i^{model}\right)^2\right]$$

We minimize the objective function, which is the sum of squared pricing errors. Implicitly we assume that our filtering procedure was successful in eliminating all market data entries which contradicted no-arbitrage considerations. Thus, we treat $Call_i^{market}$ as an unbiased estimate of the true value of a call option.

Normally, the objective function is indeed optimized with respect to the vector of parameters and the time series of spot variances V_t, which are unobservable on the market. This means that the total number of parameters would exceed 80 for each year. This approach is very computationally expensive and can yield implausible spurious results due to the large amount of parameters. Our calibration procedure uses an estimate of spot variance derived earlier in the paper to reduce the number of parameters without a significant decrease in model performance.

We suggest that one can use the formula

$${\sigma_{t,T}^{BS}}^2 = \frac{V_t - \theta^{\mathbb{Q}}}{k^{\mathbb{Q}}(T-t)}\left(1 - e^{-k^{\mathbb{Q}}(T-t)}\right) + \frac{\nu^{\mathbb{Q}}}{k^{\mathbb{Q}}(T-t)}\left(1 - e^{-k^{\mathbb{Q}}(T-t_0)}\right) + \theta^{\mathbb{Q}} + \frac{\left(\sigma^{Z(\mathbb{Q})}\right)^2}{T-t}$$

to derive a proxy for V_t. However, in practice it appears that the contribution of the term $\frac{\nu}{k(T-t)}\left(1 - e^{-k(T-t_0)}\right)$ to the total expected future variance is very low. Thus, in order to further simplify numerical calibration, we omit this term and derive our proxy from:

$${\sigma_{t,T}^{BS}}^2 = \frac{V_t - \theta^{\mathbb{Q}}}{k^{\mathbb{Q}}(T-t)}\left(1 - e^{-k^{\mathbb{Q}}(T-t)}\right) + \theta^{\mathbb{Q}} + \frac{\left(\sigma^{Z(\mathbb{Q})}\right)^2}{T-t}$$

$$\hat{V}_t = \theta + \frac{k^{\mathbb{Q}}}{1 - e^{-k^{\mathbb{Q}}(T-t)}}\left(\left({\sigma_{t,T}^{BS}}^2 - \theta^{\mathbb{Q}}\right)(T-t) - \left(\sigma^{Z(\mathbb{Q})}\right)^2\right)$$

Our optimization problem reduces to:

$$\hat{\Theta} = \underset{\hat{\Theta}}{argmin} \left[\sum_i \left(Call_i^{market} - Call\left(\hat{\Theta}, \hat{V}_t\right)_i^{model} \right)^2 \right]$$

In order to obtain the values of ${\sigma_{t,T}^{BS}}^2$ (Black-Scholes implied volatility) for each day we select one ATM-option (call), because ${\sigma_{t,T}^{BS}}^2 \approx \sigma_{t,T}^2$ for ATM-options. By using a proxy for V_t we decrease the number of estimated parameters significantly without major loss in the goodness of fit.

In [3] the following objective function was used:

$$\log\left(\mathcal{L}\left(\hat{\Theta}, V_t\right)\right) = \frac{N}{2}\log(\sigma_\varepsilon^2) + \frac{1}{2}\sum_i^N \left(\frac{Call_i^{market} - Call\left(\hat{\Theta}\right)_t^{model}}{\sigma_\varepsilon S_t} \right)^2$$

We avoided this specification and chose SSE minimization due to the following considerations. First of all, since the pricing formula is computationally heavy itself, we looked for a measure of loss as simple and intuitive as possible. SSE and MSE are frequently used by researchers to assess option pricing models ([19]). Secondly, we avoided estimating additional parameter, σ_ε. Finally, since we investigate OTM-options, we are facing the following dilemma: deep OTM-options are traded less frequently, and so they should be less important during calibration. This weighting scheme is achieved by SSE-minimization: the price of an option decreases as its moneyness decreases, and so, the same relative pricing error will be smaller in absolute terms for cheaper options with lower moneyness. Therefore, deep OTM-options have lower weights in the process of optimization.

The optimization is performed in MatLab using $fmincon$ function based on interior-point algorithm. The bounds for parameters are derived from [3].

After calibration we use Diebold-Mariano test[3] to compare the models of interest. We cannot use standard F-test for this purpose because of the nonlinear form of our pricing formula. Instead, we test the hypothesis that the mean difference between squared pricing errors of the two models is equal to zero versus the alternative hypothesis (one-sided). We apply Newey-West variance estimator to account for possible autocorrelation in differences between squared errors.

[3] We use Diebold-Mariano Test Statistic for MatLab created by Semin Ibisevic

Estimation results.

The results of calibration are presented in the table. In general, it is evident that parameters are not stable across years for the same company. Significant errors in estimated parameters might be due to invalid global minimum search, i.e. it might be the case that the numerical optimization algorithm failed to find the true global minimum of the loss function. However, since parameters are fairly consistent with the previous research, we can still make some conclusions.

AAPL		$k^{\mathbb{Q}}$	$\theta^{\mathbb{Q}}$	$\sigma^{V(\mathbb{Q})}$	$\rho^{\mathbb{Q}}$	$\sigma^{Z(\mathbb{Q})}$	$\nu^{\mathbb{Q}}$	${\rho_j}^{\mathbb{Q}}$
2002	SV	2.6466	0.0847	0.1467	-0.6357			
	SVJ	1.9975	0.0932	0.1325	-1.0000	0.0797		
	SVJJ	1.7109	0.1005	0.1319	-0.9945	0.0802	1.0E-05	-0.5502
2004	SV	5.1737	0.0613	0.0475	-0.1357			
	SVJ	3.1992	0.0962	0.0473	-0.0954	0.0613		
	SVJJ	3.4796	0.0865	0.0473	-0.1001	0.0546	0.0151	-0.0200
2006	SV	1.4655	0.2479	0.0999	-0.4222			
	SVJ	1.0463	0.2348	0.0946	-0.4538	0.0335		
	SVJJ	1.3099	0.2298	0.1023	-0.4168	0.0335	8.3E-08	-0.5017

Table 2: Estimation results; source: own calculations

AAPL		DM	p-value
2002	SV		
	SVJ	3.5494	0.0002
	SVJJ	0.7071	0.2398
2004	SV		
	SVJ	2.7368	0.0031
	SVJJ	5.1173	0.0000
2006	SV		
	SVJ	2.3599	0.0091
	SVJJ	0.7108	0.2386

Table 3: Diebold-Mariano tests; source: own calculations

The Diebold-Mariano test statistics (DM) reported in the table have the following meaning. The DM-statistic and the p-value next to the SVJ model corresponds to the null hypothesis that the SVJ model does not provide a significantly better fit than the SV model versus the alternative hypothesis that the SVJ model is better than the SV model. The DM-statistic and the p-value next to the SVJJC model corresponds to the null hypothesis that the SVJJC model does not provide a significantly better fit than the SVJ model versus the alternative hypothesis that the SVJJC model is better than the SVJ model.

First of all, it is very important to mention that the SVJ model (an analogue of Dubinsky and Johannes) outperformed the SV model in all three samples at any reasonable significance level, since the corresponding p-values are very close to zero. This result is in line with the empirical performance of the model in [3]. Another consistent result is that introducing jumps tend to decrease the estimates of long-run variance and volatility of variance. This is an intuitive result, since without jumps their effect was attributed to either of these two variables.

The evidence on the performance of the SVJJC model versus the SVJ model is somewhat mixed. In two samples out of three the SVJJC model does not provide significantly better fit than the SVJ model, and the corresponding values of the mean jump in variance is close to zero. However, in the sample (2004) the SVJJC model outperformed the SVJ model. In general, there is not enough evidence to claim that the SVJJC model is an improvement compared to the SVJ model.

This result is due to several possible reasons. First, it might be the case that in general, potential variance jumps add only a marginal effect on variance. Secondly, investors and traders might consider the risk associated with jumps in returns much more important than the risk associated with variance jumps. Third, the results may be biased due to the extreme computational complexity of numerical optimization, which might provide local extrema instead of global ones. Finally, the results are contingent on the sample under consideration. The model might perform better in the times of market turbulence. In this case, our model might be more flexible to fit the skewed pattern of returns distribution.

Conclusion

In this paper we have derived a closed-form option pricing formula for stock price process with simultaneous correlated scheduled jumps in stock price and volatility. The model can be viewed as a significant generalization of the model in [3], since it not only introduces the jump in volatility, but also can flexibly change distributions of jumps by choosing the jumps joint characteristic function $\theta(c_1, c_2)$. This model can be useful in investigating the potential impact of massive information inflows during scheduled events (such as earnings announcements).

However, within this framework further studies should be performed, namely, in the empirical part. We did not find enough evidence in favor of the model. The model should be tested more carefully on a broader sample. It could also be interesting to calibrate the model on the data during turbulent changes on the markets (during crises or turmoil related to a single company).

One further improvement can be done via implementing randomly arriving jumps, as in [4]. This can be done relatively easy by adjusting $\alpha(t)$. However, this would make the model more complicated and probably less parsimonious.

References.

[1] ***Bakshi, G., Cao, C. and Chen, Z.*** Empirical Performance of Alternative Option Pricing Models. The Journal of Finance, Vol. LII, 5, 1997. Pp. 2003 – 2049

[2] ***Black, F. and Scholes, M.*** Pricing of Options and Corporate Liabilities. The Journal of Political Economy, Vol. 81, No. 3, The University of Chicago Press, 1973. Pp. 637-654

[3] ***Dubinsky, A., Johannes, M.*** Fundamental Uncertainty, Earning Announcements and Equity Options. Graduate School of Business, Columbia University, 2006

[4] ***Duffie, D. Pan, J., Singleton, K.*** Transform Analysis and Asset Pricing for Affine-Jump Diffusions. Graduate School of Business, Stanford University, 1999

[5] ***Eraker, B.*** Do Stock Prices and Volatility Jump? Reconciling Evidence from Spot and Option Prices. The Journal of Finance, Vol. LIX, No. 3, 2004. Pp. 1367-1404

[6] ***Eraker, B., Johannes, M., Polson, N.*** The Impact of Jumps in Volatility and Returns. The Journal of Finance, Vol. LVIII, No. 3, 2003. Pp. 1269-1300

[7] ***Harrison, J. and Pliska, S.*** Martingales and Stochastic Integrals in the Theory of Continuous Trading. Stochastic Processes and Their Applications, 1981. Pp. 215-260

[8] ***Heston, S.*** A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. Review of Financial Studies, Vol. 6, No. 2, 1993. Pp. 327-343.

[9] ***Hull, J. and White, A.*** The Pricing of Options on Assets with Stochastic Volatilities. The Journal of Finance, Vol. 42, 2, 1987. Pp. 281-300

[10] ***Hull, J.*** Futures, Options, and Other Derivatives. Prentice Hall, 2009

[11] ***Jennings, R., Starks, L.*** Information Content and the Speed of Stock Price Adjustment. Journal of Accounting Research, Vol. 23, No. 1, 1985. Pp. 336-350

[12] ***Lee, R.W.*** Option Pricing by Transform Methods: Extensions, Unification, and Error Control. Journal of Computational Finance, 2004, 7(3). Pp. 51–86

[13] ***Lyuu, Y.*** Financial Engineering and Computation: Principles, Mathematics, and Algorithms. Binom, 2010

[14] ***Matsuda, K.*** Introduction to Option Pricing with Fourier Transform: Option Pricing with Exponential Lévy Models. Department of Economics, The Graduate Center, The City University of New York, 2004.

[15] ***Merton, R.C.*** Option Pricing when Underlying Stock Returns are Discontinuous. Journal of Financial Economics 3, 1976. Pp. 125-144

[16] ***Piazzesi, M.*** Bond Yields and the Federal Reserve. Journal of Political Economy, Vol. 113, No. 2, 2005. Pp. 311-344

[17] ***Shah, A.*** Black, Merton and Scholes: Their Work and Its Consequences. 1997

[18] ***Shreve, S.E.*** Stochastic Calculus for Finance II: Continuous-Time Models. Springer Science + Business Media, LLC, 2004

[19] ***Storcheus, D., Gelman, S.*** Continuous Time Option Pricing with Scheduled Jumps in the Underlying Asset. Working paper at the Laboratory of Financial Economics, International College of Economics and Finance, National Research University – Higher School of Economics, Moscow, 2012

[20] ***Todorov, V., Tauchen, G.*** Volatility Jumps. Journal of Business and Economic Statistics, Vol. 29, 2011. Pp. 356-371

[21] ***Zastawniak, T., Brzezniak, Z.,*** Basic Stochastic Processes. Springer, 2000

Appendices.

A1. Derivation of $\psi(u,S_t,V_t,t,T)$.

Suppose that $e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}\left[e^{u\log(S_T)}|\mathcal{F}_t\right]=\psi(u,S_t,V_t,t,T)=\exp[\alpha(t)+u\log(S_t)+\beta(t)V_t]$

The following shows that $e^{r(T-t)}\psi$ is a martingale:

$\mathbb{E}^{\mathbb{Q}}\left[e^{r(T-t)}\psi(u,S_t,V_t,t,T)|\mathcal{F}_s\right]=\mathbb{E}^{\mathbb{Q}}\left[\mathbb{E}^{\mathbb{Q}}\left[e^{u\log(S_T)}|\mathcal{F}_t\right]|\mathcal{F}_s\right]=\mathbb{E}^{\mathbb{Q}}\left[e^{u\log(S_T)}|\mathcal{F}_s\right]$

We now apply Ito's formula to ψ.

$$
\begin{aligned}
d\psi &= \frac{\partial\psi}{\partial t}dt+\frac{\partial\psi}{\partial\log(S_t)}d\log(S_t)^{cont}+\frac{\partial\psi}{\partial V_t}dV_t^{cont}+\frac{1}{2}\frac{\partial^2\psi}{\partial\log(S_t)^2}d\log(S_t)^{cont^2}\\
&\quad+\frac{1}{2}\frac{\partial^2\psi}{\partial {V_t}^2}\,d{V^{cont}}_t^2+\frac{\partial^2\psi}{\partial\log(S_t)\,\partial V_t}d\log(S_t)^{cont}\,dV_t^{cont}\\
&\quad+\big(\psi(u,S_t,V_t,t,T)-\psi(u,S_{t-},V_{t-},t-,T)\big)dN_t\\
&=\psi\big(\dot{\alpha_0}(t)+\dot{\beta}(t)V_t\big)dt+\psi u\,d\log(S_t)^{cont}+\psi\beta(t)\,dV_t^{cont}+\frac{1}{2}\psi u^2\,d\log(S_t)^{2^{cont}}\\
&\quad+\frac{1}{2}\psi\beta^2(t)\,dV_t^{2^{cont}}+\psi u\beta(t)d\log(S_t)^{cont}\,dV_t^{cont}\\
&\quad+\big(\psi(u,S_t,V_t,t,T)-\psi(u,S_{t-},V_{t-},t-,T)\big)\\
&=\psi\big(\dot{\alpha_0}(t)+\dot{\beta}(t)V_t\big)dt+\psi u\left(\left(r-\frac{1}{2}V_t\right)dt+\sqrt{V_t}\;dW_t^S(\mathbb{Q})\right)\\
&\quad+\psi\beta(t)\;\Big(k^{\mathbb{Q}}(\theta^{\mathbb{Q}}-V_t)dt+\sigma^{V(\mathbb{Q})}\sqrt{V_t}\,dW_t^V(\mathbb{Q})\Big)+\frac{1}{2}\psi u^2\,V_t dt\\
&\quad+\frac{1}{2}\psi\beta^2(t)\sigma^{V(\mathbb{Q})^2}V_t dt+\psi u\beta(t)\sigma^{V(\mathbb{Q})}V_t\,\rho^{\mathbb{Q}}\,dt\\
&\quad+\big(\psi(u,S_t,V_t,t,T)-\psi(u,S_{t-},V_{t-},t-,T)\big)\\
&=\psi\left(\dot{\alpha_0}(t)+\dot{\beta}(t)V_t+u\left(r-\frac{1}{2}V_t\right)+\beta(t)k^{\mathbb{Q}}(\theta^{\mathbb{Q}}-V_t)+\frac{1}{2}u^2\,V_t+\frac{1}{2}\beta^2(t)\sigma^{V(\mathbb{Q})^2}V_t\right.\\
&\quad\left.+u\beta(t)\sigma^{V(\mathbb{Q})}V_t\,\rho^{\mathbb{Q}}\right)dt+\psi u\sqrt{V_t}\;dW_t^S(\mathbb{Q})+\psi\beta(t)\,\sigma^{V(\mathbb{Q})}\sqrt{V_t}\,dW_t^V(\mathbb{Q})\\
&\quad+\big(\psi(u,S_t,V_t,t,T)-\psi(u,S_{t-},V_{t-},t-,T)\big)
\end{aligned}
$$

$\psi(u,S_t,V_t,t,T)-\psi(u,S_{t-},V_{t-},t,T)=0$ for all $t\neq t_0$

Hence,

$$
d\left(e^{r(T-t)}\psi(u,S_t,V_t,t,T)\right)=-e^{r(T-t)}r\psi dt+e^{r(T-t)}d\psi=
$$

$$= e^{r(T-t)}\psi\left(-r + \dot{\alpha_0}(t) + \dot{\beta}(t)V_t + u\left(r - \frac{1}{2}V_t\right) + \beta(t)k^{\mathbb{Q}}(\theta^{\mathbb{Q}} - V_t) + \frac{1}{2}u^2\, V_t\right.$$
$$\left. + \frac{1}{2}\beta^2(t)\sigma^{V(\mathbb{Q})^2}V_t + u\beta(t)\sigma^{V(\mathbb{Q})}V_t\, \rho^{\mathbb{Q}}\right)dt + e^{r(T-t)}\psi u\, \sqrt{V_t}\;\, dW_t^S(\mathbb{Q})$$
$$+ e^{r(T-t)}\psi\beta(t)\, \sigma^{V(\mathbb{Q})}\sqrt{V_t}\, dW_t^V(\mathbb{Q})$$
$$+ e^{r(T-t)}\big(\psi(u, S_t, V_t, t, T) - \psi(u, S_{t-}, V_{t-}, t-, T)\big)$$

Since $e^{r(T-t)}\psi$ is a martingale and $\psi(u, S_t, V_t, t, T) - \psi(u, S_{t-}, V_{t-}, t-, T) = 0$ for all $t \neq t_0$, its drift term must be equal to zero, i.e.

$$e^{r(T-t)}\psi\left(-r + \dot{\alpha_0}(t) + \dot{\beta}(t)V_t + u\left(r - \frac{1}{2}V_t\right) + \beta(t)k^{\mathbb{Q}}(\theta^{\mathbb{Q}} - V_t) + \frac{1}{2}u^2\, V_t\right.$$
$$\left. + \frac{1}{2}\beta^2(t)\sigma^{V(\mathbb{Q})^2}V_t + u\beta(t)\sigma^{V(\mathbb{Q})}V_t\, \rho^{\mathbb{Q}}\right) = 0$$
$$-r + \dot{\alpha_0}(t) + \dot{\beta}(t)V_t + u\left(r - \frac{1}{2}V_t\right) + \beta(t)k^{\mathbb{Q}}(\theta^{\mathbb{Q}} - V_t) + \frac{1}{2}u^2\, V_t + \frac{1}{2}\beta^2(t)\sigma^{V(\mathbb{Q})^2}V_t$$
$$+ u\beta(t)\sigma^{V(\mathbb{Q})}V_t\, \rho^{\mathbb{Q}} = 0$$
$$\left(\dot{\alpha_0}(t) - r(1-u) + k^{\mathbb{Q}}\theta^{\mathbb{Q}}\beta(t)\right)$$
$$+ \left(\dot{\beta}(t) - \frac{u}{2} - k^{\mathbb{Q}}\beta(t) + \frac{u^2}{2} + \frac{\beta^2(t)}{2}\sigma^{V(\mathbb{Q})^2} + u\beta(t)\sigma^{V(\mathbb{Q})}\rho^{\mathbb{Q}}\right)V_t = 0$$

Since V_t has almost surely non-differentiable paths,

$$\dot{\alpha_0}(t) - r(1-u) + k^{\mathbb{Q}}\theta^{\mathbb{Q}}\beta(t) = 0 \qquad (6.1)$$
$$\dot{\beta}(t) - \tfrac{u}{2} - k^{\mathbb{Q}}\beta(t) + \tfrac{u^2}{2} + \tfrac{\beta^2(t)}{2}\sigma^{V(\mathbb{Q})^2} + u\beta(t)\sigma^{V(\mathbb{Q})}\rho^{\mathbb{Q}} = 0 \quad (6.2)$$

For $e^{r(T-t)}\psi$ to be a martingale at the jump time $t = t_0$, the following condition must be satisfied:

$$\mathbb{E}^{\mathbb{Q}}\left[e^{r(T-t)}\psi\left(u, S_{t_0}, V_{t_0}, t_0, T\right)|\mathcal{F}_{t_0-}\right] = e^{r(T-t-)}\psi\left(u, S_{t_0-}, V_{t_0-}, t_0-, T\right)$$
$$\mathbb{E}^{\mathbb{Q}}\left[\frac{\psi\left(u, S_{t_0}, V_{t_0}, t_0, T\right)}{\psi\left(u, S_{t_0-}, V_{t_0-}, t_0-, T\right)}|\mathcal{F}_{t_0-}\right] = 1$$
$$\mathbb{E}^{\mathbb{Q}}\left[\exp\left[\alpha^j(t_0) - \alpha^j(t_0-) + uZ^S + \beta(t_0)Z^V\right]|\mathcal{F}_{t_0-}\right] = 1$$
$$\exp\left[\alpha^j(t_0) - \alpha^j(t_0-)\right]\cdot\mathbb{E}^{\mathbb{Q}}\left[\exp[uZ^S + \beta(t_0)Z^V]\,|\mathcal{F}_{t_0-}\right] = 1$$
$$\exp\left[\alpha^j(t_0) - \alpha^j(t_0-)\right]\cdot\mathbb{E}^{\mathbb{Q}}[\exp[uZ^S + \beta(t_0)Z^V]] = 1$$
$$\exp\left[\alpha^j(t_0) - \alpha^j(t_0-)\right]\cdot\theta\left(u, \beta(t_0)\right) = 1$$
$$\alpha^j(t_0) - \alpha^j(t_0-) = -\ln\left(\theta\left(u, \beta(t_0)\right)\right)$$

For any $t \neq t_0$

$$\alpha^j(t_0) - \alpha^j(t_0 -) = 0$$

For $\tau = T - t$

$$\left(\alpha^j(\tau) - \alpha^j(\tau -)\right) = -\left(\alpha^j(t) - \alpha^j(t -)\right)$$

$\alpha(\tau) = -\int_0^\tau \dot{\alpha_0}(s)ds + \sum_{0\le s\le\tau}\left(\alpha^j(\tau) - \alpha^j(\tau -)\right) = -\int_0^\tau \dot{\alpha_0}(s)ds - \sum_{0\le s\le\tau}\left(\alpha^j(t) - \alpha^j(t -)\right) = -\int_0^\tau \dot{\alpha_0}(s)ds + \ln\left(\theta\left(u,\beta(t_0)\right)\right)(1-N_t)$ (6.3)

A2. Proof that $\exp[\alpha(t) + u\log(S_t) + \beta(t)V_t]$ is indeed a discounted characteristic function $e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}\left[e^{u\log(S_T)}|\mathcal{F}_t\right]$.

Let $\Psi(t) = \exp[\alpha(t) + u\log(S_t) + \beta(t)V_t - rt]$. Let us show that Ψ is a martingale with $\alpha(t)$ and $\beta(t)$ satisfying the ODEs (6.1)-(6.2).

By Ito formula:

$$\Psi_t = \Psi_0 + \int_0^t \Psi_s\left(-r + \dot{\alpha_0}(s) + \dot{\beta}(s)V_s + u\left(r - \frac{1}{2}V_s\right) + \beta(s)k^{\mathbb{Q}}(\theta^{\mathbb{Q}} - V_s) + \frac{1}{2}u^2 V_s + \frac{1}{2}\beta^2(s){\sigma^{V(\mathbb{Q})}}^2 V_s + u\beta(s)\sigma^{V(\mathbb{Q})}V_s\,\rho^{\mathbb{Q}}\right)ds + \int_0^t \Psi_s u\sqrt{V_s}\;dW_s^S(\mathbb{Q}) + \int_0^t \Psi_s\beta(s)\,\sigma^{V(\mathbb{Q})}\sqrt{V_s}\,dW_s^V(\mathbb{Q}) + \left(\Psi_{t_0} - \Psi_{t_0-}\right)\mathbb{I}\{t \ge t_0\}$$

From ODEs (6.1)-(6.2) we have

$$\int_0^t \Psi_s\left(-r + \dot{\alpha_0}(s) + \dot{\beta}(s)V_s + u\left(r - \frac{1}{2}V_s\right) + \beta(s)k^{\mathbb{Q}}(\theta^{\mathbb{Q}} - V_s) + \frac{1}{2}u^2 V_s + \frac{1}{2}\beta^2(s){\sigma^{V(\mathbb{Q})}}^2 V_s + u\beta(s)\sigma^{V(\mathbb{Q})}V_s\,\rho^{\mathbb{Q}}\right)ds = 0$$

$\int_0^t \Psi_s u\sqrt{V_s}\;dW_s^S(\mathbb{Q})$ and $\int_0^t \Psi_s\beta(s)\,\sigma^{V(\mathbb{Q})}\sqrt{V_s}\,dW_s^V(\mathbb{Q})$ are Ito integrals, hence, they are martingales

Since (6.3) is satisfied, $\left(\Psi_{t_0} - \Psi_{t_0-}\right)\mathbb{I}\{t \ge t_0\}$ is a martingale:

$$\mathbb{E}^{\mathbb{Q}}\left[\left(\Psi_{t_0} - \Psi_{t_0-}\right)\mathbb{I}\{t \ge t_0\}|\mathcal{F}_s\right] = \left(\Psi_{t_0} - \Psi_{t_0-}\right)\mathbb{I}\{s \ge t_0\}$$

Thus, Ψ_t is a martingale, and

$$\mathbb{E}^{\mathbb{Q}}[\Psi_T|\mathcal{F}_t] = \Psi_t$$

Let us multiply both sides by $\exp[rt]$:

$$\exp[rt]\,\mathbb{E}^{\mathbb{Q}}[\Psi_T|\mathcal{F}_t] = \exp[rt]\,\Psi_t$$

$$\exp[rt]\,\mathbb{E}^{\mathbb{Q}}[\exp[\alpha(T) + u\log(S_T) + \beta(T)V_T - rT]\,|\mathcal{F}_t]$$
$$= \exp[rt]\exp[\alpha(t) + u\log(S_t) + \beta(t)V_T - rt]$$

$$\exp[-r(T-t)]\,\mathbb{E}^{\mathbb{Q}}[\exp[\alpha(T) + u\log(S_T) + \beta(T)V_T]\,|\mathcal{F}_t]$$
$$= \exp[\alpha(t) + u\log(S_t) + \beta(t)V_T]$$

$$\alpha(T) = 0$$

$$\beta(T) = 0$$

$$\exp[-r(T-t)]\,\mathbb{E}^{\mathbb{Q}}[\exp[u\log(S_T)]\,|\mathcal{F}_t] = \exp[\alpha(t) + u\log(S_t) + \beta(t)V_T]$$

QED

A3. Derivation of $\mathbb{E}^{\mathbb{Q}}\left[\int_t^T V_s\,ds|\mathcal{F}_t\right]$ with no correlations.

$$\mathbb{E}^{\mathbb{Q}}\left[\int_t^T V_s\,ds|\mathcal{F}_t\right] = \int_t^T \mathbb{E}^{\mathbb{Q}}[V_s|\mathcal{F}_t]\,ds$$

Assume that $t < t_0$ (i.e. the scheduled event has not happened by time t). Let us find the value of $\mathbb{E}^{\mathbb{Q}}[V_s|\mathcal{F}_t]$:

$$d\mathbb{E}^{\mathbb{Q}}[V_s|\mathcal{F}_t] = \mathbb{E}^{\mathbb{Q}}[dV_s|\mathcal{F}_t] = \mathbb{E}^{\mathbb{Q}}\left[k^{\mathbb{Q}}(\theta^{\mathbb{Q}} - V_s)ds + \sigma^{V(\mathbb{Q})}\sqrt{V_s}\,dW_s^V(\mathbb{Q}) + Z^V dN_s|\mathcal{F}_t\right]$$
$$= k^{\mathbb{Q}}\theta^{\mathbb{Q}}ds - k^{\mathbb{Q}}\mathbb{E}^{\mathbb{Q}}[V_s|\mathcal{F}_t]ds + \mathbb{E}^{\mathbb{Q}}[Z^V dN_s|\mathcal{F}_t]$$
$$= k^{\mathbb{Q}}\theta^{\mathbb{Q}}ds - k^{\mathbb{Q}}\mathbb{E}^{\mathbb{Q}}[V_s|\mathcal{F}_t]ds + \nu dN_s$$

Denote $\mathbb{E}^{\mathbb{Q}}[V_s|\mathcal{F}_t] = f(s)$, $s \geq t$.

$$df = k^{\mathbb{Q}}\theta^{\mathbb{Q}}ds - k^{\mathbb{Q}}f ds + \nu^{\mathbb{Q}}dN_s$$

For $t \leq s < t_0$:

$$df = k^{\mathbb{Q}}\theta^{\mathbb{Q}}ds - k^{\mathbb{Q}}f ds$$

$$\dot{f} = k^{\mathbb{Q}}\theta^{\mathbb{Q}} - k^{\mathbb{Q}}f$$

$$f(s) = (V_t - \theta^{\mathbb{Q}})e^{-k^{\mathbb{Q}}(s-t)} + \theta^{\mathbb{Q}}$$

$$f(t_0\,-) = (V_t - \theta^{\mathbb{Q}})e^{-k^{\mathbb{Q}}(t_0-t)} + \theta^{\mathbb{Q}}$$

$$f(t_0) = (V_t - \theta^{\mathbb{Q}})e^{-k^{\mathbb{Q}}(t_0-t)} + \theta^{\mathbb{Q}} + \nu^{\mathbb{Q}}$$

Hence, for $t_0 \leq s$:

$$f(s) = Ce^{-k^{\mathbb{Q}}s} + \theta^{\mathbb{Q}}$$

$$f(t_0) = Ce^{-k^{\mathbb{Q}}t_0} + \theta^{\mathbb{Q}} = (V_t - \theta^{\mathbb{Q}})e^{-k^{\mathbb{Q}}(t_0-t)} + \theta^{\mathbb{Q}} + \nu^{\mathbb{Q}}$$

$$Ce^{-k^{\mathbb{Q}}t_0} = (V_t - \theta^{\mathbb{Q}})e^{-k^{\mathbb{Q}}(t_0-t)} + \nu^{\mathbb{Q}}$$

$$C = (V_t - \theta^{\mathbb{Q}})e^{k^{\mathbb{Q}}t} + \nu^{\mathbb{Q}}e^{k^{\mathbb{Q}}t_0}$$

$$f(s) = \left((V_t - \theta^{\mathbb{Q}})e^{k^{\mathbb{Q}}t} + \nu^{\mathbb{Q}}e^{k^{\mathbb{Q}}t_0}\right)e^{-k^{\mathbb{Q}}s} + \theta^{\mathbb{Q}} = (V_t - \theta^{\mathbb{Q}})e^{-k^{\mathbb{Q}}(s-t)} + \nu^{\mathbb{Q}}e^{-k^{\mathbb{Q}}(s-t_0)} + \theta^{\mathbb{Q}}$$

Hence,

$$f(s) = \begin{cases} (V_t - \theta^{\mathbb{Q}})e^{-k^{\mathbb{Q}}(s-t)} + \theta^{\mathbb{Q}} & t \leq s < t_0 \\ (V_t - \theta^{\mathbb{Q}})e^{-k^{\mathbb{Q}}(s-t)} + \nu^{\mathbb{Q}}e^{-k^{\mathbb{Q}}(s-t_0)} + \theta^{\mathbb{Q}} & t_0 \leq s \end{cases}$$

$$\mathbb{E}^{\mathbb{Q}}\left[\int_t^T V_s\,ds|\mathcal{F}_t\right] = \int_t^T \mathbb{E}^{\mathbb{Q}}[V_s|\mathcal{F}_t]\,ds = \int_t^T f(s)\,ds = \int_t^{t_0} f(s)\,ds + \int_{t_0}^T f(s)\,ds =$$

$$= \int_t^T (V_t - \theta^{\mathbb{Q}})e^{-k^{\mathbb{Q}}(s-t)}\,ds + \int_t^T \theta^{\mathbb{Q}}\,ds + \int_{t_0}^T \nu^{\mathbb{Q}}e^{-k^{\mathbb{Q}}(s-t_0)}\,ds =$$

$$= \frac{V_t - \theta^{\mathbb{Q}}}{k^{\mathbb{Q}}}\left(1 - e^{-k^{\mathbb{Q}}(T-t)}\right) + \frac{\nu^{\mathbb{Q}}}{k^{\mathbb{Q}}}\left(1 - e^{-k^{\mathbb{Q}}(T-t_0)}\right) + \theta^{\mathbb{Q}}(T-t)$$