Advances in

OR, Optimization and Control of Stochastic Dynamics - *Applications in Finance, Economics, Biology and Environment*

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Outline

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- Bubbles, Jumps and Insiders
- Stochastic Control of Stochastic Hybrid Systems I
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- Hamilton-Jacobi-Bellman Equation
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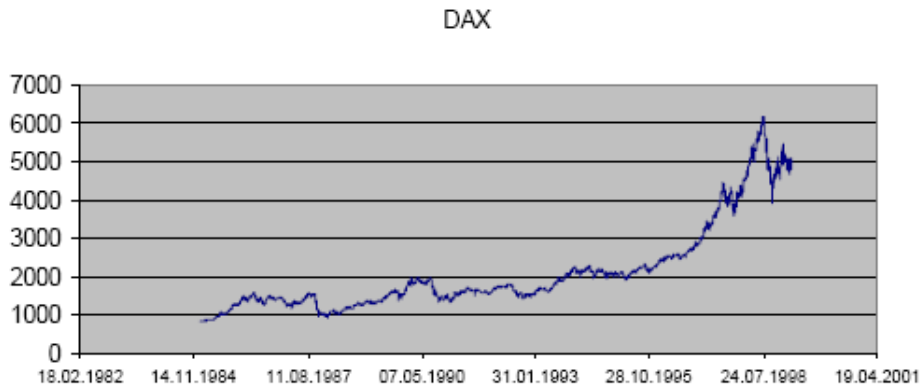
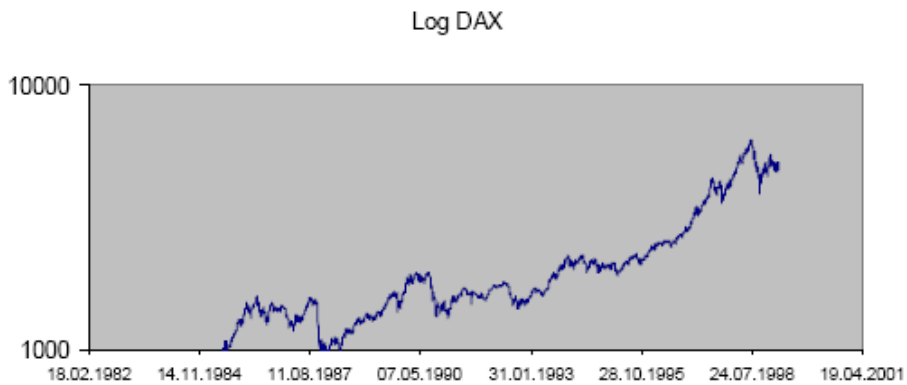
Stock Price Dynamics



$$dX_t = a(X_t, t)dt + b(X_t, t)dW_t$$



d-fine, 2002

Stock Price Dynamics

NIG Lévy Asset Price Model

Normal Inverse Gaussian Process (NIG) is the subclass of generalized hyperbolic laws and has the following representation:

$$f_{NIG}(x;\alpha,\beta,\delta,\mu) = \alpha \exp\left\{\delta\sqrt{\alpha^2-\beta^2} + \beta(x-\mu)\right\} \frac{K_1(\alpha\sqrt{\delta^2+(x-\mu)^2})}{\sqrt{\delta^2+(x-\mu)^2}},$$

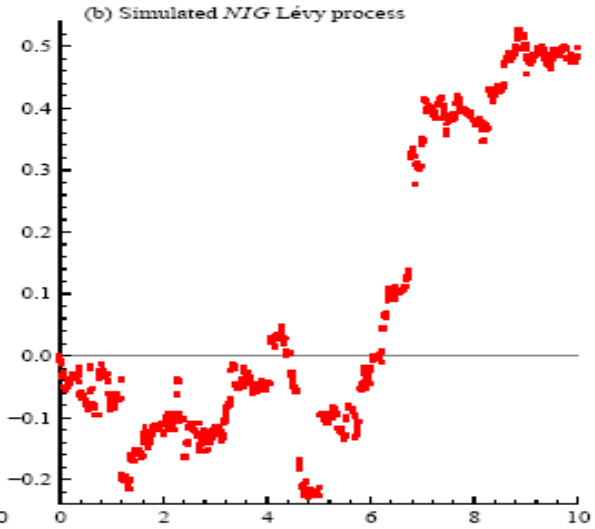
where

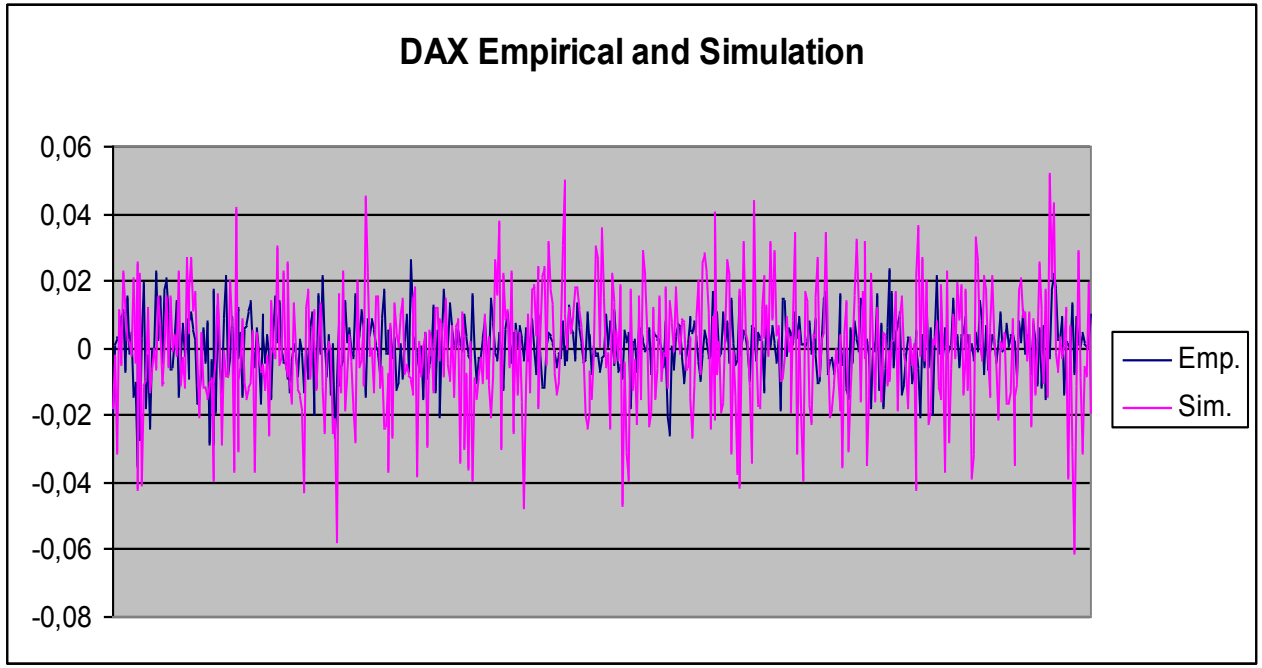
$$K_1(y) = \frac{y}{4}\int_0^\infty \exp(t+\frac{y^2}{4t})t^{-2}dt.$$

Ö. Önalan, 2006

Stock Price Dynamics

NIG Lévy Asset Price Model





Lévy Processes

Some basic Definition:

A process is said to be ***càdlàg*** ("continue à droite, limite à gauche") or RCLL ("right continuous with left limits") is a function that is right-continuous and has left limits in every point. Càdlàg functions are important in the study of stochastic processes that admit (or even require) jumps, unlike Brownian motion, which has continuous sample paths.

A process is ***adapted*** if and only if, for every realisation and every n,

this process is known at time n.

Lévy Processes

Definition: A cadlag adapted processes

$$L = (L_t \mid t \geq 0)$$

defined on a probability space (Ω, *F*, *P*) is said to be a ***Lévy processes*****,** if it possesses the following properties:

Lévy Processes

(í) $P\left(L_0 = 0 \right) = 1.$

(íí) For $0 \le s \le t,\ L_t - L_s$ is independent of F_s,
i.e., L has independent increments.

(ííí) For any $0 \le s \le t,\ L_t - L_s$ is equal in distribution to L_{t-s}
(the distribution of $L_{t+s} - L_s$ does not depend on t);
L has stationary increments.

(ív) For every $s, t \ge 0$ and $\varepsilon > 0$,
$$\lim_{s \to t} P\left(\left| L_t - L_s \right| > \varepsilon \right) = 0,$$
i.e., L is stochastically continuous.

In the presence of (í), (íí), (ííí), this is equivalent to the condition
$$\lim_{t \downarrow 0} P\left(\left| L_t \right| > \varepsilon \right) = 0.$$

Lévy Processes

- There is strong interplay between

  Lévy processes and infinitely divisible distributions.

Proposition: If L is a Lévy processes, then L_t is infinitely divisible for each $t > 0$.

Proof : For any $n \in \mathbb{N}$ and any $t \geq 0$:

$$L_t = L_{t/n} + \left(L_{2t/n} - L_{t/n} \right) + \ldots + \left(L_t - L_{(n-1)t/n} \right).$$

- Together with the stationarity and independence of increments we conclude that the random variable L_t is infinitely divisible.

Lévy Processes

- Moreover, for all $u \in \mathbf{R}$ and all $t \geq 0$ we define

$$\Psi_t(u) := \ln E\left[\, e^{i\,u\,L_t} \,\right];$$

hence, for rational $t > 0$:

$$t\,\Psi_1(u) = \Psi_t(u).$$

Lévy Processes

For every Lévy process, the following property holds:

$$E\left[e^{iuL_t} \right] = e^{t\Psi_1(u)}$$

$$= \exp\left[t\left(ibu - \frac{u^2 c}{2} + \int_{\mathbf{R}} \left(e^{iux} - 1 - iux1_{\{|x|<1\}} \right) \nu(dx) \right) \right],$$

where $\Psi(u) = \Psi_1(u)$ is the characteristic exponent of $L_1 = X$.

Lévy Processes

- The triplet (b,c,ν) is called the Lévy or characteristic triplet and

$$\Psi(u) = iub - \frac{u^2 c}{2} + \int \left(e^{iux} - 1 - iux 1_{\{|x|<1\}} \right) \nu(dx)$$

is called the Lévy or characteristic exponent,

- where $b \in \mathbf{R}$: drift term,

  $c \in \mathbf{R}^+$: diffusion coefficient and

  ν : Lévy measure.

Lévy Processes

The Lévy measure is a measure on $\mathbf{R}\backslash\{0\}$ which satisfies

$$\int_{\mathbf{R}} \left(|x|^2 \wedge 1\right) \nu(dx) < \infty.$$

- This means that a Lévy measure has no mass at the origin, but infinitely many jumps can occur around the origin.
- The Lévy measure describes the expected number of jumps of a certain size in a time in interval length 1.

Lévy Processes

- The sums of all jumps smaller than some $\varepsilon > 0$ does not converge. However, the sum of the jumps compensated by their mean does converge. This pecularity leads to the necessity of the compensator term $iux\mathbf{1}_{\{|x|<1\}}$.
- If the Lévy measure is of the form $v(dx) = f(x)dx$, then $f(x)$ is called the *Lévy density*.
- In the same way as the instant volatility describes the local uncertainty of a diffusion, the Lévy density describes the local uncertainity of a pure jump process.
- The *Lévy-Khintchine Formula* allows us to study the distributional properties of a Lévy process.

Lévy Processes

- Another key concept, the *Lévy-Ito Decomposition Theorem*, allows one to describe the structure of a Lévy process sample path.

  Lévy-Ito Decomposition Theorem:

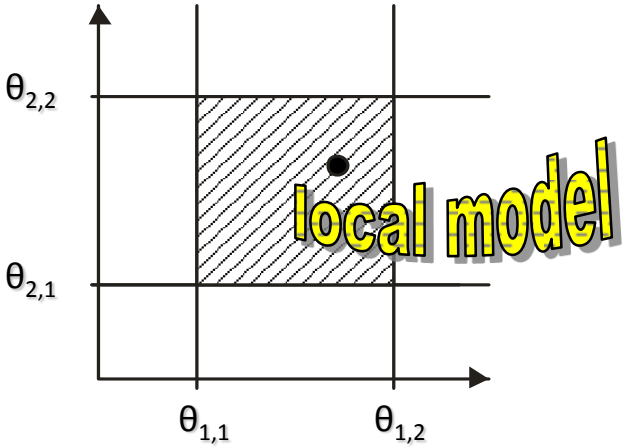
  Let X be a Lévy process, the distribution of X_1 parametrized by (β, σ^2, ν).

  Then X decomposes as $X_t = \beta t + \sigma W_t + J_t + M_t$, where W_t is a Brownian motion, and $\Delta X_t = X_t - X_{t-}$ $(t \geq 0)$ is an independent Poisson point process with intensity measure ν, $J_t = \sum_{s \leq t} \Delta X_s 1_{\{\Delta X_s > 1\}}$ and M_t is a martingale with jumps $\Delta M_t = \Delta X_t 1_{\{\Delta X_t \leq 1\}}$.

Hybrid Systems

Basic Idea

$$X(k+1) = \mathrm{M}_{s(k)} X(k) + C_{s(k)}$$



$$s(k) := F_B Q(X(k-1))$$

$$Q_i(X(k)) := \begin{cases} 1 & \text{if } X_i(k) > \theta_i \\ 0 & \text{otherwise} \end{cases}$$

hybrid

$$\min_{(m_{ij}^*),(c_{i\ell}^*),(d_i^*)} \sum_{\alpha=0}^{l^*-1} \left\| M^* \bar{E}_{\kappa_\alpha} + C^* E_{\kappa_\alpha} + D^* - \dot{\bar{E}}_{\kappa_\alpha} \right\|_\infty^2$$

subject to

$$\begin{cases} \sum_{i=1}^{n} p_{ij}(m_{ij}^*, y) \le \alpha_j(y) & (j = 1, \ldots, n) \\ \sum_{i=1}^{n} q_{i\ell}(c_{i\ell}^*, y) \le \beta_\ell(y) & (\ell = 1, \ldots, m) \\ \sum_{i=1}^{n} \zeta_i(d_i^*, y) \le \gamma(y) & \\ m_{ii} \ge \delta_{i,\min} & (i = 1, \ldots, n) \\ \& \text{ overall box constraints} & \end{cases} \qquad (y \in Y(C^*, D^*))$$

Generalized Semi-Infinite Programming

$$\mathcal{P}_{\mathcal{GSI}}(f,h,g,u,v) \quad \begin{cases} \text{Minimize} \quad f(x) \ \text{ on } \ M_{\mathcal{GSI}}[h,g], \ \text{ where} \\ M_{\mathcal{GSI}}[h,g] \ := \ \{\, x \in \mathbb{R}^n \mid \ h_i(x) = 0 \ \ (i \in I), \\ \qquad\qquad\qquad\qquad g(x,y) \geq 0 \ \ (y \in Y(x))\,\} \end{cases}$$

C^2

$$Y(x) \ = \ M_{\mathcal{F}}[u(x,\cdot), v(x,\cdot)]$$

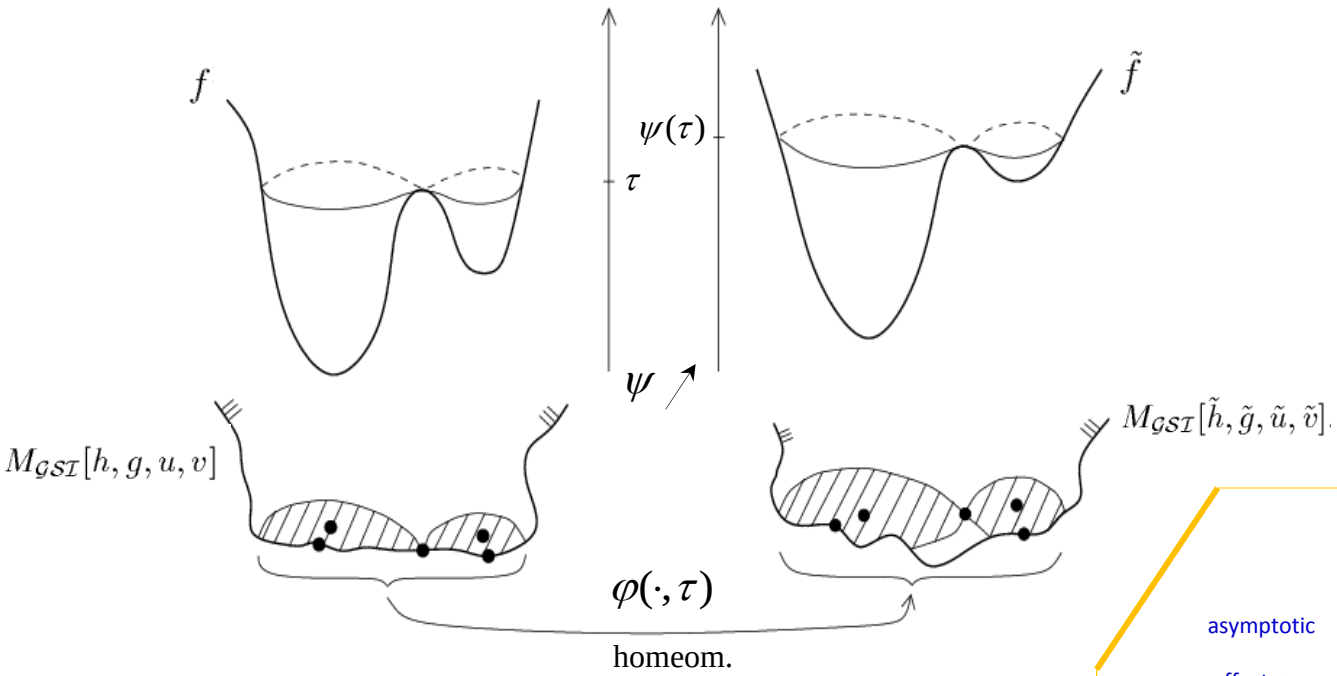
$$:= \ \{\, y \in \mathbb{R}^q \mid \ u_k(x,y) = 0 \ \ (k \in K), \ \ v_\ell(x,y) \geq 0 \ \ (\ell \in L)\,\}$$

I, K, L finite

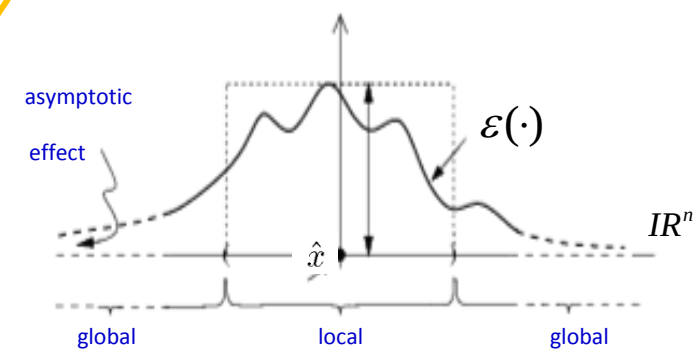
Generalized Semi-Infinite Programming

$\exists\ \mathcal{O}_{(f,h,g,u,v)}\ \forall\,(\tilde{f},\tilde{h},\tilde{g},\tilde{u},\tilde{v}) \in \mathcal{O}_{(f,h,g,u,v)}\quad \exists\ \ \psi(\cdot),\varphi(\cdot,\cdot)\in C^0:$

Jongen, Weber, Guddat et al.



$\Longleftrightarrow$: $\mathcal{P}_{\mathcal{GSI}}(f,h,g,u,v)$ structurally stable



Stochastic Hybrid Model I

$$\left(X(t),\theta(t)\right)_{t\in[0,\infty)}:$$

$$X(t)\in\mathbf{R}^d,\ \theta(t)\in\Theta=\{0,1,\ldots,N\},\quad \text{where}$$

- $\theta(t)$ is a pure jump process taking values in Θ,
- $\left(X(t),\theta(t)\right)$ is a switching jump-diffusion between jumps.

M.K. Ghosh and A. Bagchi, 2004

Stochastic Control of Hybrid Systems

The ***asset price*** dynamics is given by the following system:

$$(1)\quad \begin{cases} dX(t) = \mu(X(t),\theta(t))dt + \sigma(X(t),\theta(t))dW(t) + \int_{\mathbf{R}} g(X(t^-),\theta(t^-),u)N(dt,du), \\ d\theta(t) = \int_{\mathbf{R}} h(X(t^-),\theta(t^-),u)N(dt,du), \end{cases}$$

$$\text{for} \quad t \geq 0,$$

$$X(0) = X_0, \;\; \theta(0) = \theta_0 .$$

The price process $X(t)$ switches from one jump-diffusion path to another as the discrete component $\theta(t)$ moves from one step to another.

Stochastic Control of Hybrid Systems

$h : \mathbf{R}^d \times \Theta \times \mathbf{R} \to \mathbf{R}^N$ is defined as

$$h(x,i,u) = \begin{cases} j-i, & \text{if} \quad u \in \Delta_{ij}(x), \\ 0, & \text{otherwise,} \end{cases}$$

$N(\cdot)$: Poisson random measure,

$\tilde{N}(\cdot)$: Poisson process on $(\Omega, \mathcal{F}, P)$ corresponding to the given Poisson random measure,

$D_{\tilde{N}}$: domain of $\tilde{N}(\cdot)$,

$D = \left\{ t \in D_{\tilde{N}} \mid \tilde{N}(t) \in U \right\}$,

$\mathcal{F} = \sigma\{W(s), N(A,B) \mid s \le t,\ A \in \mathcal{B}([0,T]), B \in \mathcal{B}(\mathbf{R})\}$

Stochastic Control of Hybrid Systems

Under boundedness, measurability and Lipschitz conditions a pathwise unique solution of (1) exists.

Considering the SDE

$$Y(t) = X_0 + \int_0^t \mu(Y(s), \theta_0)\, ds + \int_0^t \sigma(Y(s), \theta_0)\, dW(s),$$

the unique solution $\{X_1(t), \theta_1(t)\}_{t\in[0,\tau_1]}$ in time interval $[0, \tau_1]$ is found as

$$X_1(t) = \begin{cases} Y(t), & \text{if} \quad 0 \le t < \tau_1, \\ Y(\tau_1^-) + g(Y(\tau_1^-), \theta_0, \tilde{N}(\tau_1)), & \text{if} \quad t = \tau_1, \end{cases}$$

$$\theta_1(t) = \begin{cases} \theta_0, & \text{if} \quad 0 \le t < \tau_1, \\ \theta_0 + h(Y(\tau_1^-), \theta_0, \tilde{N}(\tau_1)), & \text{if} \quad t = \tau_1, \end{cases}$$

M.K. Ghosh and A. Bagchi, 2004

Merton's Consumption Investment Problem

An investor with a finite lifetime must determine the amounts c_t that will be consumed and the fraction of wealth π_t that will be invested in a stock portfolio,

so as to **maximize *expected lifetime utility***.

Assuming a *relative consumption rate*

$$\lambda(t) = \frac{c(t)}{X^{(c,\pi)}(t)},$$

the ***wealth process*** evolves with the following SDE:

$$d^- X^{(\lambda,\pi)}(t,\theta_t) = X^{(\lambda,\pi)}(t,\theta_t)\Big[\Big\{r(t,\theta_t) - \lambda(t,\theta_t) + (\mu(X_t^{(\lambda,\pi)},\theta_t) - r(t,\theta_t))\pi(t,\theta_t)\Big\}dt$$

$$+\sigma(X_t^{(\lambda,\pi)},\theta_t)\pi(t,\theta_t)d^- B(t) + \pi(t,\theta_t)\int_{\mathbf{R}_0} g(X_{t^-}^{(\lambda,\pi)},\, \theta_{t^-},\, u)N(dt^-,du)\Big].$$

D. David and Y. Yolcu Okur, 2009

Merton's Consumption Investment Problem

The objective is

$$\max_{(c,\pi)} \quad E\left[\int_0^T U(c_s, s)ds \; + \; g(X^{(c,\pi)}(T), T)\right]$$

$$\text{subject to } \; c(t) \geq 0, \; X^{(c,\pi)}(t) > 0 \; (t \in [0,T]), \; X^{(c,\pi)}(0) = \upsilon.$$

Assuming a logarithmic utility function, which is *iso-elastic,* the optimization problem becomes

$$\max_{(c,\pi)} \quad E\left[\int_0^T e^{-\delta(t)} \ln c(t)dt \; + \; Ke^{-\delta(T)} \ln X^{(c,\pi)}(T)\right]$$

$$\ldots.$$

D. David and Y. Yolcu Okur, 2009

Merton's Consumption Investment Problem

We assume a constant relative risk aversion (CRRA) utility function

$$U(x,t) = \frac{x^{1-\gamma}}{1-\gamma},$$

where $\gamma \in (0,\infty) \setminus \{1\}$.

Hence, the objective takes the following form

$$\max_{(c,\pi)} E\left[\int_0^T e^{-\delta s} U(c(s),s)\,ds + e^{-\delta T} U(X^{(c,\pi)}(T),T)\right] \qquad (2)$$

$$\text{subject to } c(t) \geq 0,\ X^{(c,\pi)}(t) > 0\ (t \in [0,T]),\ X^{(c,\pi)}(0) = \upsilon.$$

Hamilton-Jacobi-Bellman Equation

In applying the ***dynamic programming*** approach, we solve the Hamilton-Jacobi-Bellman (HJB) equation associated with the utility maximization problem (2).

From (W. Fleming and R. Rishel, 1975) we have that the corresponding **HJB equation** is given by

$$\rho J(\upsilon,t) = \max_{c_t \geq 0,\, \pi_t \in \mathbf{R}^n} \Big\{ U(c_t) + J_t(\upsilon,t) + J_\upsilon(\upsilon,t)\Big(\upsilon\big[\pi_t^T(\mu - r) + r\big] - c_t\Big) + \frac{1}{2} J_{\upsilon\upsilon}(\upsilon,t)\upsilon^2 \pi_t^T \sigma\sigma^T \pi_t \Big\} \qquad (3)$$

subject to the terminal condition $J(\upsilon,T) = u(\upsilon)$,
where J the value function is given by

$$J(\upsilon,t) = \sup_{(c,\pi)} E\left[\int_t^T e^{-\delta s} U(c_s,s)\,ds + e^{-\delta T} U(X^{c,\pi}(T),T)\right].$$

Hamilton-Jacobi-Bellman Equation

In solving the HJB equation (3), the static optimization problem

$$\max_{c_t \geq 0,\, \pi_t \in \mathbf{R}^n} \left\{ U(c_t) + J_\upsilon(\upsilon,t)\left(\upsilon\left[\pi_t^T(\mu - r) + r\right] - c_t\right) + \frac{1}{2} J_{\upsilon\upsilon}(\upsilon,t)\upsilon^2 \pi_t^T \sigma\sigma^T \pi_t \right\}$$

can be handled separately to reduce the HJB equation (3) to a nonlinear partial differential equation of J only.

Introducing the Lagrange function as

$$L\left(\pi(\upsilon,t), c(\upsilon,t), \tilde{\lambda}(\upsilon,t)\right) = J_\upsilon(\upsilon,t)\left(\upsilon\left[\pi_t^T(\mu - r) + r - c_t\right]\right) + \frac{1}{2}\upsilon^2 \left\|\pi_t^T \sigma\right\|_2^2 J_{\upsilon\upsilon}(\upsilon,t) + u(c_t) - \tilde{\lambda}(\upsilon,t) c_t, \quad (4)$$

where $\tilde{\lambda}$ is the Lagrange multiplier.

Hamilton-Jacobi-Bellman Equation

Simultaneous resolution of these first-order conditions yields the optimal solutions π^{opt}, c^{opt} and $\tilde{\lambda}^{opt}$.

Substituting these into (3) gives the PDE

$$-\delta J(\upsilon,t)+\frac{\left(c^{opt}(\upsilon,t)\right)^{1-\gamma}}{1-\gamma}+J_t(\upsilon,t)+J_\upsilon(\upsilon,t)\left(\upsilon\left[\left(\pi^{opt}(\upsilon,t)\right)^T(\mu-r)+r\right]-c^{opt}(\upsilon,t)\right)$$
$$+\frac{1}{2}J_{\upsilon\upsilon}(\upsilon,t)\upsilon^2\left(\pi^{opt}(\upsilon,t)\right)^T\sigma\sigma^T\left(\pi^{opt}(\upsilon,t)\right)=0, \qquad (5)$$

with terminal condition

$$J(\upsilon,T)=e^{-\delta T}\frac{\upsilon^{1-\gamma}}{1-\gamma},$$

which can then be solved for the optimal value function $J^{opt}(\upsilon,T)$.

Hamilton-Jacobi-Bellman Equation

Because of the nonlinearity in π^{opt} and c^{opt},
the first-order conditions together with the HJB equation are a nonlinear system.

So the PDE equation (5) has no analytic solution and numerical methods such as *Newton's method* or *Sequential Quadratic Programming* (SQP) are required to solve for
$\pi^{opt}(\upsilon,t)$, $c^{opt}(\upsilon,t)$, $\tilde{\lambda}^{opt}(\upsilon,t)$ and $J^{opt}(\upsilon,t)$
iteratively.

D. Akume and G.-W. Weber, 2010

Bubbles, Jumps and Insiders

Notation:

$$(\Omega, F, P), \quad (\Omega, P) = (\Omega_B \times \Omega_\eta, P_B \times P_\eta),$$

$$\eta(t) = \int_0^t \int_{\mathbf{R}} z\, \tilde{N}(dt, dz) \geq 0, \quad \tilde{N}(dt, dz) = (N - v_F)(dt, dz) = N(dt, dz) - v_F(dz)dt,$$

$$F_t^B := \sigma\{B(s) \mid s < t, t \in [0,T]\} \vee N, \quad F_t^{\tilde{N}} := \sigma\{\tilde{N}(\Delta) \mid \Delta \in B(\mathbf{R}_0 \times (0,t)),\ t \in [0,T]\} \vee N,$$

$$F_t = F_t^B \otimes F_t^{\tilde{N}},$$

$$F_t \subseteq G_t \subseteq F \qquad \forall t \in [0,T],$$

B. Oksendal

D. Akume and G.-W. Weber, 2010

Bubbles, Jumps and Insiders

$$(\Omega, F, P), \quad (\Omega, P) = (\Omega_B \times \Omega_\eta, P_B \times P_\eta),$$

$$\eta(t) = \int_0^t \int_{\mathbf{R}} z\, \tilde{N}(dt, dz) \geq 0, \quad \tilde{N}(dt, dz) = (N - v_F)(dt, dz) = N(dt, dz) - v_F(dz)dt,$$

pure jump Lévy process compensated Poisson random measure Lévy measure

$$F_t^B := \sigma\{B(s) \mid s < t, t \in [0,T]\} \vee N, \quad F_t^{\tilde{N}} := \sigma\{\tilde{N}(\Delta) \mid \Delta \in B(\mathbf{R}_0 \times (0,t)),\ t \in [0,T]\} \vee N,$$

$$F_t = F_t^B \otimes F_t^{\tilde{N}},$$

$$F_t \subseteq G_t \subseteq F \qquad \forall t \in [0,T],$$

Bubble

Bubbles, Jumps and Insiders

$$\int_0^\infty \varphi(t,\omega)d^-B(t) = \lim_{\varepsilon\to 0}\int_0^\infty \varphi(t,\omega)\frac{B(t+\varepsilon)-B(t)}{\varepsilon}dt$$

Forward Integrals

$$\int_0^\infty \int_{\mathbf{R}_0} \varphi(t,\omega)\tilde{N}(d^-t,dz) = \lim_{n\to\infty}\int_0^\infty \int_{U_n} \varphi(t,z)\tilde{N}(dt,dz)$$

$$(U_n \subset \mathbf{R}_0 \text{ compact},\ U_n \subseteq U_{n+1}\ (n\in \mathrm{IN}),\ \nu_F\left(U_n\right)<\infty,\ \bigcup_{n=1}^\infty U_n = \mathbf{R}_0)$$

$$X(t) = X(0) + \int_0^t \alpha(s)ds + \int_0^t \beta(s)d^-B(s) + \int_0^\infty \int_{\mathbf{R}_0} \gamma(s,z)\tilde{N}(d^-s,dz) \quad \forall t\in[0,T]$$

$$d^-X(t) = \alpha(t)dt + \beta(t)d^-B(t) + \int_{\mathbf{R}_0} \gamma(t,z)\tilde{N}(d^-t,dz)$$

Forward Process

If $Y(t) = f(X(t))$:

Itô Formula for Forward Integrals

$$d^{-}Y(t) = \Big[f'(X(t))\alpha(t) + \frac{1}{2} f''(X(t))\alpha(t)\beta(t)^2 + \int_{\mathbf{R}_0} \Big\{ f(X(t^-) + \gamma(t,z))$$

$$- f(X(t^-)) - f'(X(t^-))\gamma(t,z) \Big\} \nu_F(dz) \Big] dt + f'(X(t))\beta(t) d^{-}B(t)$$

$$+ \int_{\mathbf{R}_0} \Big(f(X(t^-) + \gamma(t,z)) - f'(X(t^-)) \Big) \tilde{N}(d^{-}t, dz).$$

Bubbles, Jumps and Insiders

$$dS_0(t) = r(t)S_0(t)dt,$$
$$S_0(0) = 1,$$

$$dS_1(t) = S_1(t^-)\left[\mu(t)dt + \sigma(t)dB(t) + \int_{\mathbf{R}_0} \gamma(t,z)\tilde{N}(dt,dz)\right],$$
$$S_1(0) > 0,$$

$$d^-X^{(c,\pi)}(t) = X^{(c,\pi)}(t^-)\Big[\{r(t) + (\mu(t) - r(t))\pi(t)\}dt + \sigma(t)\pi(t)d^-B(t)$$
$$+\pi(t)\int_{\mathbf{R}_0} \gamma(t,z)\tilde{N}(d^-t,dz)\Big] - c(t)dt,$$
$$X^{(c,\pi)}(0) = \upsilon.$$

Bubbles, Jumps and Insiders

If $\lambda(t) = \dfrac{c(t)}{X^{(c,\pi)}(t)}$, then

$$d^- X^{(c,\pi)}(t) = X^{(c,\pi)}(t^-)\Big[\big\{r(t) - \lambda(t) + (\mu(t) - r(t))\pi(t)\big\}\,dt$$

$$+\sigma(t)\pi(t)d^- B(t) + \pi(t)\int_{\mathbf{R}_0} \gamma(t,z)\tilde{N}(d^- t, dz)\Big],$$

$$J(\lambda^*, \pi^*) := \sup_{(\lambda,\pi)\in A} \mathrm{IE}\left[\int_0^T e^{-\delta(t)} \ln\Big(\lambda(t) X^{(c,\pi)}(t)\Big)dt + Ke^{-\delta(T)} \ln X^{(c,\pi)}(T)\right].$$

Bubbles, Jumps and Insiders

Brownian Motion Case

$\gamma(t,z)=0, \quad \sigma(t)\neq 0 \quad$ for almost all (t,z)

π_i^* : optimal portfolios of the insider (generator of a bubble),

π_h^* : optimal portfolios of the uninformed (honest) agent

$$\pi_i^*(t)=\frac{\mu(t)-r(t)}{\sigma(t)^2}+\frac{\alpha(t)}{\sigma(t)} \qquad \pi_h^*(t)=\frac{\mu(t)-r(t)}{\sigma(t)^2}$$

$$\lambda_i^*(t)=\lambda_h^*(t)=\frac{e^{-\delta(t)}}{\int_t^T e^{-\delta(s)}ds+Ke^{-\delta(T)}},$$

$$J_i(c_{\lambda_i^*},\pi_i^*)=J_h(c_{\lambda_h^*},\pi_h^*)+\frac{1}{2}\mathrm{IE}\left[\int_0^T e^{-\delta(t)}\int_0^t \sigma(s)^2\,ds\,dt\right]+\frac{1}{2}K\,\mathrm{IE}\left[e^{-\delta(T)}\int_0^T \sigma(s)^2\,ds\right]$$

Bubbles, Jumps and Insiders

Mixed Case

$\gamma(t,z) = z, \quad \sigma(t) \neq 0 \quad$ for almost all (t,z)

$G'_t = F_t \vee \sigma(B(T_0), \eta(T_0)), \quad T_0 > T$

Assumptions : (1) Uninformed agent has access to filtration $F_t \subseteq G_t \subseteq G'_t \ (t \in [0,T])$,

(2) Lйvy measure ν_F is given by $\nu_F(ds,dz) = \rho\delta_1(dz)ds \quad (\delta_1(dz)$: unit point mass at 1),

(3) $\eta(t)$ is given by $\eta(t) = Q(t) - \rho t \quad (Q$: Poisson process of intensity $\rho)$.

$$\pi_i^*(t) = \pi_h^*(t) + \frac{\varsigma(t)}{\sigma(t)} \qquad \pi_h^*(t) = \frac{\mu(t) - r(t)}{\sigma(t)^2} - \frac{\rho}{\sigma(t)^2} \qquad \lambda_i^*(t) = \lambda_h^*(t) = \frac{e^{-\delta(t)}}{\int_t^T e^{-\delta(s)}ds + Ke^{-\delta(T)}}$$

$$\varsigma(t) = \frac{1}{2\sigma(t)}\Big[-\mu(t) + r(t) + \rho + \sigma(t)\alpha(t) - \sigma(t)^2$$

$$+\sqrt{\left(\mu(t) - r(t) - \rho + \sigma(t)\alpha(t) + \sigma(t)^2\right) + 4\sigma(t)^2\theta(t)},$$

$$\alpha(t) = \frac{\mathrm{IE}\left[B(T_0) - B(t) \mid G_t\right]_-}{T_0 - t} \qquad \theta(t) = \frac{\mathrm{IE}\left[Q(T_0) - Q(t) \mid G_t\right]_-}{T_0 - t}$$

Bubbles, Jumps and Insiders

Mixed Case

$$J_i(c_{\lambda_i^*},\pi_i^*) = J_h(c_{\lambda_h^*},\pi_h^*) + \mathrm{IE}\left[\int_0^T e^{-\delta(t)}\int_0^t\left(-\frac{1}{2}\varsigma(s)^2+\varsigma(s)\alpha(s)\right)dsdt\right]$$

$$+\mathrm{IE}\left[\int_0^T e^{-\delta(t)}\int_0^t\int_{\mathbf{R_0}}\mathbf{l}\ln\left(1+\frac{z\varsigma(s)\sigma(s)}{\sigma(s)^2+\left(\mu(s)-r(s)-\rho\right)z}\right)\nu_{\mathrm{G}}(ds,dz)dt\mathbf{l}\right]$$

$$+K\,\mathrm{IE}\left[e^{-\delta(T)}\int_0^T\left(-\frac{1}{2}\varsigma(s)^2+\varsigma(s)\alpha(s)\right)ds\right]$$

$$+K\,\mathrm{IE}\left[e^{-\delta(T)}\int_0^T\int_{\mathbf{R_0}}\mathbf{l}\ln\left(1+\frac{z\varsigma(s)\sigma(s)}{\sigma(s)^2+\left(\mu(s)-r(s)-\rho\right)z}\right)\nu_{\mathrm{G}}(ds,dz)\mathbf{l}\right]$$

D. David and Y. Yolcu Okur, 2009

Stochastic Hybrid Model II

$$dX(t) = b^n(X(t), \theta(t))dt + \sigma^n(X(t), \theta(t))dW^n(t),$$

$$P(\theta(t+\delta t) = j \mid \theta(t) = i, X(s), \theta(s), s \le t) = \lambda_{ij}^n(X(t))\delta t + \mathrm{O}(\delta t) \quad \forall i \neq j,$$

$$X(0) = X_0, \ \theta(0) = \theta_0$$

Stochastic Hybrid Model II

$$F_t^n = \sigma\{W^n(s), p(A,B) \mid \ s \le t, \ A \in B\ ([0,t]), B \in B\ (\mathbf{R})\},$$

$$\tau_n = \inf\{t \ge 0 \mid \ X(t) \in A_n\},$$

$$F_t = \bigvee_{n \in \mathrm{IN}} F_t^n,$$

$$0 = \tau_0 = \tau_1 < \tau_2 < \ldots < \tau_m < \ldots, \qquad \tau_m \to \infty \ \ (m \to \infty),$$

$$\eta(t) = n, \quad \text{if} \ \ (X(t), \theta(t)) \in S_n \times \Theta_n$$

Stochastic Hybrid Model II

$$dX(t) = \left(b(X(t),\theta(t),\eta(t)) + \sum_{m=0}^{\infty} \left(\tilde{g}_1(X(\tau_m^-),\theta(\tau_m^-),\eta(\tau_m^-)) - X(\tau_m^-) \right) \delta(t-\tau_m) \right) dt$$
$$+ \sigma(X(t),\theta(t),\eta(t)) dW^{\eta(t)}(t),$$

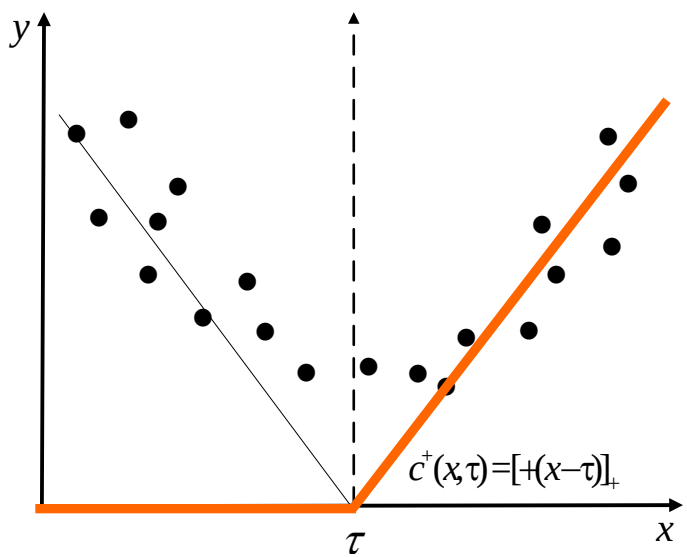
$$d\theta(t) = \int_{\mathbf{R}} h(X(t^-),\theta(t^-),\eta(t^-),u)\, p(dt,du)$$
$$+ \sum_{m=0}^{\infty} \left(\tilde{g}_2(X(\tau_m^-),\theta(\tau_m^-),\eta(\tau_m^-)) - \theta(\tau_m^-) \right) \delta(t-\tau_m) dt,$$

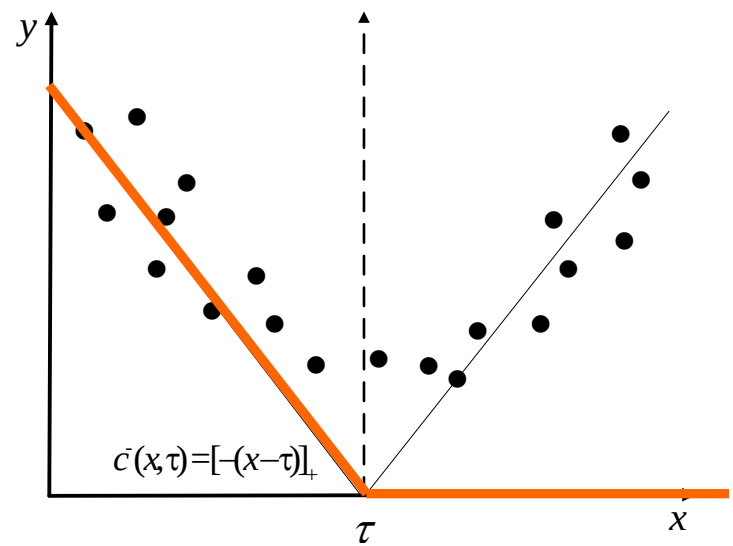
$$d\eta(t) = \sum_{m=0}^{\infty} \left(\tilde{h}(X(\tau_m^-),\theta(\tau_m^-),\eta(\tau_m^-)) - \eta(\tau_m^-) \right) I_{\{\tau_m \le t\}}$$

CMARS Method

- CMARS uses expansions in piecewise linear basis functions of the form

$$c^{+}(x,\tau) = [+(x-\tau)]_{+}\,, \qquad c^{-}(x,\tau) = [-(x-\tau)]_{+}\,.$$





Set of basis functions:

$$\wp := \left\{ (X_j - \tau)_+, (\tau - X_j)_+ \mid \tau \in \left\{ \tilde{x}_{1,j}, \tilde{x}_{2,j}, \ldots, \tilde{x}_{N,j} \right\},\ j \in \{1, 2, \ldots, p\} \right\}$$

SDEs with Fractional Brownian Motion (fBm)

$$(SDE) \qquad \dot{\boldsymbol{X}}(t) = \boldsymbol{a}(\boldsymbol{X},t) + \boldsymbol{b}(\boldsymbol{X},t)\boldsymbol{\delta}(t) \quad (t \in [0,\infty)),$$

- $\boldsymbol{a}$: deterministic part, $\boldsymbol{b}\boldsymbol{\delta}(t)$: stochastic part, $\boldsymbol{\delta}(t)$: generalized stochastic process.
- The system of stochastic differential equation is generated by fractional Brownian motion

$$d\boldsymbol{X} = \boldsymbol{a}(\boldsymbol{X},t)dt + \boldsymbol{b}(\boldsymbol{X},t)d\boldsymbol{W}^H(t),$$

- $\boldsymbol{X}(t)$: solution,
- $\boldsymbol{W}^H(t)$: **fBm**(s) with Hurst parameter H.

fBm

- Let H be between (0,1). FBm $\left(W^H(t)\right)(t \geq 0)$ with Hurst parameter H is a continuous and centered Gaussion process with covariance function

$$\mathrm{Cov}(W^H(t), W^H(s)) = \frac{1}{2}(s^{2H} + t^{2H} - |t-s|^{2H}).$$

- *s*, *t* : time.

- The Hurst parameter H of fBm explains the dependence of data:
  - Observations with $H > \frac{1}{2}$ display **long**-range dependence ("*positively correlated*").
  - Observations with $H < \frac{1}{2}$ display **short**-range dependence ("*negatively correlated*").
  - For $H = \frac{1}{2}$, $W^H(t)$ is a standard Brownian motion.

Estimation of Hurst Parameter Using CMARS

- ***penalized residual sum of squares PRRS***

$$PRSS := \sum_{i=1}^{N}\left(\dot{\bar{X}}_i - \left(\bar{G}_i + \bar{F}_i c_i\right)\right)^2 + \sum_{l=1}^{d^B}\lambda_l \sum_{\substack{|\boldsymbol{\alpha}|=1 \\ \boldsymbol{\alpha}=(\alpha_1,\alpha_2)^T}}^{2} \sum_{\substack{r<s \\ r,s\in V(l)}} \int_{Q_l^B} \alpha_l^2 \left[D_{r,s}^{\boldsymbol{\alpha}} B_l(\bar{\boldsymbol{U}}_B^l)\right]^2 d\bar{\boldsymbol{U}}_B^l$$

$$+ \sum_{m=1}^{d^C}\mu_m \sum_{\substack{|\boldsymbol{\alpha}|=1 \\ \boldsymbol{\alpha}=(\alpha_1,\alpha_2)^T}}^{2} \sum_{\substack{r<s \\ r,s\in V(m)}} \int_{Q_m^C} \beta_m^2 \left[D_{r,s}^{\boldsymbol{\alpha}} C_m(\bar{\boldsymbol{U}}_C^m)\right]^2 d\bar{\boldsymbol{U}}_C^m,$$

- $\lambda_l, \mu_m \geq 0$ (*smoothing* parameters, *tradeoff*)
- *large* values of λ_l, μ_m yield smoother curves and *smaller* ones result in more fluctuation.

$$\sum_{i=1}^{N}\left(\dot{\bar{X}}_i - \left(\bar{G}_i + \bar{F}_i c_i\right)\right)^2 =$$

$$\sum_{i=1}^{N}\left(\dot{\bar{X}}_i - \left(\alpha_0 + \sum_{l=1}^{d^B}\alpha_l B_l(\bar{\boldsymbol{U}}_{i,B}^l) + \beta_0 + \sum_{m=1}^{d^C}\beta_m C_m(\bar{\boldsymbol{U}}_{i,C}^m)\right)\right)^2.$$

Estimation of Hurst Parameter Using CMARS

- We reformulate our Tikhonov regularization as a CQP problem:

$$\min_{t,\boldsymbol{\theta}} \quad t,$$

$$\text{subject to} \quad \left\| \dot{\bar{\boldsymbol{X}}} - \bar{\boldsymbol{A}}\boldsymbol{\theta} \right\|_2 \le t,$$

$$\left\| \bar{\boldsymbol{L}}\boldsymbol{\theta} \right\|_2 \le \sqrt{M}.$$

- To write the optimality condition for this problem, we reformulate our problem as

$$\min_{t,\boldsymbol{\theta}} \quad t,$$

$$\text{such that} \quad \chi := \begin{pmatrix} \mathbf{0}_N & \bar{\boldsymbol{A}} \\ 1 & \mathbf{0}_{M_{\max}+1}^T \end{pmatrix} \begin{pmatrix} t \\ \boldsymbol{\theta} \end{pmatrix} + \begin{pmatrix} -\dot{\bar{\boldsymbol{X}}} \\ 0 \end{pmatrix},$$

$$\eta := \begin{pmatrix} \mathbf{0}_{M_{\max}+1} & \bar{\boldsymbol{L}} \\ 0 & \mathbf{0}_{M_{\max}+1}^T \end{pmatrix} \begin{pmatrix} t \\ \boldsymbol{\theta} \end{pmatrix} + \begin{pmatrix} \mathbf{0}_{M_{\max}+1} \\ \sqrt{M} \end{pmatrix},$$

$$\chi \in L^{N+1}, \ \eta \in L^{M_{\max}+2},$$

Applications

Examples: CMARS performances for fBm generated by H=0.2, H=0.3, H=0.7 and H=0.8.

Hurst index	Performance Measures					
	MAE	MSE	r	R^2	Adj-R^2	PWI
$H = 0.1$	0,8766	1,4827	0,1923	0,0370	-0,1651	1
$\boldsymbol{H = 0.2}$	0,6207*	0,7480*	0,9868*	0,9739*	0,9684*	1
$H = 0.3$	0,8861	1,5266	0,0991	0,0098	-0,1979	1
$H = 0.4$	0,8770	1,4733	0,2075	0,0430	-0,1577	1
$H = 0.5$	0,8839	1,5201	0,1281	0,0164	-0,1900	1
$H = 0.1$	0,7138	0,9776	0,2607	0,0679	-0,1276	1
$H = 0.2$	0,7162	0,9901	0,2400	0,0576	-0,1401	1
$\boldsymbol{H = 0.3}$	0,3606*	0,2516*	0,984*	0,9699*	0,9636*	1
$H = 0.4$	0,6926	0,9387	0,3249	0,1055	-0,0821	1
$H = 0.5$	0,7053	0,9763	0,2641	0,0697	-0,1254	1
$H = 0.5$	0,7031	0,9719	0,4743	0,2250	0,0623	1
$H = 0.6$	0,7048	0,9784	0,4688	0,2198	0,0560	0,9898
$\boldsymbol{H = 0.7}$	0,3634*	0,2602*	0,9582*	0,9182*	0,9010*	1
$H = 0.8$	0,7041	0,9506	0,4919	0,2419	0,0828	1
$H = 0.9$	0,7081	0,9781	0,4691	0,2200	0,0563	1
$H = 0.5$	0,6068	0,7841	0,5914	0,3498	0,2133	1
$H = 0.6$	0,6359	0,8015	0,5783	0,3345	0,1948	1
$H = 0.7$	0,6053	0,7389	0,6176	0,3815	0,2517	1
$\boldsymbol{H = 0.8}$	0,2009*	0,0822*	0,9883*	0,9768*	0,9720*	1
$H = 0.9$	0,6006	0,7294	0,6240	0,3894	0,2613	1

* indicates better performance

Conclusion and Outlook

A main **superiority** of our approach others:

- it not only estimates the Hurst parameter but also finds spline parameters of the stochastic process,
- our representation of financial processes is empowered by all the modeling and numerical advantages of CMARS.

As a **future research**, we intend

- to give a further integrated (model-based) identification of all parameters,
- to extend this approach to hybridicity and jumps,
- to do our study for the multi-dimensional systems,
- to further contribute to a deeper understanding of our modern financial markets, and in science and engineering, and to offer helpful mathematical decision tools there.

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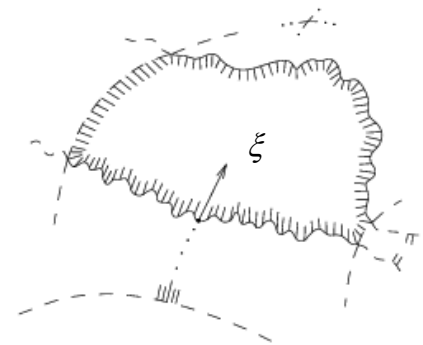
Generalized Semi-Infinite Programming

Thm. (W. 1999/2003, 2006):

Let Assumptions A and B hold f[illegible]lized semi-infinite optimization problem $\mathcal{P}_{\mathcal{GSI}}(f,h,g,u,v)$ (with [illegible]unctions).

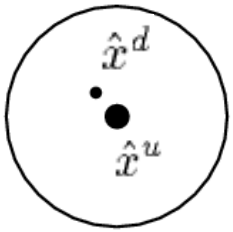
Fulfilled!

Then, $\mathcal{P}_{\mathcal{GSI}}(f,h,g,u,v)$ is *structurally stable* $\Longleftrightarrow$



$\mathbf{C}_{\mathcal{GSI}\,1}$. *EMFCQ* holds for $M_{\mathcal{GSI}}[h,g]$.

$\mathbf{C}_{\mathcal{GSI}\,2}$. All the $\mathcal{G}$-$\mathcal{O}$ Kuhn-Tucker points $\hat{x}^u$ of $\mathcal{P}_{\mathcal{GSI}}(f,h,g,u,v)$ are *strongly stable.*



$\mathbf{C}_{\mathcal{GSI}\,3}$. For each two different $\mathcal{G}$-$\mathcal{O}$ Kuhn-Tucker points $\overline{x}^1 \neq \overline{x}^2$ of $\mathcal{P}_{\mathcal{GSI}}(f,h,g,u,v)$ the corresponding critical values are different (*separate*), too: $f(\overline{x}^1) \neq f(\overline{x}^2)$. ■

Stochastic Control of SHS with Jumps

We define the SHSJ as follows:

$$(X, \Theta, U, \Omega, A, a, b, f, \delta, R, (x_0, \theta_0)), \qquad (1)$$

where

- $X \subseteq \mathbb{R}^d$ is the continuous state space,
- $\Theta = \{1, 2, \ldots, M\}$ is finite set of discrete states,
- $U = \{U_\theta\}_{\theta \in \Theta}$, where $U_\theta \subseteq \mathbb{R}^{m_\theta}$, is a finite family of sets of continuous controls,
- the partition of X is denoted by $\Omega = \{\Omega_\theta\}_{\theta \in \Theta}$, where $\Omega_\theta \subseteq \mathbb{R}^d$,
- $A = \{A_\theta\}_{\theta \in \Theta}$, where $A_\theta \subseteq \partial\Omega_\theta$, is the collection of switchings,

Stochastic Control of SHS with Jumps

- $a : \mathbb{R}^d \times \Theta \times U_\theta \to \mathbb{R}^d$ is a controlled drift term,
- $b : \mathbb{R}^d \times \Theta \times U_\theta \to \mathbb{R}^d \times \mathbb{R}^d$ is a controlled diffusion term,
- $f : \mathbb{R}^d \times \Theta \times U_\theta \times \mathbb{R} \to \mathbb{R}$ is a controlled jump function,
- $\delta : \Theta \times A \to \Theta$ is an autonomous switching map,
- $R : \Theta \times A \to P(X)$ is a reset map which assigns a reset probability kernel to each $x(\cdot) \in A_\theta$ and θ on X, where $P(X)$ is the family of probability measures on X,
- $(x_0; \theta_0)$ is the pair of initial values on $X \times \Theta$.

Stochastic Control of SHS with Jumps

Transition times $\{t_0 = 0, t_1, t_2, \ldots\}$ are defined as

$$t_{i+1} := \inf\{t > t_i : x_t \notin \Omega^0_{\theta_i}\}.$$

The state of the SHSJ at t_i is given by

$$(x_i; \theta_i) = (x(t_i); \theta(t_i)),$$

where $x_i \in \Omega^0_{\theta_i}$.

Stochastic Control of SHS with Jumps

While $x(t)$ stays in $\Omega^0_{\theta_i}$, it evolves according to the SDE

$$dx_t = a(x(t), \theta(t), u(t))dt + b(x(t), \theta(t), u(t))dW_t + \int_0^t \int_\Gamma f(x(t), \theta(t), z, u(t))N(ds, dz), \quad (2)$$

where $W(\cdot)$ is a standard Wiener process and $N(\cdot, \cdot)$ is a Poisson random measure on $\mathbb{R}^+ \times \mathbb{R}$.

Stochastic Control of SHS with Jumps

When the switching occurs

- the new discrete state is $\theta_{i+1} = \delta(\theta_i, x(t_{i+1}^-))$,
- the new continuous state $x_{t_{i+1}}$ is determined according to $R(\theta_i, x_{t_{i+1}^-})(\Xi)$,
  where $\Xi \in \Omega_{\theta_{i+1}}$ is a measurable set.

$\Rightarrow$ $x(t)$ evolves according to SDE (2) with $\theta(t) = \theta_{i+1}$ and initial condition $x(t_{i+1})$ until the next switching time.

Stochastic Control of SHS with Jumps

- $N(\cdot,\cdot)$ has intensity λ and jump size distribution $F(z)$, with compact support Γ.
- ν_n is the nth jump time with $\nu_{n+1} - \nu_n \sim exp(\lambda)$.

Making necessary assumptions, we can write the jump term as

$$J(t) = \sum_{\nu_n \leq t} f(x(\nu_n^-), \theta(\nu_n^-), z_n(\nu_n^-), u).$$

Stochastic Control of SHS with Jumps

- The functions a, b and f satisfy Lipschitz and Growth conditions, uniformly θ and u.
  $\Rightarrow$ (2) has a unique strong solution, $\forall\theta \in \Theta$, Skorohod 1972.
- Every point $x \in A_\theta$ is a *regular* point.
- The continuous control $u(t)$ is a measurable process taking values in compact set U.

Stochastic Control of SHS with Jumps

Problem: Minimizing a cost until a target set $G \subset \mathbb{R}^d$ is reached.

We assume that

- G is compact with smooth boundary,
- ∂G satisfies the regularity conditions,
- $x_0 \notin G$ and $G \subset \Omega_\theta$ for some $\theta \in \Theta$.

We define the stopping time,

$$\tau := \inf\{t : x(t) \in \partial G\}.$$

Stochastic Control of SHS with Jumps

Given an initial state (x_0, θ_0) and a discount factor $\beta \geq 0$ the cost is

$$H(x_0, \theta_0, u) := E\left(\int_0^{\tau} e^{-\beta s} k(x(s), \theta(s), u(s)) ds + e^{-\beta \tau} g(x(\tau))\right) \quad (3)$$

with respect to the admissable controls $u(t)$.
The value function is defined as

$$V(x_0, \theta_0) := inf_u H(x_0, \theta_0, u) \quad (4)$$

for $x_0 \in \Omega_{\theta_0}$.

Stochastic Control of SHS with Jumps

Theorem

Given a SHSJ with the cost (3), an optimal admissable control policy $u(x)$ must satisfy the conditions

$$inf_{u\in U}[L^u V(x,\theta) + k(x,\theta,u)] = 0, x \in G^0,$$

$$V(x,\theta) = g(x), \forall x \in \partial G, \theta \in \Theta, G \subset \Omega_\theta,$$

$$V(x',\theta') \leq V(x,\theta), \forall \theta \in \Theta, x \in A_\theta, \theta' = \delta(\theta,x), x' = R(\theta,x)(\Xi).$$

$L^\alpha h(x,\theta) := h_x(x,\theta)a(x,\theta,\alpha) + \frac{1}{2}tr[h_{xx}(x,\theta)c(x,\theta,\alpha)] + \lambda \int (h(x + f(x,\theta,z,\alpha)) - h(x,\theta))F(dz)$ with $c(x,\theta,\alpha) = b(x,\theta,\alpha)b'(x,\theta,\alpha)$ for fixed $\alpha \in U$.

Stochastic Control of SHS with Jumps

Theorem

Suppose that $V(x,\theta)$ is defined so that it is continuous from the right, twice differentiable and bounded in Ω_θ^0 and a feedback control $\overline{u}(x)$ such that the conditions of the above theorem hold and $H(x,\theta,\overline{u})$ is bounded. Then, $V(x,\theta)$ is the optimal cost and $\overline{u}(x)$ is the optimal control.