Aggregate Investment and Stock Returns

Fernando Duarte[1] Leonid Kogan[2] Dmitry Livdan[3]

[1]NY Fed

[2]MIT

[3]University of California Berkeley

November 2013

Motivation

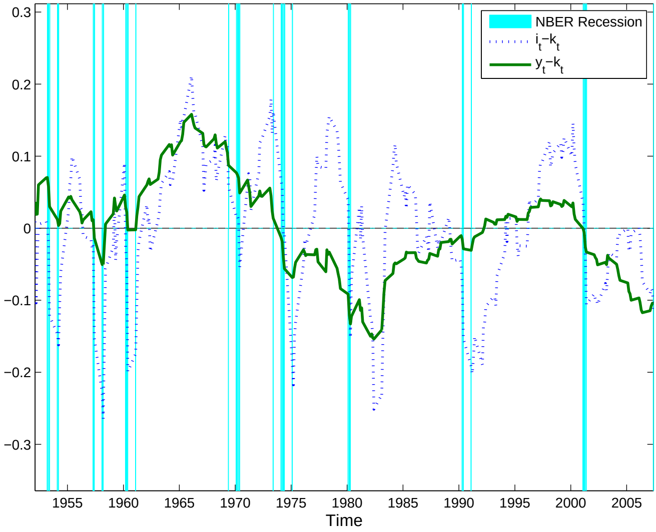
- Business cycle variables are important for predicting aggregate returns
  - Aggregate investment rate is negatively correlated with subsequent excess stock market returns (Cochrane 1991)
- Stock market volatility is important for business cycle
  - Firms scale down investment in response to high transient volatility shock (Bloom 2009)
- Our question: Can we learn something about stock market volatility from real economy?

Data
Description

- Sample from 1947Q1 to 2009Q3 for a total of 251 quarters
- Market variables:
  - CRSP daily value-weighted portfolio returns from Kenneth French's website
  - Quarterly returns, r_t, are constructed as log cumulative return
  - Quarterly volatility, vol_t, is log of the standard deviation of daily returns within quarter t
- Business cycle variables:
  - Quarterly data from U.S. National Income and Product Accounts
  - GDP, Y_t, from Table 1.1.6, item 1
  - Gross Private Domestic Investment, I_t, from Table 1.1.6, item 7
  - K_t is interpolated from annual values using the quarterly I_t and Private Fixed Assets from the FAT (Table 1.1, item 3)
  - Work with $i_t - k_t = log\frac{I_t}{K_t}$ and $y_t - k_t = log\frac{Y_t}{K_t}$

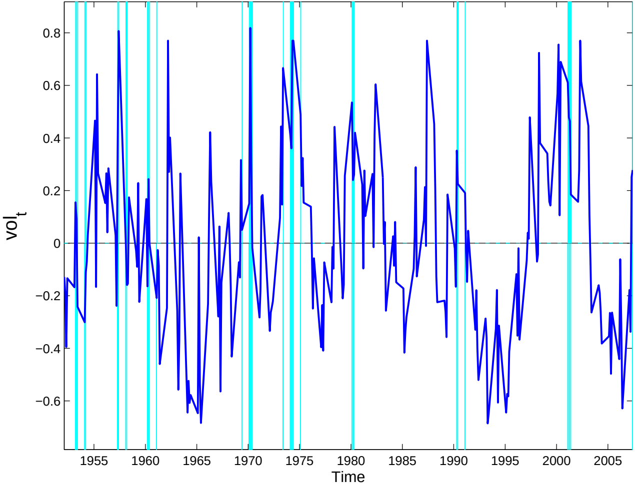
Data

$i_t - k_t$ and $y_t - k_t$



Data

vol_t


0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
vol_t
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
Time

Return predictability

Variables	Horizon h (in quarters) 1	2	3	4	5	6	7	8	9	10	11	12
$r_{t+h} - r_{f,t+h} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$												
a_1	−0.153	−0.145	−0.113	−0.058	−0.045	−0.031	−0.053	−0.096	−0.105	−0.091	−0.089	−0.093
t-stat	(−3.002)	(−2.651)	(−1.761)	(−0.952)	(−0.714)	(−0.525)	(−0.949)	(−1.777)	(−2.106)	(−1.991)	(−1.972)	(−2.413)
R^2	[0.033]	[0.029]	[0.017]	[0.005]	[0.003]	[0.001]	[0.004]	[0.013]	[0.015]	[0.011]	[0.011]	[0.012]
$r_{t+h} - r_{f,t+h} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h}$												
a_1	−0.180	−0.167	−0.132	−0.062	−0.044	−0.034	−0.058	−0.104	−0.106	−0.088	−0.074	−0.080
t-stat	(−3.101)	(−2.627)	(−1.913)	(−0.919)	(−0.626)	(−0.499)	(−0.898)	(−1.639)	(−1.775)	(−1.607)	(−1.390)	(−1.751)
a_2	0.073	0.060	0.052	0.011	−0.002	0.008	0.013	0.024	0.002	−0.008	−0.043	−0.035
t-stat	(0.839)	(0.655)	(0.586)	(0.124)	(−0.031)	(0.116)	(0.199)	(0.404)	(0.033)	(−0.155)	(−0.838)	(−0.696)
R^2	[0.031]	[0.026]	[0.014]	[0.000]	[−0.002]	[−0.003]	[−0.001]	[0.008]	[0.011]	[0.007]	[0.007]	[0.008]

Volatility predictability

Variables	Horizon h (in quarters) 1	2	3	4	5	6	7	8	9	10	11	12
	Univariate regression: $vol_{t+h} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$											
a_1	0.234	0.430	0.594	0.725	0.987	1.081	1.268	1.418	1.548	1.471	1.407	1.293
t-stat	(0.877)	(1.560)	(1.764)	(1.952)	(2.397)	(2.383)	(2.755)	(3.055)	(3.438)	(3.349)	(3.250)	(3.114)
R^2	[0.004]	[0.013]	[0.025]	[0.036]	[0.067]	[0.081]	[0.111]	[0.139]	[0.167]	[0.150]	[0.138]	[0.116]
	Multivariate regression I: $vol_{t+h} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h+1}$											
a_1	0.648	0.853	1.026	1.133	1.395	1.462	1.646	1.780	1.875	1.757	1.624	1.485
t-stat	(2.284)	(2.794)	(2.620)	(2.545)	(2.824)	(2.838)	(3.286)	(3.725)	(4.030)	(3.935)	(3.494)	(3.249)
a_2	-1.127	-1.158	-1.195	-1.140	-1.153	-1.084	-1.076	-1.031	-0.931	-0.817	-0.623	-0.549
t-stat	(−2.337)	(−2.395)	(−2.043)	(−1.796)	(−1.721)	(−1.648)	(−1.710)	(−1.768)	(−1.554)	(−1.416)	(−1.032)	(−0.944)
R^2	[0.031]	[0.042]	[0.056]	[0.064]	[0.095]	[0.105]	[0.134]	[0.160]	[0.183]	[0.162]	[0.142]	[0.119]
	Multivariate regression II: $vol_{t+h} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + a_3 vol_{t-1} + \varepsilon_{t,t+h+1}$											
a_1	0.756	0.938	1.099	1.207	1.465	1.503	1.664	1.788	1.872	1.759	1.628	1.485
t-stat	(3.120)	(3.435)	(3.166)	(3.051)	(3.197)	(3.032)	(3.399)	(3.773)	(3.947)	(3.948)	(3.502)	(3.210)
a_2	-0.777	-0.848	-0.939	-0.854	-0.892	-0.927	-0.963	-0.954	-0.937	-0.787	-0.590	-0.625
t-stat	(−1.826)	(−1.861)	(−1.766)	(−1.498)	(−1.499)	(−1.457)	(−1.531)	(−1.608)	(−1.607)	(−1.400)	(−1.076)	(−1.111)
a_3	0.423	0.344	0.283	0.287	0.260	0.153	0.089	0.053	-0.008	0.020	0.025	-0.037
t-stat	(7.102)	(4.490)	(3.314)	(3.513)	(3.297)	(1.589)	(1.162)	(0.671)	(−0.110)	(0.321)	(0.397)	(−0.523)
R^2	[0.199]	[0.151]	[0.128]	[0.137]	[0.155]	[0.123]	[0.138]	[0.159]	[0.179]	[0.158]	[0.139]	[0.117]

Cash Flows or Discount Rates?

S&P 500 Earnings Growth Volatility

Variables	Horizon h (in quarters) 1	2	3	4	5	6	7	8	9	10	11	12
$\lvert e_{t+h} - e_{t+h-1} \rvert = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$												
a_1	-0.119	-0.094	-0.072	-0.049	-0.023	-0.015	-0.004	0.001	0.003	0.016	0.029	0.035
t-stat	(-4.588)	(-3.182)	(-1.910)	(-1.089)	(-0.467)	(-0.302)	(-0.079)	(0.015)	(0.065)	(0.318)	(0.526)	(0.571)
R^2	[0.094]	[0.058]	[0.034]	[0.016]	[0.004]	[0.001]	[0.000]	[0.000]	[0.000]	[0.002]	[0.006]	[0.008]
$\lvert e_{t+h} - e_{t+h-1} \rvert = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h}$												
a_1	-0.111	-0.075	-0.047	-0.017	0.013	0.019	0.031	0.036	0.037	0.053	0.069	0.076
t-stat	(-3.696)	(-2.162)	(-1.030)	(-0.320)	(0.227)	(0.339)	(0.562)	(0.666)	(0.680)	(0.974)	(1.150)	(1.133)
a_2	-0.018	-0.042	-0.058	-0.074	-0.085	-0.080	-0.081	-0.083	-0.081	-0.088	-0.093	-0.097
t-stat	(-0.552)	(-1.181)	(-1.390)	(-1.696)	(-1.804)	(-1.632)	(-1.519)	(-1.449)	(-1.327)	(-1.431)	(-1.460)	(-1.448)
R^2	[0.090]	[0.057]	[0.037]	[0.024]	[0.016]	[0.011]	[0.010]	[0.011]	[0.010]	[0.014]	[0.020]	[0.024]

Cash Flows or Discount Rates?

S&P 500 Dividend Growth Volatility

	Horizon h (in quarters)											
Variables	1	2	3	4	5	6	7	8	9	10	11	12
	$\vert d_{t+h} - d_{t+h-1}\vert = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$											
a_1	-0.007	-0.006	-0.002	0.003	0.005	0.006	0.008	0.007	0.007	0.011	0.014	0.016
t-stat	(-1.302)	(-0.991)	(-0.326)	(0.332)	(0.598)	(0.681)	(0.816)	(0.733)	(0.775)	(1.214)	(1.678)	(2.166)
R^2	[0.010]	[0.007]	[0.001]	[0.001]	[0.004]	[0.006]	[0.010]	[0.009]	[0.010]	[0.021]	[0.033]	[0.045]
	$\vert d_{t+h} - d_{t+h-1}\vert = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h}$											
a_1	-0.010	-0.010	-0.006	0.000	0.001	0.001	0.003	0.002	0.002	0.006	0.008	0.010
t-stat	(-1.430)	(-1.221)	(-0.596)	(0.009)	(0.138)	(0.125)	(0.278)	(0.166)	(0.158)	(0.559)	(0.854)	(1.216)
a_2	0.007	0.008	0.007	0.006	0.008	0.011	0.011	0.012	0.013	0.013	0.014	0.013
t-stat	(0.722)	(0.839)	(0.600)	(0.400)	(0.525)	(0.682)	(0.684)	(0.779)	(0.857)	(0.861)	(0.955)	(0.908)
R^2	[0.007]	[0.006]	$[-0.001]$	$[-0.002]$	[0.003]	[0.008]	[0.013]	[0.013]	[0.015]	[0.026]	[0.040]	[0.050]

Story

- Take classic Gordon formula:

$$P_0 = \frac{\mathrm{E}_0[D_1]}{r - g}$$

- Dividend growth is homoscedastic
- Consider a comparative-statics experiment:
  - Hold the expected future dividends fixed
  - $\downarrow r \Rightarrow P_0 \uparrow$
  - Impact on P_0 is lager when $r - g$ is smaller

Conjecture

If discount rates experience homoscedastic shocks, an exogenous decline in discount rates should give rise to higher future return volatility

Model

Competitive Representative Firm

- Cobb-Douglas production function

$$Y_t = e^{x_t} K_t^{\alpha} L_t^{1-\alpha}$$

- Productivity shock x_t follows

$$dx_t = \left(\mu - \frac{\sigma_X^2}{2}\right) dt + \sigma_X \, dW_t$$

- Capital stock K_t

$$dK_t = (i_t - \delta) K_t \, dt$$

- Adjustment costs

$$I_t = \frac{a}{\lambda} i_t^{\lambda} K_t, \;\; \lambda > 1$$

- Profitability

$$z_t \equiv y_t - k_t = x_t + (\alpha - 1) k_t$$

- Endowed with a constant flow of labor, normalized to one, and supplied inelastically
- Has time-separable iso-elastic utility w.r.t. consumption stream C_t

$$\mathrm{E}_0\left[\int_0^{\infty} e^{-\beta t+\xi_t}\frac{C_t^{1-\gamma}}{1-\gamma}dt\right]$$

- Preference shock ξ_t evolves according to

$$d\xi_t = (b_t - \frac{\sigma_t^2}{2})dt + \sigma_t dW_t,$$

$$d\sigma_t = -\theta\sigma_t dt + v dW_{\sigma t}, \quad W_{\sigma t} = \rho W_t + \sqrt{1-\rho^2}W_t'$$

- Stochastic $\sigma_t \Rightarrow$ state-dependent marginal utility w.r.t. C_t
- Isomorphic to a model of a household with the same isoelastic preferences but distorted beliefs

- Complete set of zero-net-supply state-contingent claims with the state-price density process $\pi > 0$

- Price of any long-lived asset with cash flow X is given by

$$\mathrm{E}_t \left[\int_t^{\infty} \frac{\pi_s}{\pi_t} X_s \right] dt$$

- Representative consumer is endowed with a single stock share $\Rightarrow$ all equity financing

- Aggregate dividend flow rate

$$D_t = Y_t - \frac{a}{\lambda} i_t^{\lambda} K_t - w_t L_t$$

Definition

Competitive equilibrium is π^*, w_t^*, L^*, C^*, Y^*, K^*, i^*, and D^*, such that

1. Y^*, K^*, L^*, and i^* satisfy the technological constraints.
2. C^* and L^* maximize $\max_{\{C_t, L_t\}} \mathrm{E}_0 \left[\int_0^\infty e^{-\beta t + \xi_t} \frac{C_t^{1-\gamma}}{1-\gamma} dt \right]$

$$\text{s.t. } \mathrm{E}_0 \left[\int_0^\infty \frac{\pi_t^*}{\pi_0^*} (C_t - D_t^* - w_t^* L_t^*)\, dt \right] = 0.$$

3. i^*, L^*, and D_t^* maximize $\max_{\{i_t, L_t, D_t\}} \mathrm{E}_0 \left[\int_0^\infty \frac{\pi_t^*}{\pi_0^*} D_t\, dt, \right]$.
4. Labor market clears $L_t^* = 1$ and consumption market clears $C_t^* = D_t^* + w_t L_t^*$.

- State prices $\pi_t^* = e_t^{-\beta t + \xi_t} \left(C_t^*\right)^{-\gamma}$
- Equilibrium wages $w_t^* = (1-\alpha) Y_t^*$
- Stock price

$$S_t = K_t \left(\frac{C_t^*}{K_t}\right)^{\gamma} \left[\alpha V(\sigma_t, z_t) - (1-\alpha)\Phi(\sigma_t, z_t)\right]$$

- $V(\sigma_t, z_t)$ and $\Phi(\sigma_t, z_t)$ satisfy

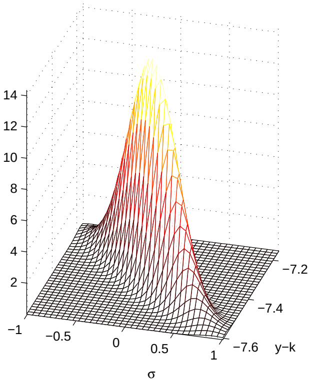
$$\left(e^z - \frac{a}{\lambda}\left(i^*\right)^{\lambda}\right)^{1-\gamma} + \mathcal{L}V = 0$$

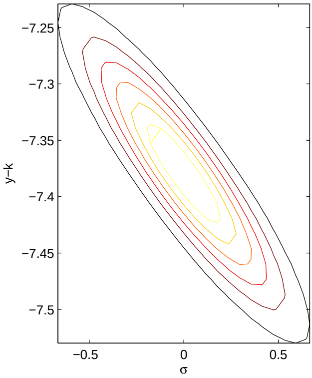
$$\frac{\frac{a}{\lambda}\left(i^*\right)^{\lambda}}{\left(e^z - \frac{a}{\lambda}\left(i^*\right)^{\lambda}\right)^{\gamma}} + \mathcal{L}\Phi = 0$$

- The optimal investment rate, i^*, satisfies

$$a\left(i^*\right)^{\lambda-1} = \left[V - \frac{1-\alpha}{1-\gamma} V_z\right] \left[e^z - \frac{a}{\lambda}\left(i^*\right)^{\lambda}\right]^{\gamma}$$

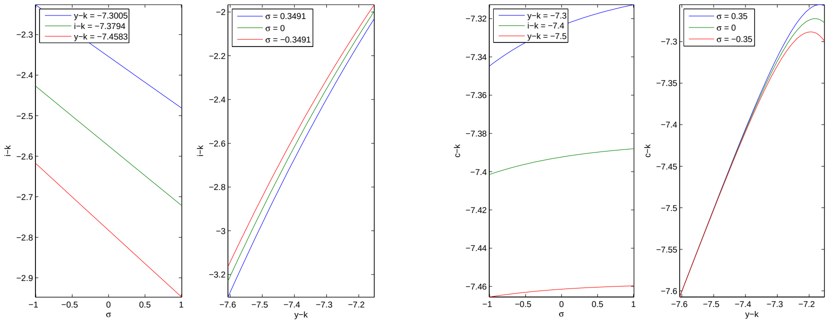
Steady-state distribution of σ and $y - k$





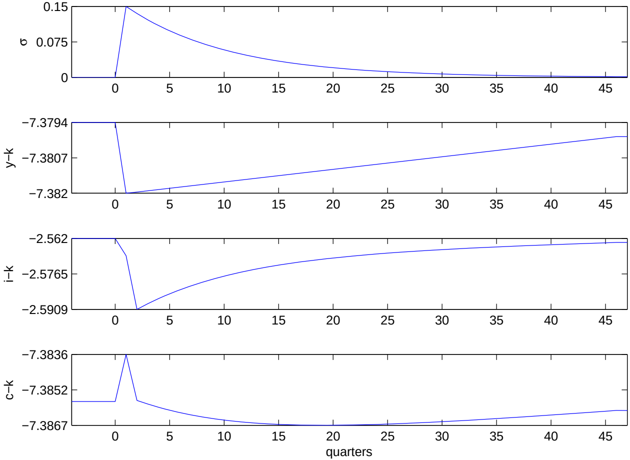
Model

Investment and Consumption



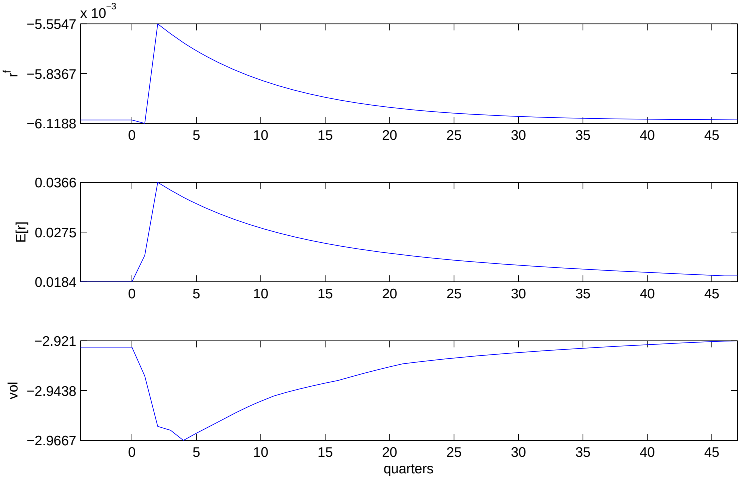
Model: IRFs to preference shock

Investment rate, profitability, and consumption



Model: IRFs to preference shock

RFR, returns, and volatility



Preferences:		
Risk Aversion	γ	10
Intertemporal Discount Parameter	β	0.02
Preference Shock ξ_t: Mean Reversion Rate	θ	0.4
Preference Shock ξ_t: Volatility	υ	0.3
Subjective Rate of Time Preferences: $b_t = B^0 + B^1\sigma_t^2$	B^0	0.10
	B^1	0.075
Technology:		
Capital Elasticity	α	0.33
Productivity Shock: Mean	μ	0.015
Productivity Shock: Variance	σ_X	0.03
Depreciation Rate	δ	0.05
Adjustment Costs Scale Parameter	a	10
Adjustment Costs Elasticity to Investment Rate	λ	5
Correlation Between dW_t and $dW_{\sigma t}$	ρ	-0.9

Data

Summary Statistics

	Data	Model
E$[i_t - k_t]$	-2.5915	-2.6562
std$[i_t - k_t]$	0.0961	0.0798
E$[y_t - k_t]$	-0.0277	-7.3794
std$[y_t - k_t]$	0.0677	0.0758
E$[r_t]$	0.0266	0.0184
std$[r_t]$	0.0806	0.0621
E$[vol_t]$	-2.7158	-2.9239
std$[vol_t]$	0.4015	0.1736
E$[r_t^f]$	0.0122	0.0061
std$[r_t^f]$	0.0068	0.0078
Sharpe Ratio	0.1787	0.1981
E$[\Delta c_t]$	0.0083	0.0054
std$[\Delta c_t]$	0.0045	0.0053
E$[\Delta y_t]$	0.0086	0.0054
std$[\Delta y_t]$	0.0124	0.0150
E$[\Delta i_t]$	0.0088	0.0052
std$[\Delta i_t]$	0.0470	0.0398

Model: Return predictability

Variables	Horizon h (in quarters)											
	1	2	3	4	5	6	7	8	9	10	11	12
						$r_{t+h} - r_{f,t+h} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$						
a_1	-0.256	-0.239	-0.207	-0.178	-0.148	-0.114	-0.130	-0.101	-0.101	-0.083	-0.067	-0.050
t-stat	(−7.407)	(−8.196)	(−6.355)	(−5.854)	(−5.078)	(−3.857)	(−3.746)	(−3.128)	(−2.874)	(−2.166)	(−1.914)	(−1.384)
R^2	[0.172]	[0.152]	[0.113]	[0.083]	[0.057]	[0.033]	[0.044]	[0.027]	[0.027]	[0.018]	[0.012]	[0.007]
						$r_{t+h} - r_{f,t+h} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h}$						
a_1	-0.264	-0.250	-0.219	-0.194	-0.168	-0.137	-0.155	-0.129	-0.127	-0.110	-0.095	-0.079
t-stat	(−8.251)	(−9.217)	(−7.213)	(−6.683)	(−6.010)	(−5.536)	(−5.872)	(−5.049)	(−5.141)	(−4.238)	(−4.164)	(−2.880)
a_2	0.055	0.071	0.081	0.096	0.113	0.118	0.129	0.140	0.136	0.140	0.144	0.145
t-stat	(2.257)	(3.129)	(3.313)	(3.992)	(4.786)	(4.959)	(5.903)	(6.182)	(5.680)	(5.384)	(5.541)	(5.475)
R^2	[0.178]	[0.166]	[0.131]	[0.109]	[0.094]	[0.073]	[0.092]	[0.082]	[0.080]	[0.073]	[0.071]	[0.066]

Model: Volatility predictability

Variables	1	2	3	4	5	6	7	8	9	10	11	12
	Horizon h (in quarters)											
	Univariate regression: $vol_{t+h} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$											
a_1	0.837	0.714	0.599	0.450	0.429	0.394	0.282	0.287	0.224	0.180	0.162	0.095
t-stat	(6.655)	(5.580)	(4.231)	(2.727)	(2.694)	(2.153)	(1.538)	(1.520)	(1.135)	(0.872)	(0.827)	(0.484)
R^2	[0.121]	[0.088]	[0.062]	[0.035]	[0.031]	[0.026]	[0.014]	[0.014]	[0.009]	[0.006]	[0.005]	[0.002]
	Multivariate regression I: $vol_{t+h} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h+1}$											
a_1	0.829	0.718	0.610	0.469	0.451	0.420	0.310	0.309	0.249	0.206	0.191	0.130
t-stat	(6.166)	(5.293)	(4.044)	(2.676)	(2.663)	(2.133)	(1.618)	(1.686)	(1.339)	(1.081)	(1.093)	(0.719)
a_2	0.053	-0.023	-0.069	-0.113	-0.123	-0.131	-0.142	-0.110	-0.127	-0.136	-0.147	-0.170
t-stat	(0.377)	(−0.148)	(−0.422)	(−0.646)	(−0.659)	(−0.676)	(−0.711)	(−0.530)	(−0.613)	(−0.653)	(−0.714)	(−0.874)
R^2	[0.117]	[0.084]	[0.059]	[0.033]	[0.030]	[0.025]	[0.013]	[0.012]	[0.007]	[0.005]	[0.004]	[0.003]

Conclusion

- We explore the relation between aggregate real investment and stock market volatility
- We document a new empirical pattern: high aggregate investment rate forecasts persistently high subsequent market volatility
- We develop a model where endogenous time-varying discount rates generate the positive relation between aggregate investment and future stock market volatility

Return predictability
Multi-Period

Variables	Horizon h (in quarters)											
	1	2	3	4	5	6	7	8	9	10	11	12
$\sum_{s=1}^{h}(r_{t+s} - r_{f,t+s}) = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$												
a_1	-0.153	-0.302	-0.418	-0.479	-0.526	-0.557	-0.610	-0.706	-0.811	-0.902	-0.991	-1.084
t-stat	(-3.002)	(-2.918)	(-2.668)	(-2.294)	(-2.009)	(-1.786)	(-1.719)	(-1.774)	(-1.874)	(-1.940)	(-2.002)	(-2.065)
R^2	[0.033]	[0.034]	[0.031]	[0.024]	[0.020]	[0.016]	[0.015]	[0.017]	[0.018]	[0.019]	[0.020]	[0.021]
$\sum_{s=1}^{h}(r_{t+s} - r_{f,t+s}) = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h}$												
a_1	-0.180	-0.347	-0.479	-0.541	-0.585	-0.619	-0.678	-0.782	-0.889	-0.978	-1.052	-1.135
t-stat	(-3.101)	(-2.920)	(-2.699)	(-2.298)	(-1.952)	(-1.707)	(-1.616)	(-1.645)	(-1.713)	(-1.739)	(-1.741)	(-1.758)
a_2	0.073	0.126	0.170	0.175	0.166	0.175	0.194	0.218	0.224	0.216	0.175	0.146
t-stat	(0.839)	(0.712)	(0.629)	(0.475)	(0.350)	(0.302)	(0.282)	(0.274)	(0.250)	(0.216)	(0.159)	(0.121)
R^2	[0.031]	[0.032]	[0.028]	[0.021]	[0.016]	[0.013]	[0.012]	[0.013]	[0.015]	[0.016]	[0.016]	[0.017]

Variables	Horizon h (in quarters) 1	2	3	4	5	6	7	8	9	10	11	12
	Univariate regression: $\sum_{s=1}^{h} vol_{t+s} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$											
a_1	0.234	0.677	1.296	2.035	3.028	4.099	5.361	6.778	8.319	9.766	11.141	12.431
t-stat	(0.877)	(1.268)	(1.633)	(1.971)	(2.414)	(2.793)	(3.254)	(3.724)	(4.204)	(4.552)	(4.857)	(5.128)
R^2	[0.004]	[0.010]	[0.020]	[0.030]	[0.046]	[0.063]	[0.085]	[0.111]	[0.140]	[0.167]	[0.192]	[0.215]
	Multivariate regression I: $\sum_{s=1}^{h} vol_{t+s} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h}$											
a_1	0.648	1.502	2.529	3.662	5.059	6.557	8.276	10.150	12.139	13.961	15.662	17.296
t-stat	(2.284)	(2.597)	(2.885)	(3.130)	(3.483)	(3.799)	(4.246)	(4.780)	(5.445)	(5.996)	(6.423)	(6.760)
a_2	-1.127	-2.264	-3.414	-4.551	-5.737	-6.982	-8.295	-9.597	-10.887	-12.005	-12.988	-13.981
t-stat	(−2.337)	(−2.337)	(−2.337)	(−2.331)	(−2.351)	(−2.401)	(−2.464)	(−2.514)	(−2.563)	(−2.578)	(−2.561)	(−2.547)
R^2	[0.031]	[0.047]	[0.063]	[0.079]	[0.100]	[0.123]	[0.150]	[0.181]	[0.216]	[0.246]	[0.273]	[0.299]
	Multivariate regression II: $\sum_{s=1}^{h} vol_{t+s} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + a_3 vol_{t-1} + \varepsilon_{t,t+h}$											
a_1	0.756	1.699	2.802	4.018	5.482	7.009	8.696	10.538	12.461	14.260	15.935	17.523
t-stat	(3.120)	(3.464)	(3.840)	(4.181)	(4.549)	(4.793)	(5.184)	(5.687)	(6.318)	(6.809)	(7.147)	(7.377)
a_2	-0.777	-1.579	-2.443	-3.177	-4.016	-5.062	-6.128	-7.266	-8.418	-9.445	-10.333	-11.363
t-stat	(−1.826)	(−1.833)	(−1.887)	(−1.856)	(−1.877)	(−1.945)	(−1.975)	(−2.012)	(−2.053)	(−2.083)	(−2.087)	(−2.112)
a_3	0.423	0.773	1.063	1.373	1.639	1.786	1.863	1.899	1.862	1.856	1.843	1.753
t-stat	(7.102)	(6.578)	(6.284)	(6.774)	(7.233)	(6.960)	(6.592)	(6.076)	(5.351)	(5.012)	(4.701)	(4.260)
R^2	[0.199]	[0.230]	[0.241]	[0.261]	[0.280]	[0.282]	[0.283]	[0.293]	[0.304]	[0.321]	[0.338]	[0.350]

Model: Return predictability

Multi-Period

Variables	Horizon h (in quarters)											
	1	2	3	4	5	6	7	8	9	10	11	12
$\sum_{s=1}^{h}(r_{t+s} - r_{f,t+s}) = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$												
a_1	-0.256	-0.495	-0.700	-0.874	-1.014	-1.115	-1.242	-1.342	-1.446	-1.535	-1.595	-1.638
t-stat	(-7.40)	(-9.43)	(-10.48)	(-11.21)	(-11.28)	(-10.37)	(-9.39)	(-8.48)	(-7.62)	(-6.88)	(-6.29)	(-5.83)
R^2	[0.17]	[0.37]	[0.49]	[0.53]	[0.56]	[0.55]	[0.56]	[0.55]	[0.55]	[0.53]	[0.50]	[0.47]
$\sum_{s=1}^{h}(r_{t+s} - r_{f,t+s}) = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h}$												
a_1	-0.264	-0.514	-0.732	-0.926	-1.087	-1.216	-1.368	-1.495	-1.621	-1.731	-1.823	-1.898
t-stat	(-8.25)	(-11.0)	(-12.9)	(-15.3)	(-17.4)	(-18.6)	(-19.6)	(-19.2)	(-17.9)	(-17.2)	(-16.8)	(-16.2)
a_2	0.055	0.126	0.205	0.301	0.408	0.519	0.636	0.767	0.889	1.016	1.154	1.292
t-stat	(2.257)	(3.220)	(4.099)	(5.249)	(6.659)	(7.889)	(9.133)	(9.740)	(9.985)	(9.729)	(9.408)	(9.082)
R^2	[0.178]	[0.400]	[0.540]	[0.604]	[0.670]	[0.694]	[0.732]	[0.761]	[0.784]	[0.795]	[0.797]	[0.800]

Model: Volatility predictability
Multi-Period

Variables	Horizon h (in quarters) 1	2	3	4	5	6	7	8	9	10	11	12
Univariate regression: $\sum_{s=1}^{h} vol_{t+s} = a_0 + a_1(i_t - k_t) + \varepsilon_{t,t+h}$												
a_1	0.837	1.552	2.152	2.608	3.040	3.404	3.669	3.953	4.188	4.394	4.531	4.601
t-stat	(6.655)	(6.674)	(6.233)	(5.424)	(4.980)	(4.528)	(4.179)	(3.996)	(3.763)	(3.500)	(3.262)	(2.983)
R^2	[0.121]	[0.182]	[0.215]	[0.211]	[0.208]	[0.203]	[0.194]	[0.188]	[0.183]	[0.177]	[0.167]	[0.155]
Multivariate regression I: $\sum_{s=1}^{h} vol_{t+s} = a_0 + a_1(i_t - k_t) + a_2(y_t - k_t) + \varepsilon_{t,t+h}$												
a_1	0.829	1.549	2.159	2.636	3.093	3.482	3.757	4.042	4.281	4.491	4.646	4.736
t-stat	(6.166)	(6.213)	(5.809)	(5.066)	(4.671)	(4.264)	(3.982)	(3.884)	(3.753)	(3.565)	(3.384)	(3.124)
a_2	0.053	0.025	-0.046	-0.165	-0.293	-0.397	-0.442	-0.448	-0.476	-0.504	-0.582	-0.670
t-stat	(0.377)	(0.087)	(−0.108)	(−0.280)	(−0.386)	(−0.429)	(−0.410)	(−0.361)	(−0.337)	(−0.316)	(−0.326)	(−0.341)
R^2	[0.117]	[0.179]	[0.212]	[0.209]	[0.207]	[0.203]	[0.193]	[0.187]	[0.182]	[0.176]	[0.166]	[0.155]