Competition among Portfolio Managers and Asset Specialization

Suleyman Basak  Dmitry Makarov

3rd International Moscow Finance Conference
ICEF  9 November 2013
Motivation

- There is substantial evidence of competition among portfolio managers—each manager attempts to outperform her peers
  - top performers attract more money flows
  - career concerns

- Managers do not hold fully diversified portfolios, but invest in a subset of all assets; different managers invest in different subsets
  - Merton’s (1987) idea: investors trade in familiar stocks

- This paper: take the classical portfolio choice setting and add the above ingredients
Main Results

- Nash equilibrium portfolio policies:
  - Risk tolerant (intolerant) manager decreases (increases) portfolio risk, relative to no competition
  - A higher risk aversion may lead to higher portfolio risk

- Competition conduce asset specialization—both managers benefit from an agreement to specialize

- We examine clients costs due to managerial turnover or changing stock characteristics
  - Client loses more when it is her manager who is replaced
  - Client’s loss is the same whether a change affects her manager’s stock or the competitor manager’s stock
Related Literature

• Dynamic models of portfolio managers’ competition: Browne (2000), Basak and Makarov (2013)


Setting

- Finite horizon \([0, T]\)

- Two sources of uncertainty, Brownian motions \(\omega_1\) and \(\omega_2\) with correlation \(\rho > 0\)

- Assets:
  - Money market account paying a constant rate \(r\)
  - Two risky assets, \(i = 1, 2\), following geometric Brownian motion

\[
dS_i = \mu_i S_i dt + \sigma_i S_i d\omega_i
\]
Portfolio managers

- Two portfolio managers $i$, $i = 1, 2$, manager $i$ is familiar only with asset $i$, so her choice variable is $\phi_{it}$—the share of wealth invested in asset $i$ at time $t$.

- Competition arises as manager $i$’s objective is given by

$$v_{it} = \frac{1}{1 - \gamma_i} \left( W_{iT}^{1 - \theta_i} R_{iT}^{\theta_i} \right)^{1 - \gamma_i},$$

where $W_{iT}$ is horizon wealth, $R_{iT}$ is relative return, $\gamma_i$ captures attitude towards risk, $\theta_i$ is relative performance bias ($\theta_i = 0$ leads to the standard CRRA function).
Nash Equilibrium

- Relative performance concerns combined with asset specialization imply that each manager faces incomplete markets

- Absent competition ($\theta_i = 0$), manager $i$’s policy is $\phi_i^* = (\mu_i - r)/(\gamma_i \sigma_i^2)$

- Use dynamic programming to solve for a Nash equilibrium: a pair $(\phi_{1t}^*, \phi_{2t}^*)$ of mutually consistent portfolio policies
Nash Equilibrium Portfolio Policies

- The equilibrium is

\[
\phi_1^* = \frac{\gamma_2 \mu_1 / \sigma_1 + \theta_1 (\gamma_1 - 1) \rho \mu_2 / \sigma_2}{\gamma_1 \gamma_2 - \rho^2 \theta_1 \theta_2 (\gamma_1 - 1) (\gamma_2 - 1)} \sigma_1
\]

\[
\phi_2^* = \frac{\gamma_1 \mu_2 / \sigma_2 + \theta_2 (\gamma_2 - 1) \rho \mu_1 / \sigma_1}{\gamma_1 \gamma_2 - \rho^2 \theta_1 \theta_2 (\gamma_1 - 1) (\gamma_2 - 1)} \sigma_2
\]

- Assume that in the above the numerators and denominators are positive, for equilibrium stability w.r.t. to best response dynamics and to preclude stock shorting.
Key Properties of Equilibrium Policies

• If a manager risk tolerant ($\gamma < 1$), she invests less in the risky asset than in the absence of competition; if a manager is risk intolerant ($\gamma > 1$), she invests more in the risky asset.

• Manager $i$'s risky investments and risk aversion are positively related provided her asset Sharpe ratio $(\mu_i - r)/\sigma_i$ is sufficiently low compared to the other asset Sharpe ratio.

• Key mechanisms: increasing stock investment reduces the volatility of relative return, decreasing it reduces the volatility of absolute wealth.
Asset Specialization versus No Specialization

- Consider the no specialization scenario where each manager can trade in both stocks.

- If each manager prefers no specialization over specialization, competition is a possible mechanism behind asset specialization (underdiversification).

- Consider parameters $\lambda_1$ and $\lambda_2$, where

$$J_i(W_{i0}) = J_i^{NoSp}((1 - \lambda_i)W_{i0}),$$

where $J_i(\cdot)$ is manager $i$’s equilibrium expected utility under asset specialization, and $J_i^{NoSp}(\cdot)$—under no specialization.

- Manager $i$ loses from specialization when $\lambda_i > 0$, but benefits from specialization when $\lambda_i < 0$. 
Client Investor

• Consider manager 1’s client investor with CRRA utility

\[ u_{cT} = \frac{W_{cT}^{1-\gamma_c}}{1 - \gamma_c}, \]

\( \gamma_c > 0 \) is the client’s relative risk aversion

• Consider parameter \( \lambda_c \) measuring the effect of specialization on the client:

\[ J_c(W_{c0}) = J_c^{NoSp}((1 - \lambda_c)W_{c0}), \]
Effect of Asset Specialization: Managers

For manager 1, we get

$$\lambda_1 = 1 - \exp((m - n)T'),$$

where \(m\) and \(n\) are

\[
m = \mu_1 \phi^*_1 - \theta_1 \mu_2 \phi^*_2 - \gamma_1 (\sigma_1 \phi^*_1)^2 / 2 \]
\[
+ \theta_1 (1 - \theta_1 (\gamma_1 - 1))(\sigma_2 \phi^*_2)^2 / 2 + \theta_1 (\gamma_1 - 1) \rho \sigma_1 \sigma_2 \phi^*_1 \phi^*_2,
\]
\[
n = \mu_1 (\phi^*_{11} - \theta_1 \phi^*_{21}) + \mu_2 (\phi^*_{12} - \theta_1 \phi^*_{22}) \]
\[
- \gamma_1 ((\sigma_1 \phi^*_{11})^2 + (\sigma_2 \phi^*_{12})^2 + 2 \rho \sigma_1 \sigma_2 \phi^*_{11} \phi^*_{12}) / 2 \]
\[
+ \theta_1 (\gamma_1 - 1) (\sigma_1^2 \phi^*_{11} \phi^*_{21} + \sigma_2^2 \phi^*_{12} \phi^*_{22} + \rho \sigma_1 \sigma_2 \phi^*_{11} \phi^*_{22} + \rho \sigma_1 \sigma_2 \phi^*_{12} \phi^*_{21}) \]
\[
+ \theta_1 (1 - \theta_1 (\gamma_1 - 1)) ((\sigma_1 \phi^*_{21})^2 + (\sigma_2 \phi^*_{22})^2 + 2 \rho \sigma_1 \sigma_2 \phi^*_{21} \phi^*_{22}) / 2.
\]


**Effect of Asset Specialization: Client**

For manager 1’s client, we get

\[
\lambda_c = 1 - \exp [\mu_1 \phi_1^* - \frac{1}{2} \gamma_c (\sigma_1 \phi_1^*)^2 - \mu_1 \phi_{11} - \mu_2 \phi_{12} + \frac{1}{2} \gamma_c ((\sigma_{11})^2 + (\sigma_{22})^2 + 2 \rho \sigma_1 \sigma_2 \phi_{11} \phi_{12})].
\]

- To obtain such expressions for manager 2 and her client, we need to switch subscripts 1 and 2 in the above expressions.

- Expressions are analytical, but it is more convenient to analyze them graphically.
Cost and benefits of transition from no specialization to specialization: for manager 1 - blue line, for manager 2 - red line, for manager 1’s client - black line
Effect of Asset Specialization: Summary

- Both managers, if risk tolerant, can benefit from asset specialization despite loss of diversification.

- Competition creates incentives to split the set of all assets into subsets, with each manager investing in her own subset.

- If managers are risk intolerant, diversification benefits are more important ⇒ both cannot prefer specialization.

- Manager 1’s ranking of specialization and no specialization scenarios could be different from the ranking of her client.
Changing Economic Environment: Effect on Clients

- Economic role of managers—save client’s time and effort by trading on her behalf. Client only needs to pick the right manager at the beginning.

- Will the initially “right” manager remain the “right” one if economic environment changes?

- If we switch off the competition channel, the answer is yes
  - Manager 1 is the right one if $\gamma_1 = \gamma_c$
  - If stock characteristics $\mu_1$ or $\sigma_1$ change, manager 1 will adjust her strategy, but the new strategy $(\mu_1 - r)/(\gamma_1 \sigma_1^2)$ will be optimal for the client.
  - The client can lose only if her manager is replaced by a new one with different risk aversion.
Client’s Cost from Changing Environment

- Denoting by hat \( \hat{\phi} \) strategies after a model parameter changes, the client’s cost \( \lambda_c \) is

\[
J_c((1 - \lambda_c)W_0, \hat{\phi}_c) = J_c(W_0, \hat{\phi}_1),
\]

The value \( \lambda_c > 0 \) is the percentage of wealth the client is prepared to pay to induce her manager to follow the desired (by the client) strategy \( \hat{\phi}_c \). We obtain:

\[
\lambda_c = 1 - \exp \left( \mu_1(\hat{\phi}_1^* - \hat{\phi}_c) - \frac{1}{2} \gamma_c \hat{\sigma}_1^2((\hat{\phi}_1^*)^2 - (\hat{\phi}_c)^2) \right),
\]
(c) Changing relative concern $\theta_1$ and $\theta_2$

Client’s cost when managers’ characteristics change: blue line corresponds to manager 1, red line to manager 2
Changing expected return $\mu_1$ and $\mu_2$

Changing volatility $\sigma_1$ and $\sigma_2$
Changing Environment: Key Results

• Client is more sensitive to (change in) characteristics of her own manager than those of competitor manager

• Client is equally sensitive to characteristics of her manager’s stock and those of the other manager’s stock, the one in which her manager does not trade

• Key intuition: in the former case, the client’s desired strategy remains the same, in the latter case, it does change

• Without competition, the “right” manager 1 remains “right” for the client for any change in stock 1 characteristics change; under competition, this is no longer the case
## Conclusion

- We provide closed-form expressions for Nash equilibrium portfolios under competition and asset specialization, and examined their properties.

- We examine conditions under which managers prefer asset specialization over no specialization—possible mechanism behind underdiversification.

- We argue that under competition clients need to pay more attention to their managers, as managers are likely to deviate from the clients’ desired policies when the environment changes.
(g) risk tolerant managers

(h) risk intolerant managers
Summary

- Relative performance concern $\theta$ needs to be sufficiently high for the risk tolerant manager to prefer specialization over no specialization.

- If manager is risk intolerant, she values diversification (no specialization) more when her relative concern $\theta$ increases.

- Under competition, the client can benefit if her risk intolerant manager specializes rather than invests in all assets.