

## **Cooperative games, solutions and applications**

### **Syllabus:**

1. Cooperative games with transferable utility (TU games) and their interpretation.

The characteristic function, relation between TU games and noncooperative normal form games, saving games and cost games. Types of TU games (essential and inessential games, superadditive games, convex games, monotonic games, simple games, constant sum games. Strategic equivalence, normalization. Basis of unanimity games, Harsanyi dividends.

Examples of TU games: glove games, a game "landlord and peasants", bankruptcy games, airport games, weighted majority games, market games, veto rich games, assignments games, big boss games.

2. Main solution concepts, their properties and axiomatic characterization.

The imputation set.

Domination. Stable sets (von Neumann and Morgenstern solutions).

The core, balanced games, necessary and sufficient conditions for the nonemptiness of the core. Totally balanced games, the  $\varepsilon$ -core and the least core. Core catchers, in particular the Weber set.

$M$  and  $M_1^i$ -bargaining sets and the kernel, their properties.

The nucleolus, existence and uniqueness, relation to the core and the kernel, characterization via balancedness (the Kohlberg's theorem).

The Shapley value, different formula representations and their interpretation. Axiomatic characterization: axiomatic of Shapley and axiomatic of Young. The potential of the Shapley value. Simple games, the Shapley-Shubik power index. Properties, in particular the null-player out property. Asymmetric extensions of the Shapley value—probabilistic values, random-order values, the weighted Shapley value.

Peculiarities of different solution concepts in particular classes of applied TU games.

3. Classes of games with a nonempty core: convex games and 1-convex/1-concave games.

Necessary and sufficient conditions for the convexity of a game, the Shapley's lemma and the Ichiishi's theorem.

1-convex and 1-concave games and their properties, 1-concave basis in the space of all TU games.

Applied models of 1-convex/1-concave games: library games, data games, co-insurance games.

4. TU games with limited cooperation and their solutions.

Games with coalition structures. The Aumann-Drèze value and Owen value.

Games with undirected graph communication structures. The Myerson value and its efficient modification, the average tree solution.

Games with directed communication structures and their solutions for particular case of forest games.

TU games endowed with both coalition and communication structures.

Applications: the social capital index; the water distribution problem of a river with multiple sources, a delta and possible islands along the river bed, and a river with multiple users.

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