ALGEBRAIC GEOMETRY: START-UP COURSE (SPRING 2014)

Outline. Any system of polynomial equations can be viewed as functor from a category of commutative algebras to the category of sets (by associating to an algebra the set of solutions over this algebra). This point of view can be refined to a very geometric notion of scheme. The language of schemes is the bulk of the course. Another theme is Grothendieck topologies.

To illustrate the notions, results, etc., there will be a detailed duscussion of curves and abelian varieties.

Syllabus. 1. Categories and functors and homological algebra; adjoint functors; representable functors; Yoneda's Lemma; natural transformations of functors; limits, colimits and their examples (fibre products); relativization; examples of categories: Sets, Groups, Abelian Groups, Rings, Commutative (unital) rings; colimits and gluing of functors

2. Grothendieck (pre-)topologies: basic definitions; presheaves and sheaves; first examples: topological space, various types of varieties; direct images and inverse images of sheaves; ring objects, e.g., $\mathbb{A}^1$, functions and structure sheaves; sheafification of presheaves.

3. Basic constructions in various geometric categories:

- of (locally) ringed spaces;
- of affine schemes, the anti-equivalence Spec : $\{$commutative rings$\} \to \{$affine schemes$\}$, the Hilbert's Nullstellensatz;
- of schemes,
- of projective and proper schemes,
- of algebraic varieties, affine and projective algebraic varieties,

4. Relative versions and corresponding classes of morphisms (affine, quasi-compact, projective, proper, finite, quasi-finite, finitely presented, flat, faithfully flat, étale, (locally) complete intersection, smooth, regular...); (sub)canonical Grothendieck topologies on a category; subcanonicity of the fpqc topology

5. (Quasi-)coherent sheaves; direct images and inverse images of coherent sheaves

6. Group schemes and torsors (principal bundles); vector bundles and locally free sheaves; line bundles, invertible sheaves and Cartier divisors; abelian varieties; Picard group scheme

7. Local and infinitesimal study: local rings; tangent spaces to abstract and embedded varieties; (non)singularity, regularity, normality; completion; regular maps their differential and their fibres; Bertini–Sard theorem.

8. Homological algebra and cohomology of sheaves: derived functors; the long exact cohomology sequence; the functor Hom and its derivatives; the global sections functor and its derivative (cohomology); flasque (or flabby) sheaves and resolutions; Čech cohomology of presheaves; vanishing theorems for Zariski cohomology of affine and projective varieties; ample, very ample sheaves and projective embeddings

Prerequisites. Basic knowledge of Commutative Algebra and of Homological Algebra is desirable (e.g., [AM, Weibel] more than suffices). Some experience in geometry and topology (projective spaces, topological spaces) would also be helpful.

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Teaching Assistant. P. Sechin

Examination type. There will be several homeworks and a final exam.

References.

Schemes

[1] Ulrich Görtz, Torsten Wedhorn, *Algebraic Geometry I*, Vieweg+Teubner Verlag, 2010.
[2] David Mumford, *Red Book of Varieties and Schemes*, 1999, Springer, Lecture Notes in Math., 1358. (first textbook of this type).
[3] Ravi Vakil, Math 216 (Foundations of Algebraic Geometry), `http://math.stanford.edu/ vakil/216blog/` (this is very comprehensive).
[4] Robin Hartshorne, *Algebraic Geometry,* 1977, Springer. (A standard reference.)
[5] J.S.Milne, *Algebraic Geometry,* `http://www.jmilne.org/math/CourseNotes/`.
[6] I.R.Shafarevich, *Basic Algebraic Geometry,* 1994, Springer.
[7] Yu.I.Manin, *Lectures on Algebraic Geometry,* 1966-1968; reprinted in Yu.I.Manin, Introduction to scheme theory and quantum groups, MCCME, 2012. (In Russian)

Grothendieck Topologies

[1] M. Artin, *Grothendieck Topologies,* Spring, 1962, Harvard
[2] J.S.Milne, *Étale Cohomology,* Princeton Univ., 1980, see also `http://www.jmilne.org/math/CourseNotes/`.

Commutative Algebra and Homological algebra

[AM] M.F.Atiyah, I.G.MacDonald, *Introduction to commutative algebra,* 1969, Addison-Wesley.
[Weibel] C.Weibel, *Introduction to homological algebra,* Cambridge University Press, 1994.