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IN PURSUIT OF A COMPREHENSIVE AND CUSTOMISABLE FRAMEWORK FOR THE ACCOUNTING OF VALUE ADDED IN INTERNATIONAL TRADE

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Gross exports accounting is a novel sub-area of research that seeks to allocate the value added in gross trade flows to its true country and sector of origin and country or sector of destination. Various frameworks have been recently proposed to perform such decompositions. This paper reviews and classifies these into two broad categories: gross exports accounting and cumulative value added accounting. While the former was exhaustively explored by Wang, Wei and Zhu (2013), this paper attempts to refine and generalise the latter. The results are mostly identical to those of Koopman, Wang and Wei (2012) and Stehrer (2013), but the generalised framework attains a desired level of computational efficiency and is highly customisable for specific purpose of global value chain analyses. The refined formulations are applied to describe Russia's export performance from global value chain perspective using the data from the World Input-Output Database (WIOD) for 2000, 2005 and 2010.

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Keywords: gross exports accounting, value added in trade, global value chains, inter-country input-output tables

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1. Introduction

Since a while, simple concepts and measurements have not sufficed to describe and explain the patterns of international trade. Fragmentation of production in late 20th and early 21st century which some analysts consider as significant as 19th century's industrial revolution (Baldwin, 2011) led to a persistent increase of trade in intermediate goods and services. It's more common nowadays that a particular product purchased for final use is the result of interactions within an inter-industry network where a multitude of producers acquire inputs from one another and add value on each subsequent production stage. Likewise, a raw material or primary input may have to virtually travel along a complex chain of industrial interactions until it is finally consumed or invested as part of a much more sophisticated product. As these interactions span across multiple borders, they are treated as global value chains or global supply chains – a term which has become a representative if not synonymous to international trade (Park *et al.*, 2013).

Case studies have shown that the traditional gross trade statistics give a misleading picture of "who produces what and for whom" (an expression from Daudin *et al.*, 2009) and how the benefits from trade in the form of value added or job creation are allocated. This is perhaps most manifest in the technology-intensive industries that typically outsource many operations and rely on cross-border supplies of parts and components. In the often cited case of Apple's iPhone, the assembling and exporting economy (China) was found to directly contribute only 2% of the retail price in the destination market (U.S.), while Apple's own contribution was thought to be around 58%. For an iPad, the assembler in China earned about 2% of the retail price in the U.S. and Apple retained 30% (Kraemer *et al.*, 2011). Interestingly, this is also typical for labour-intensive industries. A cost breakdown of a jacket manufactured in China and sold in the U.S. attributed only 5% to China and 86% to the U.S. (Low, 2013). A considerable body of similar case studies based on micro data (see an overview in Ali-Yrkko and Rouvinen, 2013) provide useful insights but not a comprehensive solution to the issue of identifying value added in gross trade flows. For the latter, economists and statisticians now employ inter-country input-output (ICIO) tables that link flows of goods and services for intermediate and final use between industries and countries in a consistent framework originally proposed by Leontief (1936) and refined by Isard (1951), Moses (1955), Leontief and Strout (1963) and others. The input-output model is capable to capture the infinite series of interactions among suppliers and consumers along the whole value chain.

The application of input-output techniques to international trade analyses eventually led to the emergence of a novel sub-area of research – tracing value added in trade, or gross exports accounting. The core objective is to separate net value added flows from gross trade flows as is usually done in national accounting (e.g. for GDP estimates) and to identify the origin and destination of value added in international trade. Various frameworks have been recently proposed

to measure value added in trade, and the most influential contributions include Daudin *et al.* (2009), Johnson and Noguera (2012), Koopman *et al.* (2010, 2012), Stehrer (2012, 2013). Many of the proposed concepts have been utilised by the OECD and WTO that jointly introduced a "Trade in Value Added" database[1] in 2013, a stepping stone in the production of alternatively measured trade statistics.

Koopman, Powers, Wang, Wei and Zhu authored several papers in 2010-2013 introducing the most elaborate framework for the gross exports accounting up to date. Their work has in fact been the major inspiration behind this paper. Although these authors proposed many important methodological advances, there is still room for generalisation of the gross exports accounting equations.

This paper therefore makes the following contributions. First, it introduces a relatively simple way to derive the formulae for the decomposition of the cumulative value added flows embodied in international trade and normalisation with respect to gross exports. The proposed formulations are intended to link various known and new terms using a minimal setup and providing maximum information on the creation and capture of value added. Hence, an important motivation of this work is to attain computational efficiency.

Second, the derived formulations are used to further generalise an accounting approach that largely builds on Koopman *et al.* (2012) and Stehrer (2013) in pursuit of a more complete and easily customisable framework. This paper also offers an explicit derivation of two matrices that account for the inter-sectoral transfer of value added in the production process on the exporting country's side and on the partner country's side. Various concepts known from previous studies are then discussed to help better interpret the data obtained.

Complementary to another complete framework by Wang *et al.* (2013) for the decomposition of direct export flows into value added parts, this is probably what an interested analyst or data user may expect from the discussion on value added in international trade. Indeed, generalisation converts a research framework into a customisable multi-purpose analytical tool.

The paper is organised as follows. Section 2 briefly overviews the frameworks proposed so far and discusses their core conceptual differences. Section 3 explains the derivation of the formulae to capture value added in international trade and generalisation of Koopman *et al.* (2012) and Stehrer (2013) framework. Section 4 applies some of the derived formulae to real world production and trade data and briefly discusses the results. Finally, Section 5 concludes. Important summaries and additional explanations are provided in the Appendices.

[1] OECD-WTO Trade in Value Added (TiVA), http://stats.oecd.org/Index.aspx?DataSetCode=TIVA_OECD_WTO.

2. Overview of the existing frameworks for the gross exports accounting

Renewed interest in the ICIO analysis, partly reinforced by the release of new global IO databases[2], has been fuelling the discussion on the gross exports accounting and tracing value added in international trade. A number of frameworks have been recently proposed, so it's essential to understand their contributions and conceptual differences to properly put this paper in the context of the ongoing discussion.

Most studies in our sub-area of research refer to Hummels *et al.* (1999) as the point of departure. Hummels and his co-authors did not provide a method for the complete decomposition of gross trade flows, but proposed first measures of vertical specialisation in trade that have effectively become building blocks for the subsequent research efforts and are still widely used for the global value chains analysis.[3] These measures, known as VS and VS1 can be described for any single country as follows:

- VS accounts for the import content of a country's exports, or "how much foreign value added is required to produce a unit of direct exports?",
- VS1 accounts for a country's domestic value added in partners' exports, or "how much domestic value added is required to produce partner countries' exports, per unit of direct exports of the country in focus?"

VS depicts a country as a recipient of foreign value added to be further processed for exports, or its relative position with respect to the upstream value chain. VS1 depicts a country as a supplier of domestic value added to be used in partners' exports, or its relative position with respect to the downstream value chain. VS therefore relates to the backward perspective and VS1 to the forward perspective in the global value chain analysis, as sketched in Figure 1.

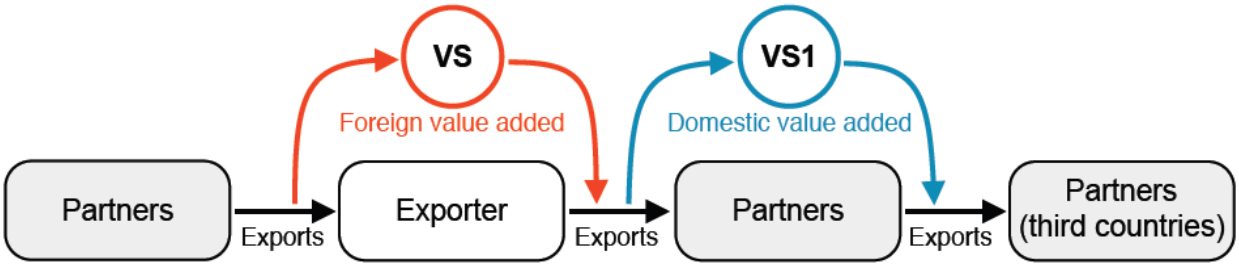

Figure 1. Vertical specialisation measures VS and VS1 proposed by Hummels *et al.* (1999)

[2] See the special issue of *Economic Systems Research*, 2013, Vol.25, No.1 for an overview.

[3] Examples include OECD (2013a), OECD, WTO and UNCTAD (2013), UNCTAD (2013). Note that the measures of vertical specialisation appear in those publications under different names, e.g. "backward/forward participation" or "upstream/downstream component".

Daudin *et al.* (2009) proposed an additional measure that is in fact a subset of VS1: domestic value added used in partners' exports that ultimately returns home in final products. They call it VS1* and develop an ICIO for their computations to correct the inaccuracies in the measures derived by Hummels *et al.* (1999) from single-country IO tables.

Johnson and Noguera (2012)[4] are usually credited for the introduction of a consistent multi-country framework for the computation of the value added content of bilateral trade, or "value added exports" that describes value added produced in a source country and finally absorbed in a destination country. They proposed a ratio of value added exports to gross exports at the sector and aggregate levels called VAX ratio as a way to address the "double-counting" problem and measure the intensity of production sharing. Johnson and Noguera generalised the computation procedures in an ICIO setting. Their contribution is also intimately related to the measurement of the factor content of trade as in Trefler and Zhu (2010).

Koopman *et al.* (2010) developed a framework that was in many respects similar to that of Johnson and Noguera but shifted the focus of their analysis to the complete decomposition of gross exports. Their effort integrated previous literature on vertical specialisation with newer literature on value added content of trade. In brief, Koopman *et al.* (2010) core contribution is:

- a consistent and relatively simple method of computation of the true VS and VS1 values in an inter-country setting,
- a decomposition that attributes all value added in a country's exports to its sources and destinations.

Koopman *et al.* (2010) proposed a breakdown of gross exports into three basic components: domestic value added destined for direct importing partners or third countries, domestic value added that returns home from abroad and foreign value added. A more detailed breakdown splits these three basic components into seven more detailed components.

Koopman *et al.* (2012) provided a unified framework that breaks up a country's gross exports into the sum of various components that are similar to their 2010 results. They show that the value added exports, VS, VS1, and VS1* are linear combinations of these components. Their new generalised version of the gross exports accounting equation contained nine terms. Though not explicitly observed (but recognised in Wang *et al.*, 2013), the 2012 version of Koopman and his co-authors' gross exports decomposition contained a conceptual deviation from the 2010 version. The 2010 paper focused on breaking down direct gross exports flows into value added components, whereas the 2012 paper focused on capturing both direct and indirect value added flows and normalisation with respect to gross exports. This is an important distinction between the two inter-

[4] First draft manuscript of Johnson and Noguera dates back to 2008.

related frameworks that will become more apparent in the later sections of this paper. Figures 2.1 and 2.2 present in a simplified form the two frameworks, but should be treated with care because some components in both frameworks, though carrying equivalent titles, are results of different computational procedures and are not directly comparable.

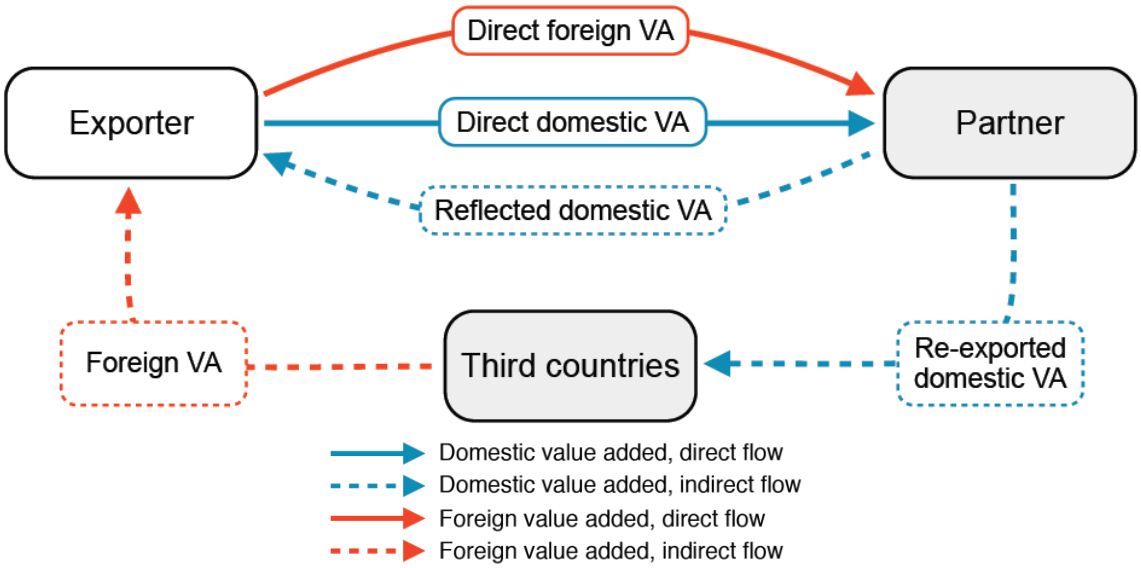

Figure 2.1. A simplified representation of the gross exports accounting framework (as in Koopman *et al.*, 2010)

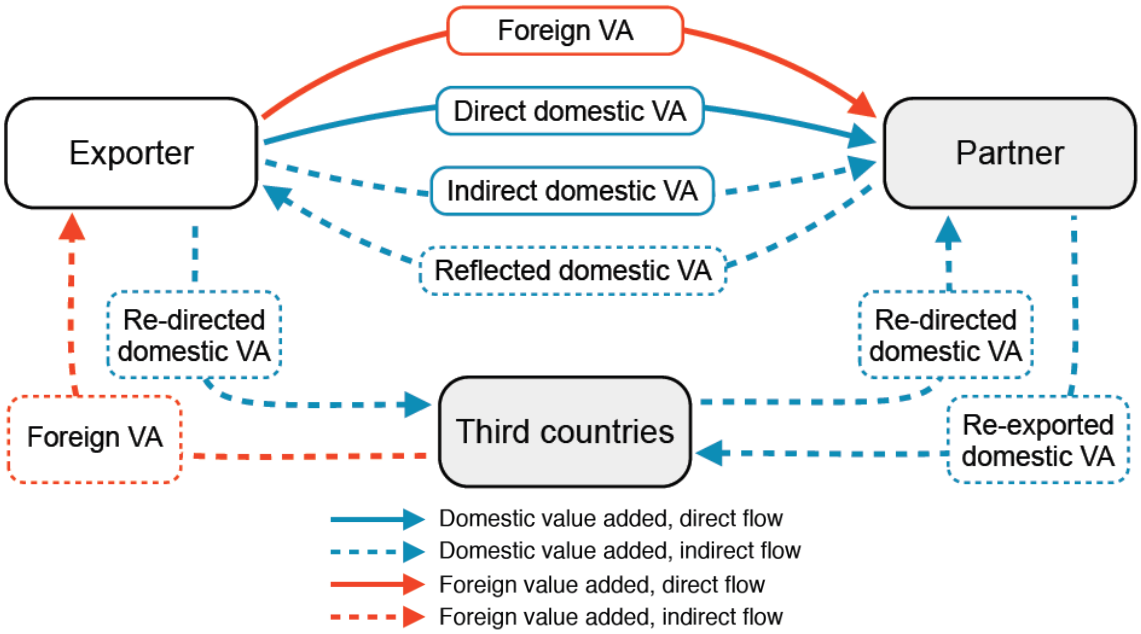

Figure 2.2. A simplified representation of the value added in gross exports accounting framework (as in Koopman *et al.*, 2012)

This is also a point where interpretation becomes critical to understand the distinctive features of the existing frameworks. Stehrer (2012) drew a borderline between the concepts of "trade in value added" and "value added in trade" that was rather helpful to structure readers' thoughts about

the subject. In bilateral trade relations, the first concept – "trade in value added" – accounts for the value added of one country directly and indirectly contained in final consumption of another country. VAX proposed by Johnson and Noguera (2012) is a good example of the application of this concept. The second concept – "value added in trade" – calculates the value added contained in gross trade flows between two countries. Examples include VS and VS1. The two concepts address different questions and may be used for different purposes. For trade policy which usually applies to the gross trade flows, the results of "value added in trade" may be more enlightening. For global value chain analysis, "trade in value added" may be a more relevant concept. Stehrer (2012) also carefully studied the properties of bilateral and overall trade balances in net and gross terms.

Stehrer (2013) applied the framework of Koopman *et al.* (2012) at the bilateral level. This allowed for a detailed account of the relationships between the two concepts from Stehrer (2012) and the role of third countries in bilateral value added trade. Many results in the present paper are identical to those in Stehrer (2013), while the derivation of the relevant formulae hopefully offers certain improvements.

A number of recent studies review and elaborate the frameworks mentioned so far for specific analytical purposes. Meng *et al.* (2012) apply "trade in value added" and "value added in trade" concepts to measure the progress of regional economic integration through cross-border value chains. They make useful observations on the calculation of "trade in value added" at the sectoral level for their alternative version of the revealed comparative advantage indicator.

Kuroiwa (2014) applies the framework of Koopman *et al.* (2012) to derive a gross exports accounting equation for the special case of IDE-JETRO's Asian Input-Output Tables which, unlike the global ICIO tables, contain exogenous vectors of imports from and exports to the Rest of the World. He then uses the equation to assess the technological intensity of China's exports.

Kuboniwa (2014a, 2014b, 2014c) develops a theoretical discussion on the relationship between trade balances in value added and gross terms building on many of the previously discussed concepts.

In sum, complete frameworks for the gross exports accounting combine both "trade in value added" and "value added in trade" concepts. However, the exact combination varies. It is important to discern two types of such frameworks for a clearer understanding of their applications.

- The first type builds on Koopman *et al.* (2010) and decomposes direct exports into value added terms. Normalisation with respect to total gross exports is common and each component will be bound between 0 and 100%. Wang *et al.* (2013) provide the most complete generalisation of such framework up to date with a breakdown of gross exports into sixteen components at the bilateral sectoral level.

- The second type of frameworks builds on Koopman *et al.* (2012) and in fact decomposes direct and indirect flows of value added, not exactly gross exports, into value added components. Normalisation with respect to gross exports is possible, but some components may well exceed 100% at the sectoral and bilateral level. Normalisation with respect to total exported value added is more natural and will bind the components between 0 and 100%. It is this type of decomposition that this paper intends to further generalise. It is suggested that this type be better described as "cumulative value added accounting" rather than "gross exports accounting" of type one.

It appears that there is no single framework that would offer all-in-one solution for all analytical purposes. Each type of decomposition has its own advantages and can be best applied in certain situations but can yield less useful results in other. A researcher or data user has therefore to carefully consider the purpose of the analysis to select proper application. Besides, as in many input-output based applications, meaningful economic interpretation of the proposed formulations is key to their proper use for specific analysis.

3. Refined generalised framework for the cumulative value added accounting

This section first introduces the notation and the "minimal matrix setup" which is thought to be one of the distinctive features of the proposed generalised accounting framework. The reader is expected to be familiar with the input-output analysis, so the core concepts and the inter-country input-output tables are only briefly reviewed. This is followed by the derivation of the "basic accounting relationship" which then undergoes various algebraic manipulations to provide decompositions in the sectoral and aggregate country dimensions. The "basic accounting relationship" is also used for a surprisingly compact proof of the equality of total (i.e. aggregated across partner countries) trade balances in gross and value added terms and for a concise decomposition of the bilateral trade balances. One implication of the global value chains is that value added is traded not only among countries, but also among economic sectors or industries in each country. The final part of this section discusses how the proposed framework can be customised to identify the final destination of value added at the product or sector level. The concepts of "value added at origin" and "value added at destination" are then explicitly formalised. A summary of all formulae obtained and a description of the matrices therein are provided in Appendices A and B.

3.1. Concepts, notations and the minimal matrix setup

For the purpose of a holistic value chain analysis, the global economy is modeled by a global input-output table as shown in Figure 3.[5] In this table, each product flow is attributed to a selling industry/country and purchasing industry/country for intermediate use or purchasing country for final use. There are also primary inputs to production or value added that is usually assumed to be immobile across borders. So, directly, a country only uses value added from the factors of production confined to its own territory.[6] In principle, final demand and value added that both give GDP estimates can be split into various sub-categories, but such level of detail is not necessary for the generalised framework in this paper and can be easily added by an interested researcher.

		Intermediate demand			Final demand			Total output
		Country 1	Country 2	Country k	Country 1	Country 2	Country k	
Supply	Country 1	Z_{11}	Z_{12}	Z_{1k}	f_{11}	f_{12}	f_{1k}	x_1
	Country 2	Z_{21}	Z_{22}	Z_{2k}	f_{21}	f_{22}	f_{2k}	x_2
	Country k	Z_{k1}	Z_{k2}	Z_{kk}	f_{k1}	f_{k2}	f_{kk}	x_k
Value added		v_1	v_2	v_k				
Total input		x_1	x_2	x_k				

Figure 3. Simplified version of an inter-country input-output table

Each cell Z in the table corresponds to the intermediate demand of the purchasing country from the selling country and represents a matrix of the inter-industry transactions while f, v and x represent vectors, respectively, of final demand, value added and total output.

The matrix representation of the global ICIO table for K countries appears as follows:

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z_{11}} & \mathbf{Z_{12}} & \cdots & \mathbf{Z_{1k}} \\ \mathbf{Z_{21}} & \mathbf{Z_{22}} & \cdots & \mathbf{Z_{2k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Z_{k1}} & \mathbf{Z_{k2}} & \cdots & \mathbf{Z_{kk}} \end{bmatrix}, \qquad \mathbf{F} = \begin{bmatrix} \mathbf{f_{11}} & \mathbf{f_{12}} & \cdots & \mathbf{f_{1k}} \\ \mathbf{f_{21}} & \mathbf{f_{22}} & \cdots & \mathbf{f_{2k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{f_{k1}} & \mathbf{f_{k2}} & \cdots & \mathbf{f_{kk}} \end{bmatrix}, \qquad \mathbf{x} = \begin{bmatrix} \mathbf{x_1} \\ \mathbf{x_2} \\ \vdots \\ \mathbf{x_k} \end{bmatrix},$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v_1} & \mathbf{v_2} & \cdots & \mathbf{v_n} \end{bmatrix}$$

In an economy of N economic sectors, each block element $\mathbf{Z}_{rs}$ is a N×N matrix, $\mathbf{f}_{rs}$ and $\mathbf{x}_r$ are N×1 vectors and $\mathbf{v}_s$ is a 1×N vector. Henceforth, indices *r* and *s* denote, respectively, selling and purchasing countries, and indices *i* and *j* denote selling and purchasing sectors in each country. So,

[5] Explaining the rationale for the use of global input-output tables is beyond the scope of this paper. Interested reader may refer to Murray and Lenzen (2013) for mostly non-technical introduction, or to the special issue of *Economic Systems Research*, 2013, Vol. 25, No.1 for a more scholarly discussion.

[6] Overcoming this limitation and accounting for the income flows, beyond trade flows, is an important part of OECD and WTO joint efforts, see OECD and WTO (2012).

Z is a KN×KN matrix of intermediate demand, **F** is a KN×K matrix of final demand, x is a KN×1 column vector of total output, and **v** is a 1×KN row vector of value added.

In the next few steps, we will work out our "minimal setup" of only five basic matrices that will be required for most subsequent computations.

Define a KN×KN diagonal matrix of value added coefficients:

$$\mathbf{V_c} = \begin{bmatrix} \hat{\mathbf{v}}_{\mathbf{c,1}} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{v}}_{\mathbf{c,2}} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \hat{\mathbf{v}}_{\mathbf{c,k}} \end{bmatrix}$$

where each block $\hat{\mathbf{v}}_{\mathbf{c,s}}$ is a N×N diagonalised vector of value added coefficients, $v_{c,j} = \dfrac{v_j}{x_j}$.

In matrix notation, $\mathbf{V_c} = \hat{\mathbf{v}}\hat{\mathbf{x}}^{-1}$.

We will also need two KN×K matrices representing total gross exports and bilateral gross exports:

$$\mathbf{E}_{tot} = \begin{bmatrix} \mathbf{e_1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{e_2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{e_k} \end{bmatrix}, \qquad \mathbf{E}_{bil} = \begin{bmatrix} \mathbf{0} & \mathbf{e_{12}} & \cdots & \mathbf{e_{1k}} \\ \mathbf{e_{21}} & \mathbf{0} & \cdots & \mathbf{e_{2k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{e_{k1}} & \mathbf{e_{k2}} & \cdots & \mathbf{0} \end{bmatrix}$$

In $\mathbf{E}_{tot}$, each N×1 block element $\mathbf{e}_r$ is equal to the sum of exports for intermediate and final use over all trading partners, $\mathbf{e}_{tot,r} = \sum_{s \neq r}^{K} \mathbf{Z}_{rs}\mathbf{i}_n + \sum_{s \neq r}^{K} \mathbf{f}_{rs}$. In $\mathbf{E}_{bil}$, each N×1 block element $\mathbf{e}_{rs}$ only accounts for bilateral flows, $\mathbf{e}_{bil,rs} = \mathbf{Z}_{rs}\mathbf{i}_n + \mathbf{f}_{rs}$, $r \neq s$, and $\mathbf{i}_n$ is a N×1 vector of ones for the summation across sectors.

Next we find the inter-country version of the Leontief inverse which is key to demand-driven input-output analysis:

$$\mathbf{(I - A)^{-1}} = \begin{bmatrix} \mathbf{I} - \mathbf{A_{11}} & -\mathbf{A_{12}} & \cdots & -\mathbf{A_{1k}} \\ -\mathbf{A_{21}} & \mathbf{I} - \mathbf{A_{22}} & \cdots & -\mathbf{A_{2k}} \\ \vdots & \vdots & \ddots & \vdots \\ -\mathbf{A_{k1}} & -\mathbf{A_{k2}} & \cdots & \mathbf{I} - \mathbf{A_{kk}} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{L_{11}} & \mathbf{L_{12}} & \cdots & \mathbf{L_{1k}} \\ \mathbf{L_{21}} & \mathbf{L_{22}} & \cdots & \mathbf{L_{2k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{L_{k1}} & \mathbf{L_{k2}} & \cdots & \mathbf{L_{kk}} \end{bmatrix} = \mathbf{L}$$

$\mathbf{A}_{rs}$ blocks are N×N technical coefficient matrices that relate intermediate inputs to total output $a_{ij} = \dfrac{z_{ij}}{x_j}$ (so $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$). Leontief inverse is a KN×KN multiplier matrix that allows total

output to be expressed as a function of final demand: $\mathbf{x} = \mathbf{Ax} + \mathbf{Fi} = \mathbf{(I - A)^{-1}Fi} = \mathbf{LFi}$ (where **i** is an appropriately sized summation vector).

This completes the setup for the derivation of the basic value added accounting formulae. It is indeed minimal and only consists of five matrices: $\mathbf{V}_c$, **L**, **F**, $\mathbf{E}_{tot}$ and $\mathbf{E}_{bil}$. One more matrix, $\hat{\mathbf{x}}$ that is the diagonalised total output, will occasionally feature the interim formulations but will eventually disappear from the final ones.

Note that the accounting formulae that use the matrices defined above will yield results in the KN×K dimension, that is [country/sector]×country. For an extension of the framework to the full KN×KN dimension and discussion of the "value added at origin" and "value added at destination" concepts, an additional matrix will need to be constructed from **Z** and added to the setup.

Throughout algebraic manipulations, some matrices may have to be redefined in dimensions other that the above. In such cases, for clarity, we will specify the new dimension by a subscript, e.g. $\mathbf{Z}_{(KN\times K)}$ or $\mathbf{F}_{(KN\times KN)}$.

Besides usual matrix summation and multiplication, the framework will also require using the following operators:

- $\hat{}$ – extracting a block-diagonal matrix from a block matrix or creating a block-diagonal matrix from a vector,
- $\check{}$ – extracting a matrix with off-diagonal block elements from a block matrix,
- $\circ$ – block-element by block-element multiplication, known as Hadamard product in case of element-by-element multiplication,
- $'$ – transposition of a block matrix.

These four operators should be applied block-wise to the KN×KN **L** matrix and KN×K **F**, $\mathbf{E}_{tot}$ and $\mathbf{E}_{bil}$ matrices and **x** vector. This is an essential pre-requisite for all further manipulations with block matrices throughout this paper.

The important distinction that this setup implies in relation to most similar studies can be summarised as follows.

- This paper does not discuss a simplified representation for the case of two- or three-country world. The starting point of the discussion is the general case of K countries and N sectors. Various parts of this section and Appendix B provide a zoom in view on the matrices obtained and help with their proper economic interpretation. So the setup is thought to be convenient for both reader's understanding and the implementation in a matrix computation software.
- There are only five core matrices and a number of common matrix operators that are required for decompositions of cross-border flows of value added.

An example of similar work is Wang *et al.* (2013) who suggest an implementation of their framework into computer code in Appendix O of their paper.

3.2. The basic accounting relationship and the "value added at origin" concept

Note that, by definition of gross exports:

$$\mathbf{E}_{bil} - \mathbf{E}_{tot} = \mathbf{F} + \mathbf{Z}_{(KN\times K)} - \hat{\mathbf{x}}_{(KN\times K)} \tag{1}$$

where $\mathbf{Z}_{(KN\times K)}$ is the $\mathbf{Z}$ matrix condensed to the KN×K dimension (aggregated across the partner country *s*' sectors), and $\hat{\mathbf{x}}_{(KN\times K)}$ is a KN×K block-diagonalised vector of total output arranged in a similar way, to conform with the $\mathbf{E}_{tot}$ and $\mathbf{E}_{bil}$ matrix dimensions. The resulting matrix on both sides of (1) has bilateral trade flows in the off-diagonal block elements and total exports with the negative sign in the diagonal block elements.

Using that $\mathbf{Z}_{(KN\times K)} = \mathbf{A}\hat{\mathbf{x}}_{(KN\times K)}$, rewrite the right part of the equation as follows:

$$\mathbf{F} - \hat{\mathbf{x}}_{(KN\times K)} + \mathbf{Z}_{(KN\times K)} = \mathbf{F} - \hat{\mathbf{x}}_{(KN\times K)} + \mathbf{A}\hat{\mathbf{x}}_{(KN\times K)} = \mathbf{F} - (\mathbf{I} - \mathbf{A})\hat{\mathbf{x}}_{(KN\times K)} = \mathbf{F} - \mathbf{L}^{-1}\hat{\mathbf{x}}_{(KN\times K)}$$

Then multiply both sides of (1), including the rewritten right side, by the value added multipliers matrix $\mathbf{V_c L}$:

$$\mathbf{V_c L}(\mathbf{E}_{bil} - \mathbf{E}_{tot}) = \mathbf{V_c L}(\mathbf{F} - \mathbf{L}^{-1}\hat{\mathbf{x}}_{(KN\times K)})$$

A simple rearrangement gives:

$$\mathbf{V_c L E}_{bil} = \mathbf{V_c L F} + \mathbf{V_c L E}_{tot} - \mathbf{V_c}\hat{\mathbf{x}}_{(KN\times K)} \tag{2}$$

And the zoom in view on this equation is given below:

$$\begin{bmatrix} \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{e}_{t1} & \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{e}_{t2} & \cdots & \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{e}_{tk} \\ \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{e}_{t1} & \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{e}_{t2} & \cdots & \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{e}_{tk} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{e}_{t1} & \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{e}_{t2} & \cdots & \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{e}_{tk} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{f}_{t1} & \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{f}_{t2} & \cdots & \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{f}_{tk} \\ \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{f}_{t1} & \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{f}_{t2} & \cdots & \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{f}_{tk} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{f}_{t1} & \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{f}_{t2} & \cdots & \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{f}_{tk} \end{bmatrix} +$$

$$+\begin{bmatrix} \mathbf{V}_{c,1}\mathbf{L}_{11}\mathbf{e}_1 & \mathbf{V}_{c,1}\mathbf{L}_{12}\mathbf{e}_2 & \cdots & \mathbf{V}_{c,1}\mathbf{L}_{1k}\mathbf{e}_k \\ \mathbf{V}_{c,2}\mathbf{L}_{21}\mathbf{e}_1 & \mathbf{V}_{c,2}\mathbf{L}_{22}\mathbf{e}_2 & \cdots & \mathbf{V}_{c,2}\mathbf{L}_{2k}\mathbf{e}_k \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\mathbf{L}_{k1}\mathbf{e}_1 & \mathbf{V}_{c,k}\mathbf{L}_{k2}\mathbf{e}_2 & \cdots & \mathbf{V}_{c,k}\mathbf{L}_{kk}\mathbf{e}_k \end{bmatrix} - \begin{bmatrix} \mathbf{V}_{c,1}\mathbf{x}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_{c,2}\mathbf{x}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{V}_{c,k}\mathbf{x}_k \end{bmatrix}$$

Each of the matrices above deserves a stand-alone interpretation. $\mathbf{V_c L E}_{bil}$ can be treated as a "bilateral value added in trade matrix". Each element in this matrix corresponds to both direct and indirect flows of value added that originates in sector *i* of country *r* and "lands" in country *s* to satisfy aggregate (intermediate plus final) demand in country *s*:

$$\mathbf{V_c LE}_{bil} = \begin{bmatrix} \mathbf{V}_{c,1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_{c,2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{V}_{c,k} \end{bmatrix} \begin{bmatrix} \mathbf{L}_{11} & \mathbf{L}_{12} & \cdots & \mathbf{L}_{1k} \\ \mathbf{L}_{21} & \mathbf{L}_{22} & \cdots & \mathbf{L}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{L}_{k1} & \mathbf{L}_{k2} & \cdots & \mathbf{L}_{kk} \end{bmatrix} \begin{bmatrix} \mathbf{0} & \mathbf{e}_{12} & \cdots & \mathbf{e}_{1k} \\ \mathbf{e}_{21} & \mathbf{0} & \cdots & \mathbf{e}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{e}_{k1} & \mathbf{e}_{k2} & \cdots & \mathbf{0} \end{bmatrix} =$$

$$= \begin{bmatrix} \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{e}_{t1} & \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{e}_{t2} & \cdots & \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{e}_{tk} \\ \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{e}_{t1} & \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{e}_{t2} & \cdots & \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{e}_{tk} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{e}_{t1} & \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{e}_{t2} & \cdots & \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{e}_{tk} \end{bmatrix}$$

Usually, $\mathbf{V_c LE}_{bil}$ matrix does not feature "value added in trade" calculations and does not explicitly appear in any of the existing frameworks. However, this matrix is useful to estimate domestic value added embodied in direct and indirect gross trade flows.

Note that the columns of $\mathbf{V_c LE}_{bil}$ (and of $\mathbf{E}_{bil}$) sum to total imports of country *s*. The rows of $\mathbf{V_c LE}_{bil}$ sum to domestic value added in total gross exports of country *r*, sectoral or aggregated.

$\mathbf{V_c LF}$ matrix is much more familiar as it is equal to Johnson and Noguera's bilateral "value added exports" matrix used for their derivation of the VAX measure. So it can be called "trade in value added" matrix. Each element here represents both direct and indirect flows of value added that originates in sector *i* of country *r* and "ends up" in country *s* to satisfy its final demand:

$$\mathbf{V_c LF} = \begin{bmatrix} \mathbf{V}_{c,1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_{c,2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{V}_{c,k} \end{bmatrix} \begin{bmatrix} \mathbf{L}_{11} & \mathbf{L}_{12} & \cdots & \mathbf{L}_{1k} \\ \mathbf{L}_{21} & \mathbf{L}_{22} & \cdots & \mathbf{L}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{L}_{k1} & \mathbf{L}_{k2} & \cdots & \mathbf{L}_{kk} \end{bmatrix} \begin{bmatrix} \mathbf{f}_{11} & \mathbf{f}_{12} & \cdots & \mathbf{f}_{1k} \\ \mathbf{f}_{21} & \mathbf{f}_{22} & \cdots & \mathbf{f}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{f}_{k1} & \mathbf{f}_{k2} & \cdots & \mathbf{f}_{kk} \end{bmatrix} =$$

$$= \begin{bmatrix} \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{f}_{t1} & \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{f}_{t2} & \cdots & \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{f}_{tk} \\ \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{f}_{t1} & \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{f}_{t2} & \cdots & \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{f}_{tk} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{f}_{t1} & \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{f}_{t2} & \cdots & \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{f}_{tk} \end{bmatrix}$$

The rows of $\mathbf{V_c LF}$ sum to total value added (sectoral or aggregated) produced in country *r*. The columns of $\mathbf{V_c LF}$ sum to total value added absorbed in country *s*.

$\mathbf{V_c LE}_{tot}$ is the "value added in total trade" matrix. Koopman *et al.* (2010) proposed to use it for the computation of the multilateral VS and VS1 measures, as the column sums of the off-

diagonal elements give VS and the row sums of the off-diagonal (aggregated) elements give VS1 in monetary terms:

$$\mathbf{V_c L E}_{tot} = \begin{bmatrix} \mathbf{V}_{\mathbf{c},1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_{\mathbf{c},2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{V}_{\mathbf{c},\mathbf{k}} \end{bmatrix} \begin{bmatrix} \mathbf{L}_{11} & \mathbf{L}_{12} & \cdots & \mathbf{L}_{1\mathbf{k}} \\ \mathbf{L}_{21} & \mathbf{L}_{22} & \cdots & \mathbf{L}_{2\mathbf{k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{L}_{\mathbf{k}1} & \mathbf{L}_{\mathbf{k}2} & \cdots & \mathbf{L}_{\mathbf{kk}} \end{bmatrix} \begin{bmatrix} \mathbf{e}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{e}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{e}_\mathbf{k} \end{bmatrix} =$$

$$= \begin{bmatrix} \mathbf{V}_{\mathbf{c},1}\mathbf{L}_{11}\mathbf{e}_1 & \mathbf{V}_{\mathbf{c},1}\mathbf{L}_{12}\mathbf{e}_2 & \cdots & \mathbf{V}_{\mathbf{c},1}\mathbf{L}_{1\mathbf{k}}\mathbf{e}_\mathbf{k} \\ \mathbf{V}_{\mathbf{c},2}\mathbf{L}_{21}\mathbf{e}_1 & \mathbf{V}_{\mathbf{c},2}\mathbf{L}_{22}\mathbf{e}_2 & \cdots & \mathbf{V}_{\mathbf{c},2}\mathbf{L}_{2\mathbf{k}}\mathbf{e}_\mathbf{k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{\mathbf{c},\mathbf{k}}\mathbf{L}_{\mathbf{k}1}\mathbf{e}_1 & \mathbf{V}_{\mathbf{c},\mathbf{k}}\mathbf{L}_{\mathbf{k}2}\mathbf{e}_2 & \cdots & \mathbf{V}_{\mathbf{c},\mathbf{k}}\mathbf{L}_{\mathbf{kk}}\mathbf{e}_\mathbf{k} \end{bmatrix}$$

Note also that the columns of $\mathbf{V_c L E}_{tot}$ (and of $\mathbf{E}_{tot}$) sum to total exports of country *s* and its rows sum to total exports of country *r*'s value added (sectoral or aggregated) in gross trade flows.

$\mathbf{V_c}\hat{\mathbf{x}}_{(KN\times K)}$ is a block-diagonal matrix of sectoral value added.

As our primary interest is international trade, it's legitimate to consider the off-diagonal block elements only from equation (2). $\mathbf{V_c}\hat{\mathbf{x}}_{(KN\times K)}$ then disappears from our basic accounting relationship since it has diagonal block element only:

$$\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}\right] = \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right] + \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}\right] \tag{3}$$

The right side of equation (3) can be recognised as the sum of the multilateral VAX and VS1 measures in monetary terms. This basic accounting relationship implies a straightforward interpretation: value added that originates in sector *i* of country *r* and "lands" in country *s* via direct and indirect trade flows is equal to the value added that "ends up" in country *s* plus value added that is re-exported by country *s* to third countries. $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right]$ is therefore a net term and $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}\right]$ is a double-counted term. This gives a basic decomposition of bilateral value added in total trade into two components that can be expressed as its shares.

One implicit aggregation concept behind this basic accounting relationship needs clarifying. Equation (3) and its more detailed variants in the KN×K or [country/sector]×country dimension correspond to an aggregation concept that may be reasonably called "value added at origin" as it attributes each aggregated flow destined for country *s* to each sector *i* in country *r* which sources value added. An alternative concept is "value added at destination" which assigns sectoral or aggregated value added from country *r* to product of sector *j* consumed or re-exported by country *s*. That will allow for further generalisation of the framework but will require an extension of the initial minimal setup and will be explored later in this section. Meanwhile, Figure 4 visualises the

distinction between these two concepts which is essential for the proper interpretation of the formulations below.

Value added at origin

Destination of value added		Country *s*: imports	
Origin of value added		For consumption	For re-exports
Country *r*: exports, N sectors (*i*)	Sector *i*1	$V_cLF(i1)$	$V_cLE_{tot}(i1)$
	Sector *i*2	$V_cLF(i2)$	$V_cLE_{tot}(i2)$
	Sector *i*n	$V_cLF(in)$	$V_cLE_{tot}(in)$

Value added at destination

Destination of value added	Country *s*: imports, products of N sectors (*j*)					
	For consumption			For re-exports		
Origin of value added	Product of sector *j*1	Product of sector *j*2	Product of sector *j*n	Product of sector *j*1	Product of sector *j*2	Product of sector *j*n
Country *r*: exports	$V_cLF(j1)$	$V_cLF(j2)$	$V_cLF(jn)$	$V_cLE_{tot}(j1)$	$V_cLE_{tot}(j2)$	$V_cLE_{tot}(jn)$

Figure 4. Value added at origin and value added at destination concepts

In just two steps, equation (3) can be rearranged to express gross bilateral exports as a sum of value added components. First, note that the "bilateral value added in trade matrix" $\mathbf{V_cLE}_{bil}$ can be decomposed into two matrices that represent direct and indirect bilateral flows of value added in trade:

$$\mathbf{V_cLE}_{bil} = \mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil} + \mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil} \tag{4}$$

The zoom in view on (4) is as follows:

$$\begin{bmatrix} \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{e}_{t1} & \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{e}_{t2} & \cdots & \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\mathbf{e}_{tk} \\ \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{e}_{t1} & \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{e}_{t2} & \cdots & \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\mathbf{e}_{tk} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{e}_{t1} & \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{e}_{t2} & \cdots & \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\mathbf{e}_{tk} \end{bmatrix} =$$

$$= \begin{bmatrix} \mathbf{0} & \mathbf{V}_{c,1}\mathbf{L}_{11}\mathbf{e}_{12} & \cdots & \mathbf{V}_{c,1}\mathbf{L}_{11}\mathbf{e}_{1k} \\ \mathbf{V}_{c,2}\mathbf{L}_{22}\mathbf{e}_{21} & \mathbf{0} & \cdots & \mathbf{V}_{c,2}\mathbf{L}_{22}\mathbf{e}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\mathbf{L}_{kk}\mathbf{e}_{k1} & \mathbf{V}_{c,k}\mathbf{L}_{kk}\mathbf{e}_{k2} & \cdots & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{V}_{c,1}\sum_{t\neq 1}^{K}\mathbf{L}_{1t}\mathbf{e}_{t1} & \mathbf{V}_{c,1}\sum_{t\neq 1}^{K}\mathbf{L}_{1t}\mathbf{e}_{t2} & \cdots & \mathbf{V}_{c,1}\sum_{t\neq 1}^{K}\mathbf{L}_{1t}\mathbf{e}_{tk} \\ \mathbf{V}_{c,2}\sum_{t\neq 2}^{K}\mathbf{L}_{2t}\mathbf{e}_{t1} & \mathbf{V}_{c,2}\sum_{t\neq 2}^{K}\mathbf{L}_{2t}\mathbf{e}_{t2} & \cdots & \mathbf{V}_{c,2}\sum_{t\neq 2}^{K}\mathbf{L}_{2t}\mathbf{e}_{tk} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\sum_{t\neq k}^{K}\mathbf{L}_{kt}\mathbf{e}_{t1} & \mathbf{V}_{c,k}\sum_{t\neq k}^{K}\mathbf{L}_{kt}\mathbf{e}_{t2} & \cdots & \mathbf{V}_{c,k}\sum_{t\neq k}^{K}\mathbf{L}_{kt}\mathbf{e}_{tk} \end{bmatrix}$$

Insert equation (4) in (2) and retain direct flows only on the left side:

$$\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil} = \mathbf{V_c}\mathbf{L}\mathbf{F} + \mathbf{V_c}\mathbf{L}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil} - \mathbf{V_c}\hat{\mathbf{x}}_{(KN\times K)}$$

Take the off-diagonal block elements:

$$\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil} = \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right] + \mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right] \tag{5}$$

Equation (5) therefore provides for the decomposition of domestic value added in direct bilateral exports. The last term $\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]$ represents a matrix of domestic value added exported indirectly from sector *i* in country *r* to country *s*.

Next, a more labourious procedure will result in a matrix of foreign value added in direct bilateral exports. In reverse order, the construction of this KN×K matrix is as follows:

$$\begin{bmatrix} \mathbf{0} & \left[\sum_{t\neq 1}^{K}\mathbf{V_{c,t}}\mathbf{L_{t1}}\right]\mathbf{e_{12}} & \cdots & \left[\sum_{t\neq 1}^{K}\mathbf{V_{c,t}}\mathbf{L_{t1}}\right]\mathbf{e_{13}} \\ \left[\sum_{t\neq 2}^{K}\mathbf{V_{c,t}}\mathbf{L_{t2}}\right]\mathbf{e_{21}} & \mathbf{0} & \cdots & \left[\sum_{t\neq 2}^{K}\mathbf{V_{c,t}}\mathbf{L_{t2}}\right]\mathbf{e_{23}} \\ \vdots & \vdots & \ddots & \vdots \\ \left[\sum_{t\neq k}^{K}\mathbf{V_{c,t}}\mathbf{L_{tk}}\right]\mathbf{e_{k1}} & \left[\sum_{t\neq k}^{K}\mathbf{V_{c,t}}\mathbf{L_{tk}}\right]\mathbf{e_{k2}} & \cdots & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \sum_{t\neq 1}^{K}\mathbf{V_{c,t}}\mathbf{L_{t1}} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \sum_{t\neq 2}^{K}\mathbf{V_{c,t}}\mathbf{L_{t2}} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \sum_{t\neq k}^{K}\mathbf{V_{c,t}}\mathbf{L_{tk}} \end{bmatrix}\mathbf{E}_{bil} =$$

$$= diag\left(\begin{bmatrix}\hat{\mathbf{v}}_{\mathbf{c,1}} & \hat{\mathbf{v}}_{\mathbf{c,2}} & \cdots & \hat{\mathbf{v}}_{\mathbf{c,k}}\end{bmatrix}\begin{bmatrix} 0 & \mathbf{L_{12}} & \cdots & \mathbf{L_{1k}} \\ \mathbf{L_{21}} & 0 & \cdots & \mathbf{L_{2k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{L_{k1}} & \mathbf{L_{k2}} & \cdots & 0 \end{bmatrix}\right)\mathbf{E}_{bil} = \left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil}$$

where $\mathbf{V_{c}}_{(N\times KN)} = [\hat{\mathbf{v}}_{\mathbf{c,1}} \quad \hat{\mathbf{v}}_{\mathbf{c,2}} \quad \cdots \quad \hat{\mathbf{v}}_{\mathbf{c,k}}]$ and $\hat{\mathbf{v}}_{\mathbf{c,s}}$ are N×N diagonalised vectors of value added coefficients as in previous formulations.

$\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil}$ is analogous to $\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil}$, but captures all value added other than domestic in direct bilateral trade flows. The sum of domestic and foreign value added in direct exports is equal to total gross exports (provided that $\mathbf{v} = \mathbf{x'} - \mathbf{i'Z}$), but this only holds for the aggregated gross exports. As is known, sectoral value added in exports is not equal to sectoral gross exports because of the inter-sectoral transfer of value added throughout the production process. So in the KN×K or [country/sector]×country dimension, the sum of $\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil}$ and $\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil}$ will not yield gross sectoral exports. To attain an identity with gross exports at the sectoral level, we will need

a correcting term that accounts for the difference between gross sectoral exports and sectoral value added in exports:

$$\mathbf{E}_{bil} - \left(\mathbf{V_c} \hat{\mathbf{L}} \mathbf{E}_{bil} + \left[\mathbf{V}_{\mathbf{c}(N\times KN)} \check{\mathbf{L}} \right] \mathbf{E}_{bil} \right) = \mathbf{E}_{bil} - \left[\mathbf{V}_{\mathbf{c}(N\times KN)} \hat{\mathbf{L}} \right] \mathbf{E}_{bil} = \left(\mathbf{I} - \left[\mathbf{V}_{\mathbf{c}(N\times KN)} \hat{\mathbf{L}} \right] \right) \mathbf{E}_{bil} \qquad (6)$$

The following formulation allows one to see that this is a matrix of the inter-sectoral transfer of value added created in sector *i* of the exporting country *r*:

$$\left(\mathbf{I} - \left[\mathbf{V}_{\mathbf{c}(N\times KN)} \hat{\mathbf{L}} \right] \right) \mathbf{E}_{bil} = \left(\left[\mathbf{i}' \hat{\mathbf{V}}_{\mathbf{c}} \mathbf{L} \right] - \left[\mathbf{V}_{\mathbf{c}(N\times KN)} \hat{\mathbf{L}} \right] \right) \mathbf{E}_{bil} \qquad (7)$$

Note that (7) holds if $\mathbf{i}'\mathbf{V_c}\mathbf{L} = \mathbf{i}'$, or $\mathbf{v} = \mathbf{x}' - \mathbf{i}'\mathbf{Z}$. Appendix C contains a more detailed discussion of this matrix for the interested reader.

Finally, add the matrix of foreign value added in direct bilateral exports and the correcting term to both sides of equation (5):

$$\mathbf{V_c} \hat{\mathbf{L}} \mathbf{E}_{bil} + \left[\mathbf{V}_{\mathbf{c}(N\times KN)} \check{\mathbf{L}} \right] \mathbf{E}_{bil} + \left(\mathbf{I} - \left[\mathbf{V}_{\mathbf{c}(N\times KN)} \hat{\mathbf{L}} \right] \right) \mathbf{E}_{bil} =$$

$$= \left[\mathbf{V_c} \check{\mathbf{L}} \mathbf{F} \right] + \mathbf{V_c} \check{\mathbf{L}} \mathbf{E}_{tot} - \left[\mathbf{V_c} \check{\mathbf{L}} \mathbf{E}_{bil} \right] + \left[\mathbf{V}_{\mathbf{c}(N\times KN)} \check{\mathbf{L}} \right] \mathbf{E}_{bil} + \left(\mathbf{I} - \left[\mathbf{V}_{\mathbf{c}(N\times KN)} \hat{\mathbf{L}} \right] \right) \mathbf{E}_{bil}$$

The left side adds up to gross sectoral exports:

$$\mathbf{E}_{bil} = \left[\mathbf{V_c} \check{\mathbf{L}} \mathbf{F} \right] + \mathbf{V_c} \check{\mathbf{L}} \mathbf{E}_{tot} - \left[\mathbf{V_c} \check{\mathbf{L}} \mathbf{E}_{bil} \right] + \left[\mathbf{V}_{\mathbf{c}(N\times KN)} \check{\mathbf{L}} \right] \mathbf{E}_{bil} + \left(\mathbf{I} - \left[\mathbf{V}_{\mathbf{c}(N\times KN)} \hat{\mathbf{L}} \right] \right) \mathbf{E}_{bil} \qquad (8)$$

Let us jump for a while into the K×K or country×country dimension to see that the correcting term then disappears. Construct a sector-wise aggregation matrix $\mathbf{S_n}$:

$$\mathbf{S}_n = \begin{bmatrix} \mathbf{i} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{i} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{i} \end{bmatrix}$$

where $\mathbf{i}$ are N×1 summation vectors. Aggregate with respect to the exporting sectors. Since the columns of the value added multiplier matrix $\mathbf{V_c}\mathbf{L}$ theoretically add up to one, the last term in (8) becomes zero:

$$\mathbf{S}_n' \mathbf{E}_{bil} = \mathbf{S}_n' \left[\mathbf{V_c} \check{\mathbf{L}} \mathbf{F} \right] + \mathbf{S}_n' \mathbf{V_c} \check{\mathbf{L}} \mathbf{E}_{tot} - \mathbf{S}_n' \left[\mathbf{V_c} \check{\mathbf{L}} \mathbf{E}_{bil} \right] + \mathbf{S}_n' \left[\mathbf{V}_{\mathbf{c}(N\times KN)} \check{\mathbf{L}} \right] \mathbf{E}_{bil} \qquad (9)$$

which translates our basic relationship into a decomposition of bilateral gross exports at country level, in a way largely similar to Koopman *et al.* (2012) and Stehrer (2013). The components can

now be expressed as ratios (rather than shares) to gross exports and some of these ratios may well exceed 1. This formulation also links various measures known from the literature on gross exports accounting or vertical specialisation in their monetary form: the first term on the right side is the bilateral VAX measure, the second is VS1, and the fourth is VS. The third term may be treated as a "reversed VS1" because it represents the exporter's value added that flows from third countries to partners, i.e. in a direction that is opposite to VS1. So these four measures from (9) add up to the aggregate gross exports:

GROSS EXPORTS = VAX + VS1 – "reversed VS1" + VS

The next subsection will discuss a more profound decomposition of the bilateral value added flows using the off-diagonal block elements of the matrices involved. However, diagonal elements will also be useful to explain certain properties of individual terms.

3.3. The itemised accounting of the bilateral value added flows

It is possible to split $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right]$ and $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}\right]$ terms from the basic accounting relationship (3) into various components. For clarity of notation, start with the same terms from (2), i.e. before removing diagonal block elements. Then the "trade in value added" matrix $\mathbf{V_c}\mathbf{L}\mathbf{F}$ can be expressed as follows:

$$\mathbf{V_c}\mathbf{L}\mathbf{F} = \mathbf{V_c}\hat{\mathbf{L}}\mathbf{F} + \mathbf{V_c}\mathbf{L}\hat{\mathbf{F}} + \left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{F}} - \mathbf{V_c}\hat{\mathbf{L}}\hat{\mathbf{F}}\right]$$

Then, remove diagonal block elements:

$$\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right] = \left[\mathbf{V_c}\check{\hat{\mathbf{L}}}\mathbf{F}\right] + \left[\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}}\right] + \left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{F}} - \mathbf{V_c}\hat{\mathbf{L}}\hat{\mathbf{F}}\right] = \mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}} + \mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}} + \left[\mathbf{V_c}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right] \qquad (10)$$

The resulting terms on the right side need careful interpretation. The first term $\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}}$ captures the value added that originates in sector *i* of country *r* and is embodied in products made in country *r* for final demand in country *s*. The second term $\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}}$ captures the value added that originates in sector *i* of country *r* and is embodied in products made in country *s* for final demand in country *s*. The third term $\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right]$ captures the value added that originates in sector *i* of country *r* and is embodied in products made in third countries for final demand in country *s*. The principal distinction between these terms is therefore in the place where intermediate products are transformed into final products: in the exporting country *r*, partner country *s* or third countries. Note that only the first term $\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}}$ is in fact an export flow of final products while two other terms

represent exports of intermediates that are finally absorbed in the partner country. Appendix B contains zoom in views on these matrices.

Another manipulation will discern two components in $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}\right]=\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}$ from equation (3) that is the matrix of domestic value added re-exported by trading partners. That can be more conveniently handled after removing diagonal block elements:

$$\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}=\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}+\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}-\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right] \tag{11}$$

The first term, $\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}$, is a matrix of value added that originates in sector *i* of the exporting country *r* and returns home via gross exports from the partner country *s* ("reflected value added"):

$$\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}=\begin{bmatrix}\mathbf{V_{c,1}} & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & \mathbf{V_{c,2}} & \cdots & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{V_{c,k}}\end{bmatrix}\begin{bmatrix}\mathbf{0} & \mathbf{L_{12}} & \cdots & \mathbf{L_{1k}}\\ \mathbf{L_{21}} & \mathbf{0} & \cdots & \mathbf{L_{2k}}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{L_{k1}} & \mathbf{L_{k2}} & \cdots & \mathbf{0}\end{bmatrix}\circ\begin{bmatrix}\mathbf{0} & \mathbf{e_{21}} & \cdots & \mathbf{e_{k1}}\\ \mathbf{e_{12}} & \mathbf{0} & \cdots & \mathbf{e_{k2}}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{e_{1k}} & \mathbf{e_{2k}} & \cdots & \mathbf{0}\end{bmatrix}=$$

$$=\begin{bmatrix}\mathbf{0} & \mathbf{V_{c,1}L_{12}e_{21}} & \cdots & \mathbf{V_{c,1}L_{1k}e_{k1}}\\ \mathbf{V_{c,2}L_{21}e_{12}} & \mathbf{0} & \cdots & \mathbf{V_{c,2}L_{2k}e_{k2}}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{V_{c,k}L_{k1}e_{1k}} & \mathbf{V_{c,k}L_{k2}e_{2k}} & \cdots & \mathbf{0}\end{bmatrix}$$

Note again that $\circ$ and $'$ signify, respectively, block-by-block multiplication and block-by block transposition.

The second term in (11), $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}-\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]$, is a matrix of value added that originates in sector *i* of the exporting country *r* and is re-exported by partner country *s* to third countries ("redirected value added").

Finally, compile a new equation for a more profound decomposition of bilateral domestic value added in trade flows from (3), (10) and (11):

$$\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}\right]=\mathbf{V_c}\hat{\check{\mathbf{L}}}\check{\mathbf{F}}+\mathbf{V_c}\check{\mathbf{L}}\hat{\check{\mathbf{F}}}+\left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{F}}\right]+\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}+\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}-\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right] \tag{12}$$

Further decomposition is possible using that $\mathbf{E}_{bil}=\check{\mathbf{F}}+\check{\mathbf{Z}}_{(KN\times K)}$ and $\mathbf{E}_{tot}=\left[\hat{\check{\mathbf{F}}}\mathbf{i}\right]+\left[\hat{\check{\mathbf{Z}}}\mathbf{i}\right]$. However, we don't go beyond this point as the two last terms in (12) are double counted terms and

their present form is perhaps sufficient for a general analysis of global value chains. The interested reader can decompose these terms and see the result.

The next exercise will repeat the steps in equations (5) to (8), that is removing bilateral value added in indirect exports and adding up foreign value added in direct exports, to obtain an itemised accounting formulation for gross exports.

$$\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil} = \mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}} + \mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}} + \left[\mathbf{V_c}\check{\hat{\mathbf{L}}}\check{\mathbf{F}}\right] + \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil} + \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right] - \left[\mathbf{V_c}\check{\hat{\mathbf{L}}}\mathbf{E}_{bil}\right] \quad (13)$$

$$\begin{aligned}\mathbf{E}_{bil} &= \mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}} + \mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}} + \left[\mathbf{V_c}\check{\hat{\mathbf{L}}}\check{\mathbf{F}}\right] + \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil} + \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right] - \left[\mathbf{V_c}\check{\hat{\mathbf{L}}}\mathbf{E}_{bil}\right] + \\ &+ \left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{}\check{\mathbf{L}}\right]\mathbf{E}_{bil} + \left(\mathbf{I} - \left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{}\mathbf{L}\right]\right)\mathbf{E}_{bil}\end{aligned} \quad (14)$$

Similarly to (11), the matrix of foreign value added may be further decomposed given that $\mathbf{E}_{bil} = \check{\mathbf{F}} + \check{\mathbf{Z}}_{(KN\times K)}$. The last term in (14), again, accounts for the inter-sectoral transfer of value added from the sector of origin throughout the production process on the exporting country's side. The sector-wise aggregation to the K×K or country×country dimension will drop this term:

$$\begin{aligned}\mathbf{S}'_n\mathbf{E}_{bil} &= \mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}} + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}} + \mathbf{S}'_n\left[\mathbf{V_c}\check{\hat{\mathbf{L}}}\check{\mathbf{F}}\right] + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil} + \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right] - \\ &- \mathbf{S}'_n\left[\mathbf{V_c}\check{\hat{\mathbf{L}}}\mathbf{E}_{bil}\right] + \mathbf{S}'_n\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{}\check{\mathbf{L}}\right]\mathbf{E}_{bil}\end{aligned} \quad (15)$$

Equation (15) yields the results that are identical to those in Stehrer (2013, see equation 9), and the only difference is that in the latter study, double counted terms are split into final and intermediate components using that $\mathbf{E}_{bil} = \check{\mathbf{F}} + \check{\mathbf{Z}}_{(KN\times K)} = \check{\mathbf{F}} + \left[\mathbf{A}\hat{\mathbf{x}}_{(KN\times K)}\check{}\right]$.

The above derivation of the gross exports accounting equation reveals that it is in fact a result of the decomposition of cumulative value added not direct exports flows. That's the reason why bilateral trade between the partner country and third countries – captured by $\left[\mathbf{V_c}\check{\hat{\mathbf{L}}}\check{\mathbf{F}}\right]$, $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]$ and $\left[\mathbf{V_c}\check{\hat{\mathbf{L}}}\mathbf{E}_{bil}\right]$ – appears in this formula for bilateral gross exports which may first seem counter-intuitive (see Figure 5 for a visualisation). This is also the reason why the range of individual components in (14) and (15) expressed as ratios to gross exports is not confined to 0-100%, so a normalisation with respect to gross exports will give ratios rather than shares.

As noted in section 2, for a decomposition of direct bilateral exports into detailed value added components that are bound between 0 and 100% one should use the framework developed in Wang *et al.* (2013).

Lastly, note that the difference between $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]$ and $\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]$ gives the balance of trade in the exporting country's value added between the partner country and third countries. The next subsection will reveal that the difference between these terms equals zero after aggregating across partner countries.

3.4. Aggregation across partner countries: the decomposition of value added in total exports

As noted earlier, row sums of $\mathbf{V_c}\mathbf{L}\mathbf{E}_{bil}$, or the aggregation across partner countries, equal domestic value added in total gross exports of country *r*, inclusive of reflected value added given by the diagonal block elements. Removing the reflected value added results in $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}\right]$. At the sectoral level, the aggregation of this matrix leads to a decomposition of value added that originates in sector *i* of country *r* and is sent to all partner countries via direct and indirect exports. Below, the aggregation results are shown for both basic accounting equation (3) and the itemised version (12):

$$\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}\right]\mathbf{i} = \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right]\mathbf{i} + \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}\right]\mathbf{i} \tag{16}$$

$$\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}\right]\mathbf{i} = \mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}}\mathbf{i} + \mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}}\mathbf{i} + \left[\mathbf{V_c}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right]\mathbf{i} + \left[\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i} + \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i} \tag{17}$$

where **i** is a K×1 summation vector. The interpretation of the individual terms is similar to (3) and (12), with respect to all trading partners. Pre-multiplication by the sector-wise aggregation matrix $\mathbf{S}'$ will yield a country level decomposition.

The same type of aggregation applies to equations (8) and (14) to express sectoral total gross exports as a sum of value added components. However, the aggregation at the country level, based on equations (9) and (15), is of particular interest. The condensed form is:

$$\mathbf{S}'_n\mathbf{E}_{bil}\mathbf{i} = \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right]\mathbf{i} + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}\mathbf{i} - \mathbf{S}'_n\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i} + \mathbf{S}'_n\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil}\mathbf{i} \tag{18}$$

And the itemised form is given by:

$$\mathbf{S}'_n\mathbf{E}_{bil}\mathbf{i} = \mathbf{S}'_n\mathbf{V}_\mathbf{c}\hat{\mathbf{L}}\check{\mathbf{F}}\mathbf{i} + \mathbf{S}'_n\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\hat{\mathbf{F}}\mathbf{i} + \mathbf{S}'_n\left[\mathbf{V}_\mathbf{c}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right]\mathbf{i} + \mathbf{S}'_n\left[\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i} +$$
$$+\mathbf{S}'_n\left[\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V}_\mathbf{c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i} - \mathbf{S}'_n\left[\mathbf{V}_\mathbf{c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i} + \mathbf{S}'_n\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil}\mathbf{i} \tag{19}$$

The fifth term in the above equation $\mathbf{S}'_n\left[\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V}_\mathbf{c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i}$ is country *r*'s value added re-exported by partner country *s* to third countries. The sixth term $\mathbf{S}'_n\left[\mathbf{V}_\mathbf{c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i}$ is country *r*'s value added indirectly exported via third countries to partner country *s*. At the aggregate country level, these terms equal each other, and hence the balance of trade in exporting country *r*'s value added among all partners is zero (for an explicit proof, see Appendix E). So these terms eventually vanish from the final version of this equation:

$$\mathbf{S}'_n\mathbf{E}_{bil}\mathbf{i} = \mathbf{S}'_n\mathbf{V}_\mathbf{c}\hat{\mathbf{L}}\check{\mathbf{F}}\mathbf{i} + \mathbf{S}'_n\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\hat{\mathbf{F}}\mathbf{i} + \mathbf{S}'_n\left[\mathbf{V}_\mathbf{c}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right]\mathbf{i} + \mathbf{S}'_n\left[\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i} + \mathbf{S}'_n\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil}\mathbf{i} \tag{20}$$

Lastly, let us derive one more remarkable result using an aggregation across partner countries of the equation (5):

$$\mathbf{V}_\mathbf{c}\hat{\mathbf{L}}\mathbf{E}_{bil}\mathbf{i} = \left[\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\mathbf{F}\right]\mathbf{i} + \mathbf{V}_\mathbf{c}\check{\mathbf{L}}\mathbf{E}_{tot}\mathbf{i} - \left[\mathbf{V}_\mathbf{c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i} = \left[\mathbf{V}_\mathbf{c}\check{\mathbf{L}}\mathbf{F}\right]\mathbf{i} + \left[\mathbf{V}_\mathbf{c}\hat{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i} \tag{21}$$

This equation may also be derived from (20) by subtracting the matrix of foreign value added from both parts or by taking diagonal block elements from (2) followed by a simple rearrangement. Read this equation as follows: domestic value added in total direct exports equals total exported domestic value added plus the sum of all reflected exports.

3.5. Aggregation across exporting countries: the decomposition of value added in total imports

The columns of $\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{E}_{bil}$ sum to total imports of country *s*. The reader can confirm this by looking at the structure of the "bilateral value added in trade matrix" or using the known property of the value added multiplier matrix: $\mathbf{i}'\mathbf{V}_\mathbf{c}\mathbf{L} = \mathbf{i}'$ and $\mathbf{i}'\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{E}_{bil} = \mathbf{i}'\mathbf{E}_{bil}$. Then the aggregation across all exporting countries, or summing the columns of $\mathbf{V}_\mathbf{c}\mathbf{L}\mathbf{E}_{bil}$ and its components will provide a decomposition of total imports of country *s*.

To obtain a workable formulation, first split $\mathbf{V_c L E}_{bil}$ into diagonal and off-diagonal block elements as in (4) and aggregate the diagonal block elements across exporting countries' sectors using the aggregation matrix $\mathbf{S}_n$. So, first aggregate (13):

$$\mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil} = \mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}} + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}} + \mathbf{S}'_n\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right] + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil} +$$

$$+\mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right] - \mathbf{S}'_n\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]$$

Then note that $\mathbf{S}'_n\left(\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil} + \mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}\right) = \mathbf{S}'_n\mathbf{V_c L E}_{bil}$ and insert $\mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}$ in both sides of the previous equation to obtain:

$$\mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil} + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil} = \mathbf{S}'_n\mathbf{V_c L E}_{bil} = \mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}} + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}} + \mathbf{S}'_n\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right] + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil} +$$

$$+\mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right] - \mathbf{S}'_n\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right] + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}$$

Finally, using that $\mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil} - \mathbf{S}'_n\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right] = \mathbf{S}'_n\left[\mathbf{V_c}\hat{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]$ rewrite this equation as follows:

$$\begin{aligned}\mathbf{S}'_n\mathbf{V_c L E}_{bil} &= \mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}} + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}} + \mathbf{S}'_n\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right] + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil} + \\ &+\mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right] + \mathbf{S}'_n\left[\mathbf{V_c}\hat{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]\end{aligned} \tag{22}$$

In a condensed form the same equation is:

$$\mathbf{S}'_n\mathbf{V_c L E}_{bil} = \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right] + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} + \mathbf{S}'_n\left[\mathbf{V_c}\hat{\check{\mathbf{L}}}\mathbf{E}_{bil}\right] \tag{23}$$

Further aggregation into 1×K dimension leads to a decomposition of total gross imports. For notational simplicity, a 1×KN summation vector $\mathbf{i}'$ is then used to aggregate across both countries and sectors:

$$\mathbf{i}'\mathbf{V_c}\mathbf{L}\mathbf{E}_{bil} = \mathbf{i}'\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}} + \mathbf{i}'\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}} + \mathbf{i}'\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right] + \mathbf{i}'\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil} +$$
$$+\mathbf{i}'\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right] + \mathbf{i}'\left[\mathbf{V_c}\hat{\check{\mathbf{L}}}\mathbf{E}_{bil}\right] \tag{24}$$

In the itemised form above, 1×K vectors on the right side depict the use of foreign value added by the importing country *s* for domestic consumption (first three terms), re-exports back to the partners (fourth term), re-exports to third countries (fifth term) and importing country's value added that was first used by the exporting country and finally returns home (sixth term).

Grouping the terms will result in a condensed form:

$$\mathbf{i}'\mathbf{V_c}\mathbf{L}\mathbf{E}_{bil} = \mathbf{i}'\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right] + \mathbf{i}'\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} + \mathbf{i}'\left[\mathbf{V_c}\hat{\check{\mathbf{L}}}\mathbf{E}_{bil}\right] \tag{25}$$

3.6. A note on total and bilateral trade balances

Recall that the columns of $\mathbf{V_c}\mathbf{L}\mathbf{E}_{tot}$ sum to total exports of country *s* (and also value added in total exports, provided again that $\mathbf{i}'\mathbf{V_c}\mathbf{L} = \mathbf{i}'$ or $\mathbf{v} = \mathbf{x}' - \mathbf{i}'\mathbf{Z}$) whereas the columns of $\mathbf{V_c}\mathbf{L}\mathbf{E}_{bil}$ sum to total imports (also value added in total imports) of *s*. Also note that the diagonal block elements which are the only elements in the columns of $\mathbf{V_c}\hat{\mathbf{x}}_{(KN\times K)}$ can be interpreted as the sum of value added generated in country *s* by both domestic and foreign final demand (which is equal to the sum across the rows of $\mathbf{V_c}\mathbf{L}\mathbf{F}$). And the columns of $\mathbf{V_c}\mathbf{L}\mathbf{F}$ sum to total value added absorbed in country *s*. Then the basic equation (2) can be simply rearranged in 1×K dimension to show:

$$\mathbf{i}'\mathbf{V_c}\mathbf{L}\mathbf{E}_{tot} - \mathbf{i}'\mathbf{V_c}\mathbf{L}\mathbf{E}_{bil} = \mathbf{i}'\mathbf{V_c}\hat{\mathbf{x}}_{(KN\times K)} - \mathbf{i}'\mathbf{V_c}\mathbf{L}\mathbf{F}$$

or

$$\mathbf{i}'\mathbf{E}_{tot} - \mathbf{i}'\mathbf{E}_{bil} = \mathbf{i}'\mathbf{V_c}\hat{\mathbf{x}}_{(KN\times K)} - \mathbf{i}'\mathbf{V_c}\mathbf{L}\mathbf{F} \tag{26}$$

The left side of the equation (26) is a difference between the total gross exports and total gross imports of country *s*, or a 1×K vector of trade balances in gross terms. Likewise, the right side gives the difference between total value added generated and total value added absorbed in country s, i.e. a 1×K vector of trade balances in value added terms. This completes the proof of the equality of trade balances in gross and net terms and succinctly confirms the earlier results of Stehrer (2012, 2013) and Kuboniwa (2014b, 2014c).

Bilateral trade balances in K×K matrix form can be calculated as differences between relevant bilateral matrices and their transposes. For gross trade balances this can be expressed as:

$$\mathbf{S}'_n\mathbf{E}_{bil} - \left(\mathbf{S}'_n\mathbf{E}_{bil}\right)' = \mathbf{S}'_n\mathbf{E}_{bil} - \mathbf{E}'_{bil}\mathbf{S}_n \tag{27}$$

Using (2), (4) and (9) gross trade balance can be decomposed into various components:

$$\mathbf{S}_n'\mathbf{E}_{bil} - \left(\mathbf{S}_n'\mathbf{E}_{bil}\right)' = \left[\mathbf{S}_n'\mathbf{V_c}\mathbf{LF} - \left(\mathbf{S}_n'\mathbf{V_c}\mathbf{LF}\right)'\right] + \left[\mathbf{S}_n'\mathbf{V_c}\mathbf{LE}_{tot} - \left(\mathbf{S}_n'\mathbf{V_c}\mathbf{LE}_{tot}\right)'\right] -$$

$$-\left[\mathbf{S}_n'\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil} - \left(\mathbf{S}_n'\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}\right)'\right] + \left[\mathbf{S}_n'\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\check{\mathbf{L}}}\right]\mathbf{E}_{bil} - \left(\mathbf{S}_n'\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\check{\mathbf{L}}}\right]\mathbf{E}_{bil}\right)'\right]$$

The matrix of bilateral balances of value added in gross trade can be computed as:

$$\mathbf{S}_n'\mathbf{V_c}\mathbf{LE}_{bil} - \left(\mathbf{S}_n'\mathbf{V_c}\mathbf{LE}_{bil}\right)' = \mathbf{S}_n'\mathbf{V_c}\mathbf{LE}_{bil} - \mathbf{E}_{bil}'\mathbf{L}'\mathbf{V_c}\mathbf{S}_n \tag{28}$$

or in the decomposed form,

$$\mathbf{S}_n'\mathbf{V_c}\mathbf{LE}_{bil} - \left(\mathbf{S}_n'\mathbf{V_c}\mathbf{LE}_{bil}\right)' = \left[\mathbf{S}_n'\mathbf{V_c}\mathbf{LF} - \left(\mathbf{S}_n'\mathbf{V_c}\mathbf{LF}\right)'\right] + \left[\mathbf{S}_n'\mathbf{V_c}\mathbf{LE}_{tot} - \left(\mathbf{S}_n'\mathbf{V_c}\mathbf{LE}_{tot}\right)'\right]$$

Finally, the matrix of bilateral balances of trade in value added is:

$$\mathbf{S}_n'\mathbf{V_c}\mathbf{LF} - \left(\mathbf{S}_n'\mathbf{V_c}\mathbf{LF}\right)' = \mathbf{S}_n'\mathbf{V_c}\mathbf{LF} - \mathbf{F}'\mathbf{L}'\mathbf{V_c}\mathbf{S}_n \tag{29}$$

It is therefore evident that these three types of bilateral trade balances for a pair of countries are not equal unless under very special conditions. For example, gross trade balance (27) equals balance of trade in value added (29) only if the sum of respective elements in the matrices of bilateral balances of domestic re-exported value added, domestic re-directed value added and foreign value added is zero.

3.7. Extension to the KN×KN and K×KN dimensions and the "value added at destination" concept

The discussion has so far focused on the decomposition of value added flows that originate in sector *i* of exporting country *r* and "end up" or "land" in partner country *s*. It therefore attributed all value added component flows to their country/sector of origin. As briefly outlined in subsection 3.2, this decomposition in the KN×K dimension implicitly relates to the "value added at origin" concept. A reasonable question is whether an extension to the KN×KN dimension and then an aggregation to the K×KN dimension is possible, to obtain a "value added at destination" decomposition? That would capture all value added created in country *r* embodied in products of sector *j* consumed or re-exported by partner country *s*. The answer is definitely positive, but such change of perspective is not a trivial exercise and requires an extension to our "minimal setup".

Koopman *et al.* (2010) first propose to aggregate across exporting country sectors and disaggregate the partner country sectors in a matrix similar to the "value added in total trade" $\mathbf{V_c}\mathbf{LE}_{tot}$ matrix in this paper. They treat it as a "sectoral measure of value-added trade in global value chains" (see formula 12 in Koopman *et al.*, 2010 for the two-country case). Meng *et al.*

(2012) briefly discuss similar type of disaggregation applied to their sectoral "trade in value added" measure that they use to derive alternative, TiVA-based version of revealed comparative advantage indicators (equations 12-13 in Meng *et al.*, 2012).

A systematic effort to sort out the implicit aggregation in previously used measures can be found in Wang *et al.* (2013). They propose a distinction between a "forward-linkage based measure" which includes indirect exports of a sector's value added via gross exports from other sectors of the same exporting country, and a "backward-linkage based measure" which is value added from all sectors of a given exporting country embodied in a given sector's gross exports. In other words, their "forward-linkage based measure" treats a sector as a source of value added while the "backward-linkage based measure" treats a sector as a recipient of value added. The terms Wang and his co-authors propose may be confusing because "forward" linkages in input-output analysis usually relate to Ghosh model. So in this paper, related terms, similar to those of Wang *et al.* (2013) in concept yet different in computation, are called "value added at origin" and "value added at destination".

First, convert **F**, $\mathbf{E}_{tot}$ and $\mathbf{E}_{bil}$ matrices from the KN×K to the KN×KN dimension:

$$\mathbf{F}_{(KN\times KN)} = \begin{bmatrix} \hat{\mathbf{f}}_{\mathbf{11}} & \hat{\mathbf{f}}_{\mathbf{12}} & \cdots & \hat{\mathbf{f}}_{\mathbf{1k}} \\ \hat{\mathbf{f}}_{\mathbf{21}} & \hat{\mathbf{f}}_{\mathbf{22}} & \cdots & \hat{\mathbf{f}}_{\mathbf{2k}} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\mathbf{f}}_{\mathbf{k1}} & \hat{\mathbf{f}}_{\mathbf{k2}} & \cdots & \hat{\mathbf{f}}_{\mathbf{kk}} \end{bmatrix},$$

$$\mathbf{E}_{tot\,(KN\times KN)} = \begin{bmatrix} \hat{\mathbf{e}}_{\mathbf{1}} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{e}}_{\mathbf{2}} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \hat{\mathbf{e}}_{\mathbf{k}} \end{bmatrix}, \qquad \mathbf{E}_{bil\,(KN\times KN)} = \begin{bmatrix} \mathbf{0} & \hat{\mathbf{e}}_{\mathbf{12}} & \cdots & \hat{\mathbf{e}}_{\mathbf{1k}} \\ \hat{\mathbf{e}}_{\mathbf{21}} & \mathbf{0} & \cdots & \hat{\mathbf{e}}_{\mathbf{2k}} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\mathbf{e}}_{\mathbf{k1}} & \hat{\mathbf{e}}_{\mathbf{k2}} & \cdots & \mathbf{0} \end{bmatrix}$$

The matrix of diagonalised total output that has been used for deriving the basic accounting relationship, is constructed as follows:

$$\hat{\mathbf{x}}_{(KN\times KN)} = \begin{bmatrix} \hat{\mathbf{x}}_{\mathbf{1}} & 0 & \cdots & 0 \\ 0 & \hat{\mathbf{x}}_{\mathbf{2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{\mathbf{x}}_{\mathbf{k}} \end{bmatrix}$$

The above conversion of **F**, $\mathbf{E}_{tot}$ and $\mathbf{E}_{bil}$ and **x** is for computational purpose only, to keep the sectoral dimension of results, and does not involve a meaningful interpretation.

Now $\mathbf{Z}$, $\mathbf{F}_{(KN\times KN)}$, $\mathbf{E}_{tot(KN\times KN)}$, $\mathbf{E}_{bil(KN\times KN)}$ and $\hat{\mathbf{x}}_{(KN\times KN)}$ are all KN×KN matrices. Owing to the above specification, all blocks in $\mathbf{F}_{(KN\times KN)}$, $\mathbf{E}_{tot(KN\times KN)}$, $\mathbf{E}_{bil(KN\times KN)}$ and $\hat{\mathbf{x}}_{(KN\times KN)}$ contain either diagonal elements only or zeros except $\mathbf{Z}$ where blocks contain nonnegative values in all or many of the elements. For the equation (1) to hold, we'll need one more matrix that would account for the presence of the off-diagonal elements in each block of $\mathbf{Z}$. Appendix D discusses in detail the construction of this matrix of the inter-sectoral transfer of value added that is denoted as $\mathbf{Z}^*$. Then the equation (1) in KN×KN dimension is as follows:

$$\mathbf{E}_{bil\,(KN\times KN)} - \mathbf{E}_{tot\,(KN\times KN)} = \mathbf{F}_{(KN\times KN)} + \mathbf{Z} - \hat{\mathbf{x}}_{(KN\times KN)} - \mathbf{Z}^* \tag{30}$$

The same manipulation applies as in subsection 3.2, and the difference is that one more term appears on the right side:

$$\mathbf{E}_{bil\,(KN\times KN)} - \mathbf{E}_{tot\,(KN\times KN)} = \mathbf{F}_{(KN\times KN)} - \mathbf{L}^{-1}\hat{\mathbf{x}}_{(KN\times KN)} - \mathbf{Z}^*$$

$$\mathbf{V_c L}\left(\mathbf{E}_{bil\,(KN\times KN)} - \mathbf{E}_{tot\,(KN\times KN)}\right) = \mathbf{V_c L}\left(\mathbf{F}_{(KN\times KN)} - \mathbf{L}^{-1}\hat{\mathbf{x}}_{(KN\times KN)} - \mathbf{Z}^*\right)$$

$$\mathbf{V_c L E}_{bil\,(KN\times KN)} = \mathbf{V_c L F}_{(KN\times KN)} + \mathbf{V_c L E}_{tot\,(KN\times KN)} - \mathbf{V_c}\hat{\mathbf{x}}_{(KN\times KN)} - \mathbf{V_c L Z}^* \tag{31}$$

The last term accounts for the inter-sectoral transfer of value added embodied in intermediate products on their way from the sector of origin *i* to the sector of destination *j*. So, this is the inter-sectoral transfer of value added on the partner countries' side.

Finally, removing the diagonal block elements yields the basic accounting relationship in the KN×KN or full [country/sector]×[country/sector] dimension:

$$\left[\mathbf{V_c L}\check{\mathbf{E}}_{bil\,(KN\times KN)}\right] = \left[\mathbf{V_c L}\check{\mathbf{F}}_{(KN\times KN)}\right] + \left[\mathbf{V_c L}\check{\mathbf{E}}_{tot\,(KN\times KN)}\right] - \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{Z}^*\right] \tag{32}$$

All subsequent equations for the value added or gross exports accounting as in subsections 3.2-3.5 in the KN×KN dimension should include the last term from (32). In the first two matrices on the right side of the equation (32), each element should be interpreted as the value added originating in sector *i* of country *r* embodied in product of sector *j* used by country *s* for domestic consumption or re-exports. The last term accounts for the re-allocation of value added originating in sector *i* of country *r* resulting from inter-sectoral flows of intermediates for which country *s* is responsible. Appendix D offers more profound explanation of the meaning of the elements in this re-allocation matrix. The interested reader may also wish to compare this with the $\left(\mathbf{I} - \left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\mathbf{L}\right]\right)\mathbf{E}_{bil}$ matrix of the inter-sectoral transfer of value added on the exporting countries' side. Both are double-counted terms.

Pre-multiplication of (32) and any derivative equation by the sector-wise aggregation matrix $\mathbf{S}'_n$ will condense the results to the K×KN or country×[country/sector] dimension:

$$\mathbf{S}'_n\left[\mathbf{V_c}\mathbf{L}\overset{\vee}{\mathbf{E}}_{bil\,(KN\times KN)}\right]=\mathbf{S}'_n\left[\mathbf{V_c}\mathbf{L}\overset{\vee}{\mathbf{F}}_{(KN\times KN)}\right]+\mathbf{S}'_n\left[\mathbf{V_c}\mathbf{L}\overset{\vee}{\mathbf{E}}_{tot\,(KN\times KN)}\right]-\mathbf{S}'_n\left[\mathbf{V_c}\overset{\vee}{\mathbf{L}}\mathbf{Z}^*\right] \tag{33}$$

where each element in any resulting matrix should be interpreted as the value added originating from (all sectors of) country *r* embodied in product of sector *j* used by country *s* for domestic consumption or re-exports. Again, the re-allocation term accounts for the inter-sectoral value added flows on the partner countries' side.

Note that the aggregation of (32) and (33) across recipient country's sectors, respectively, to the KN×K and K×K dimensions, i.e. post multiplication by $\mathbf{S}_n$, will make the last term equal to zero.

One should also note that a transformation of (32) using the steps in (5-8) into a gross exports accounting equation in the KN×KN dimension will have no meaningful interpretation because of the presence of zeroes in the off-diagonal elements of each block in $\mathbf{E}_{bil(KN\times KN)}$. However, an aggregated version in K×KN dimension can be interpreted in terms of the total value added components embodied in products received at the partners' side.

This completes the discussion on the framework proposed in this paper for the generalised value added in trade accounting. The list of the formulae obtained can be found in the Appendix A and a detailed description and interpretation of the matrices is in the Appendix B. Meanwhile, Figure 5 below visualises bilateral value added flows embodied in direct and indirect exports as discussed in this section. Next section will put the generalised framework to test using real data on production and trade.

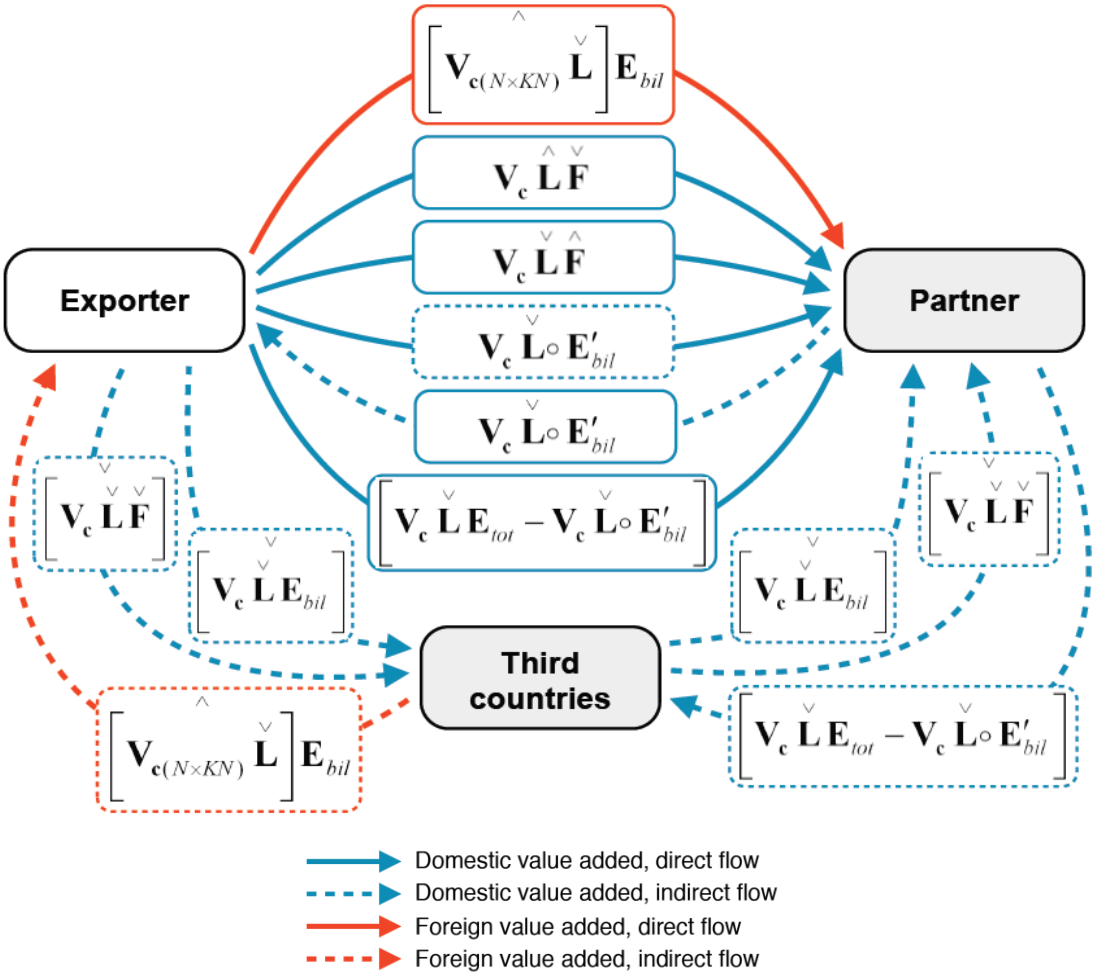

Figure 5. Schematic representation of bilateral value added flows in direct and indirect exports

Note: For brevity, this figure shows matrix terms that correspond to the value added flows in the KN×K dimension. The terms for the inter-sectoral transfer of value added on the exporting country or partner country's side are dropped.

4. Application and results

For an application of the generalised cumulative value added accounting framework, this section focuses on Russia's global and bilateral exports. Previous studies have revealed an extremely high content of domestic value added in Russia's gross exports. As an upstream natural resource producer Russia was found to have a large share of its domestic value added absorbed by direct importers, delivered via intermediate rather than final products. A significant portion of Russia's intermediate exports were also used by other countries to produce their intermediate goods exports (Koopman *et al.*, 2010, 2012; OECD 2013b).

In conventional analysis of international trade, exports and exporters of energy and other natural resources are often isolated or neglected. However, a brief discussion below shows that the natural resource exporters like Russia may be an interesting case for the analysis of global value

chains. In particular, Russia's case will show how the value added generated in a resource-extracting sector is circulated through partners' and third countries' trade, or downstream value chain.

The formulae derived through the previous section are tested here with data from the World Input-Output Database (WIOD). The WIOD database is the outcome of a project funded by the European Commission and implemented by a consortium of 11 international partners. It contains a series of national and inter-country supply, use and input-output tables supplemented by sets of socio-economic and environmental indicators for the period from 1995 to 2011. WIOD covers 27 European Union member states, 13 other major non-European economies plus estimates for the rest of the world and discerns 35 industries based on NACE revision 1 which corresponds to ISIC revision 3.[7]

The following subsections pose some of the questions on Russia's role in global value chains that the discussed framework is capable to address.

4.1. Does the application of alternative accounting methods change perception of countries' relative importance in global and Russia's bilateral trade?

To see whether the relative importance of individual exporting countries in global trade changes when measured with the alternative accounting frameworks, and to estimate the magnitude of such changes, let us consider three concepts:

- "gross exports" – that is usual gross trade statistics,
- "value added in exports" – that is gross trade flows reallocated to the countries of origin of value added contained therein, and
- "exports of value added" – that is gross trade flows reallocated to the countries of origin of value added with the double-counted flows removed.

This involves computing three vectors, $\mathbf{S}_n'\mathbf{E}_{bil}\mathbf{i}$, $\mathbf{S}_n'\left[\mathbf{V}_c\check{\mathbf{L}}\mathbf{E}_{bil}\right]\mathbf{i}$ and $\mathbf{S}_n'\left[\mathbf{V}_c\check{\mathbf{L}}\mathbf{F}\right]\mathbf{i}$, and the shares of each element with respect to the column sums. Note that the reflected exports (diagonal elements) are not accounted for, to conform with the basic accounting relationship (3). Accounting for the reflected exports may be important for such country as the United States, but does not affect the order of the results.

Using these three concepts and WIOD data for 2000, 2005 and 2010, Table 1 ranks world top 20 exporters. The year 2010 seems to introduce the most significant changes when moving from gross to value added measurements. China overtook the United States as the largest exporter in terms of gross exports (10.9% vs. 10.2% of global exports), but lacked behind the United States as

[7] For more information on the WIOD project, see Timmer (2012), Dietzenbacher *et al.* (2013) and the WIOD website www.wiod.org.

the country of origin of value added embodied in those exports (11.1% vs. 11.9%). Finally, China and the United States performed almost equally well in generating net value added for consumption abroad (11.7% of global traded value added, but China actually led with a negligible 0.02% advantage). Similar comparisons for 2000 and 2005 don't affect the top exporter's position, but switch the countries that appear as the second or third top exporters.

Table 1. Twenty largest global exporters and three measurement concepts
(million US$ and percentage of total world exports, current prices)

Gross exports				Domestic value added in all exports				Exports of domestic value added			
#	Exporter	US$ mln	%	#	Exporter	US$ mln	%	#	Exporter	US$ mln	%
					2000						
1.	United States	982,509	14.2	1.	United States	1,046,097	16.3	1.	United States	763,314	15.5
2.	Germany	614,537	8.9	2.	Japan	585,710	9.2	2.	Japan	456,314	9.3
3.	Japan	512,775	7.4	3.	Germany	555,318	8.7	3.	Germany	422,937	8.6
4.	United Kingdom	378,672	5.5	4.	United Kingdom	369,769	5.8	4.	United Kingdom	278,810	5.7
5.	France	349,817	5.1	5.	France	303,855	4.7	5.	France	235,032	4.8
6.	Canada	319,788	4.6	6.	China	277,098	4.3	6.	China	225,223	4.6
7.	China	279,547	4.0	7.	Canada	253,328	4.0	7.	Canada	215,835	4.4
8.	Italy	271,817	3.9	8.	Italy	246,581	3.9	8.	Italy	195,319	4.0
9.	Netherlands	214,498	3.1	9.	Netherlands	163,775	2.6	9.	Netherlands	125,783	2.6
10.	Korea	199,000	2.9	10.	Korea	157,674	2.5	10.	Korea	123,628	2.5
11.	Chinese Taipei	171,724	2.5	11.	Mexico	132,213	2.1	11.	Mexico	110,435	2.2
12.	Mexico	170,880	2.5	12.	Chinese Taipei	132,164	2.1	12.	Chinese Taipei	102,780	2.1
13.	Belgium	161,850	2.3	13.	Russia	121,135	1.9	13.	Spain	93,068	1.9
14.	Spain	140,904	2.0	14.	Spain	118,484	1.9	14.	Russia	84,498	1.7
15.	Sweden	110,833	1.6	15.	Belgium	113,672	1.8	15.	Belgium	83,964	1.7
16.	Russia	98,757	1.4	16.	Australia	93,915	1.5	16.	Australia	73,567	1.5
17.	Australia	90,274	1.3	17.	Sweden	93,400	1.5	17.	Sweden	70,118	1.4
18.	Ireland	88,634	1.3	18.	Austria	70,036	1.1	18.	India	52,883	1.1
19.	Austria	78,443	1.1	19.	Indonesia	65,997	1.0	19.	Austria	52,031	1.1
20.	Denmark	68,027	1.0	20.	India	64,760	1.0	20.	Indonesia	51,951	1.1
					2005						
1.	United States	1,187,011	10.5	1.	United States	1,286,023	12.3	1.	United States	921,964	11.8
2.	Germany	1,096,000	9.7	2.	Germany	979,745	9.4	2.	Germany	732,559	9.4
3.	China	836,719	7.4	3.	Japan	751,602	7.2	3.	China	591,321	7.6
4.	Japan	653,687	5.8	4.	China	746,283	7.1	4.	Japan	559,326	7.1
5.	United Kingdom	547,245	4.8	5.	United Kingdom	559,711	5.3	5.	United Kingdom	411,107	5.3
6.	France	517,610	4.6	6.	France	454,849	4.3	6.	France	345,993	4.4
7.	Italy	428,302	3.8	7.	Italy	393,796	3.8	7.	Italy	302,136	3.9
8.	Canada	416,185	3.7	8.	Canada	355,549	3.4	8.	Canada	295,525	3.8
9.	Netherlands	341,872	3.0	9.	Russia	309,071	3.0	9.	Netherlands	202,278	2.6
10.	Korea	327,910	2.9	10.	Netherlands	274,293	2.6	10.	Korea	197,238	2.5
11.	Belgium	247,066	2.2	11.	Korea	268,302	2.6	11.	Russia	197,180	2.5
12.	Spain	245,986	2.2	12.	Spain	211,901	2.0	12.	Spain	161,834	2.1
13.	Russia	226,895	2.0	13.	Belgium	177,745	1.7	13.	Mexico	142,664	1.8
14.	Chinese Taipei	226,721	2.0	14.	Mexico	176,545	1.7	14.	Belgium	129,379	1.7
15.	Mexico	218,310	1.9	15.	Chinese Taipei	170,747	1.6	15.	Australia	122,701	1.6
16.	Sweden	170,943	1.5	16.	Australia	168,512	1.6	16.	Chinese Taipei	118,523	1.5
17.	Ireland	159,912	1.4	17.	India	147,270	1.4	17.	India	115,326	1.5
18.	India	157,728	1.4	18.	Sweden	145,643	1.4	18.	Sweden	108,006	1.4
19.	Australia	149,343	1.3	19.	Brazil	136,639	1.3	19.	Brazil	102,800	1.3
20.	Austria	144,306	1.3	20.	Austria	122,567	1.2	20.	Austria	89,397	1.1

						2010					
1.	China	1,743,486	10.9	1.	United States	1,767,249	11.9	1.	China	1,300,561	11.7
2.	United States	1,634,458	10.2	2.	China	1,645,795	11.1	2.	United States	1,298,542	11.7
3.	Germany	1,391,739	8.7	3.	Germany	1,195,077	8.0	3.	Germany	903,105	8.1
4.	Japan	835,356	5.2	4.	Japan	918,590	6.2	4.	Japan	690,182	6.2
5.	United Kingdom	617,535	3.9	5.	United Kingdom	620,217	4.2	5.	United Kingdom	452,148	4.1
6.	France	609,074	3.8	6.	Russia	528,076	3.6	6.	France	394,262	3.6
7.	Korea	519,545	3.2	7.	France	513,184	3.5	7.	Italy	346,051	3.1
8.	Italy	514,168	3.2	8.	Italy	444,477	3.0	8.	Canada	337,358	3.0
9.	Netherlands	468,328	2.9	9.	Canada	422,439	2.8	9.	Russia	330,619	3.0
10.	Canada	449,279	2.8	10.	Korea	386,228	2.6	10.	Korea	287,267	2.6
11.	Russia	371,743	2.3	11.	Netherlands	352,855	2.4	11.	Netherlands	258,148	2.3
12.	Belgium	322,585	2.0	12.	Australia	323,310	2.2	12.	Australia	229,948	2.1
13.	Spain	322,167	2.0	13.	Spain	276,740	1.9	13.	India	220,449	2.0
14.	Chinese Taipei	311,633	1.9	14.	India	275,697	1.9	14.	Spain	212,852	1.9
15.	India	308,576	1.9	15.	Brazil	248,603	1.7	15.	Mexico	185,057	1.7
16.	Mexico	286,285	1.8	16.	Mexico	231,554	1.6	16.	Brazil	183,369	1.7
17.	Australia	273,733	1.7	17.	Chinese Taipei	222,378	1.5	17.	Belgium	161,346	1.5
18.	Brazil	232,982	1.5	18.	Belgium	221,631	1.5	18.	Chinese Taipei	155,067	1.4
19.	Sweden	212,123	1.3	19.	Indonesia	199,799	1.3	19.	Indonesia	145,827	1.3
20.	Ireland	197,741	1.2	20.	Sweden	177,034	1.2	20.	Sweden	130,802	1.2

Note: the Rest of the World is dropped from the list of exporters.
Source: WIOD database, author's calculations.

In Table 1, moving from gross exports to domestic value added in exports measurement means removing foreign value added in national exports and adding domestic value added in partners' exports (less those that return home). In the language of the literature on vertical specialisation, this means subtracting VS and adding VS1 (again, corrected for reflected exports). So net (indirect) exporters of value added – those who indirectly supply more domestic value added than directly receive foreign value added – usually raise to higher ranks. In 2010, Russia is the 11th global exporter in gross terms and 6th in terms of domestic value added in exports with the total exports indicator raised by 42%, or from 2.3% to 3.6% of the global exports. Similarly, Australia climbs up from the 17th to 12th position and its contribution to the global exports raises from 1.7% to 2.2%. The opposite examples are given most notably by Belgium and Chinese Taipei. Note that the results for 2010 show that China is about to establish itself as a net (indirect) exporter of value added, i.e. to provide more domestic value added to downstream producers than use foreign value added for its exports.

Moving further right in the table, from domestic value added in exports to exports of domestic value added means removing the domestic value added that circulates in downstream value chain via intermediate products and is therefore double counted. Then net exporters of value added would typically step back or, at best, retain their ranks. In 2010, Russia being the 6th largest country in terms of domestic value added in exports (contribution 3.6%) becomes the 9th exporter of net value

added (contribution 3.0%). Australia remains to be 12th largest exporter with slightly lower contribution in terms of net value added (2.2% turns to 2.1%).

In Table 2, same measurement concepts are applied to Russia's bilateral exports. Again, the largest visible differences appear in 2010 when the three approaches to identify the largest export partners yield three different results: the principal export market in gross terms is Italy (8.8%), the principal destination for Russia's value added embodied in gross exports is China (7.0%), and the most important final destination for Russia's value added is the United States (9.6%). The differences as we will see in more detail below stem from the relative position of Russia's trading partners as absorbers or re-exporters of value added.

A striking fact reported by Table 2 is that the importance of the United States as an export destination for Russia is much higher than revealed by the traditional trade statistics. In 2005, for example, 4.4% of Russia's gross exports were directly sent to the United States, but the share of total exported value added from Russia that eventually ended up in the United States was 10.4%. In 2010, respective percentages were 4.9% and 9.6% that effectively made the United States the largest consumer of the value added of Russian origin.

The application of alternative measurement concepts to rank exporters and export markets therefore allows an analyst to see whether those who export the most are the same who generate most of the value added. The reassessment of the exporter ranking mainly concerns upstream value added suppliers including Russia. A similar application to bilateral exports may discover that some seemingly unimportant markets are in fact important driving forces for a country's exports via indirect final demand.

Table 2. Russia's ten largest trade (export) partners in gross and value added terms (million US$ and percentage of Russia's total exports, current prices)

Gross exports				Domestic value added in all exports				Exports of domestic value added			
#	Partner	US$ mln	%	#	Partner	US$ mln	%	#	Partner	US$ mln	%
					2000						
1.	Germany	16,447	16.7	1.	Germany	18,495	15.3	1.	Germany	12,798	15.1
2.	Italy	8,507	8.6	2.	Italy	9,189	7.6	2.	United States	7,026	8.3
3.	France	5,304	5.4	3.	United States	8,098	6.7	3.	Italy	6,755	8.0
4.	Poland	4,069	4.1	4.	France	6,835	5.6	4.	France	4,685	5.5
5.	United States	3,844	3.9	5.	Poland	4,152	3.4	5.	Japan	3,261	3.9
6.	Finland	3,263	3.3	6.	Japan	3,853	3.2	6.	Poland	2,999	3.5
7.	China	3,008	3.0	7.	United Kingdom	3,852	3.2	7.	United Kingdom	2,907	3.4
8.	Japan	2,915	3.0	8.	Spain	3,489	2.9	8.	China	2,645	3.1
9.	Spain	2,593	2.6	9.	China	3,397	2.8	9.	Spain	2,433	2.9
10.	Turkey	2,542	2.6	10.	Finland	3,088	2.5	10.	Turkey	2,310	2.7
					2005						
1.	Germany	28,683	12.6	1.	Germany	36,822	11.9	1.	Germany	22,921	11.6
2.	Italy	14,145	6.2	2.	United States	23,057	7.5	2.	United States	20,424	10.4
3.	China	13,118	5.8	3.	Italy	17,095	5.5	3.	Italy	12,143	6.2
4.	France	10,993	4.8	4.	China	16,571	5.4	4.	France	11,022	5.6

5.	Netherlands	10,109	4.5	5.	France	15,817	5.1	5.	China	10,889	5.5
6.	United States	10,016	4.4	6.	Netherlands	11,704	3.8	6.	United Kingdom	8,432	4.3
7.	Turkey	8,395	3.7	7.	United Kingdom	11,220	3.6	7.	Turkey	6,762	3.4
8.	Poland	7,373	3.2	8.	Turkey	8,934	2.9	8.	Japan	6,402	3.2
9.	Finland	6,268	2.8	9.	Poland	8,279	2.7	9.	Spain	5,866	3.0
10.	United Kingdom	6,081	2.7	10.	Spain	8,205	2.7	10.	Poland	5,080	2.6
					2010						
1.	Italy	32,783	8.8	1.	China	37,078	7.0	1.	United States	31,735	9.6
2.	China	24,188	6.5	2.	United States	36,873	7.0	2.	China	26,515	8.0
3.	Germany	19,459	5.2	3.	Italy	36,678	6.9	3.	Italy	25,290	7.6
4.	United States	18,206	4.9	4.	Germany	35,010	6.6	4.	Germany	20,833	6.3
5.	Netherlands	16,826	4.5	5.	France	25,315	4.8	5.	France	17,655	5.3
6.	France	16,773	4.5	6.	Japan	21,859	4.1	6.	Japan	17,260	5.2
7.	Japan	14,975	4.0	7.	Netherlands	20,236	3.8	7.	United Kingdom	10,302	3.1
8.	Poland	11,813	3.2	8.	Korea	14,212	2.7	8.	Spain	8,275	2.5
9.	Finland	9,369	2.5	9.	United Kingdom	14,147	2.7	9.	Poland	7,817	2.4
10.	Korea	8,496	2.3	10.	Poland	13,601	2.6	10.	Korea	7,535	2.3

Note: the Rest of the World is dropped from the list of exporters.
Source: WIOD database, author's calculations.

4.2. What drives value added flows from Russia and other countries?

The basic accounting relationship (3) links together two concepts from Tables 1 and 2 – "domestic value added in exports" ($\mathbf{S}'_n\left[\mathbf{V}_c\check{\mathbf{L}}\mathbf{E}_{bil}\right]\mathbf{i}$) and "exports of domestic value added" ($\mathbf{S}'_n\left[\mathbf{V}_c\check{\mathbf{L}}\mathbf{F}\right]\mathbf{i}$) – and relates the difference between the two to the domestic value added re-exported by trading partners ($\mathbf{S}'_n\left[\mathbf{V}_c\check{\mathbf{L}}\mathbf{E}_{tot}\right]\mathbf{i}$), which is hereafter referred to as "re-exported value added" for brevity. This term statistically captures cumulative double counted flows of value added that circulate across borders and are largely responsible for the growth of global value chains. It is also known as VS1 (Hummels *et al.*, 1999), or an indicator of "forward participation" in global value chains (various OECD publications), or "downstream component" (UNCTAD 2013). Application of the accounting relationship (3) allows one to see which of the two components – exports of value added for final demand or for re-exports – and to what extent is responsible for cumulative value added in global exports that can be attributed to a single country. Table 3 reports the results as ratios to gross exports in 2000, 2005 and 2010 that partly isolates the effect of different years' prices. Top 20 exporters are ranked by their total gross exports in 2010. Ratios of value added are highlighted to help the viewer easily interpret their relative intensities across countries and across time.

Table 3. Twenty largest global exporters from the global value chains perspective (million US$ and ratios, current prices)

	Total gross exports, US$ mln			Domestic value added in all exports / total gross exports			Exports of domestic value added / total gross exports			Re-exported domestic value added / total gross exports		
Exporter	**2000**	**2005**	**2010**	**2000**	**2005**	**2010**	**2000**	**2005**	**2010**	**2000**	**2005**	**2010**
China	279,547	836,719	1,743,486	0.99	0.89	0.94	0.81	0.71	0.75	0.19	0.19	0.20
United States	982,509	1,187,011	1,634,458	1.06	1.08	1.08	0.78	0.78	0.79	0.29	0.31	0.29
Germany	614,537	1,096,000	1,391,739	0.90	0.89	0.86	0.69	0.67	0.65	0.22	0.23	0.21
Japan	512,775	653,687	835,356	1.14	1.15	1.10	0.89	0.86	0.83	0.25	0.29	0.27
United Kingdom	378,672	547,245	617,535	0.98	1.02	1.00	0.74	0.75	0.73	0.24	0.27	0.27
France	349,817	517,610	609,074	0.87	0.88	0.84	0.67	0.67	0.65	0.20	0.21	0.20
Korea	199,000	327,910	519,545	0.79	0.82	0.74	0.62	0.60	0.55	0.17	0.22	0.19
Italy	271,817	428,302	514,168	0.91	0.92	0.86	0.72	0.71	0.67	0.19	0.21	0.19
Netherlands	214,498	341,872	468,328	0.76	0.80	0.75	0.59	0.59	0.55	0.18	0.21	0.20
Canada	319,788	416,185	449,279	0.79	0.85	0.94	0.67	0.71	0.75	0.12	0.14	0.19
Russia	98,757	226,895	371,743	1.23	1.36	1.42	0.86	0.87	0.89	0.37	0.49	0.53
Belgium	161,850	247,066	322,585	0.70	0.72	0.69	0.52	0.52	0.50	0.18	0.20	0.19
Spain	140,904	245,986	322,167	0.84	0.86	0.86	0.66	0.66	0.66	0.18	0.20	0.20
Chinese Taipei	171,724	226,721	311,633	0.77	0.75	0.71	0.60	0.52	0.50	0.17	0.23	0.22
India	67,708	157,728	308,576	0.96	0.93	0.89	0.78	0.73	0.71	0.18	0.20	0.18
Mexico	170,880	218,310	286,285	0.77	0.81	0.81	0.65	0.65	0.65	0.13	0.16	0.16
Australia	90,274	149,343	273,733	1.04	1.13	1.18	0.81	0.82	0.84	0.23	0.31	0.34
Brazil	64,412	134,030	232,982	1.00	1.02	1.07	0.77	0.77	0.79	0.22	0.25	0.28
Sweden	110,833	170,943	212,123	0.84	0.85	0.83	0.63	0.63	0.62	0.21	0.22	0.22
Ireland	88,634	159,912	197,741	0.64	0.67	0.66	0.51	0.53	0.53	0.13	0.14	0.13

Note: the Rest of the World is dropped from the list of exporters.
Source: WIOD database, author's calculations.

It appears that six countries in the WIOD database – the United States, Japan, United Kingdom, Russia, Australia and Brazil – have total domestic value added circulating in global exports that is equal or exceeds their total gross exports. By the way, China is quite close to assume this pattern if it beefs up its downstream value added exports. Note that the United States, Japan and United Kingdom have relatively stable ratios of domestic value added in global exports to total gross exports. Visible changes, if any, e.g. decline in Japan's ratio in 2000-2010 by 0.04, mostly correspond to the respective change in its "final", or "absorbed" component. Meanwhile, Brazil, Australia and Russia experienced an increase in their ratios through 2000-2010, respectively by 0.07, 0.14 and 0.19. The source of the increase was the "re-exported" component which rose by 0.06 in Brazil, by 0.11 in Australia and by 0.16 in Russia. This effectively discerns a group of the upstream suppliers of services in global value chains (United States, Japan, United Kingdom) and a group of the upstream suppliers of natural resources (Russia, Australia, Brazil).

Russia stands out for the extremely high contribution of both value added that ends up in final demand and value added that is further re-exported, if the domestic gross exports is used as a benchmark. The re-exported component expressed as the ratio to gross exports shows the magnitude and growth rate that is unparalleled in the WIOD database. However, the use of other databases

with superior country coverage, e.g. OECD-WTO TiVA database built on the OECD ICIO system, would reveal similar pattern for such resource-rich countries as Norway, Saudi Arabia and Chile.

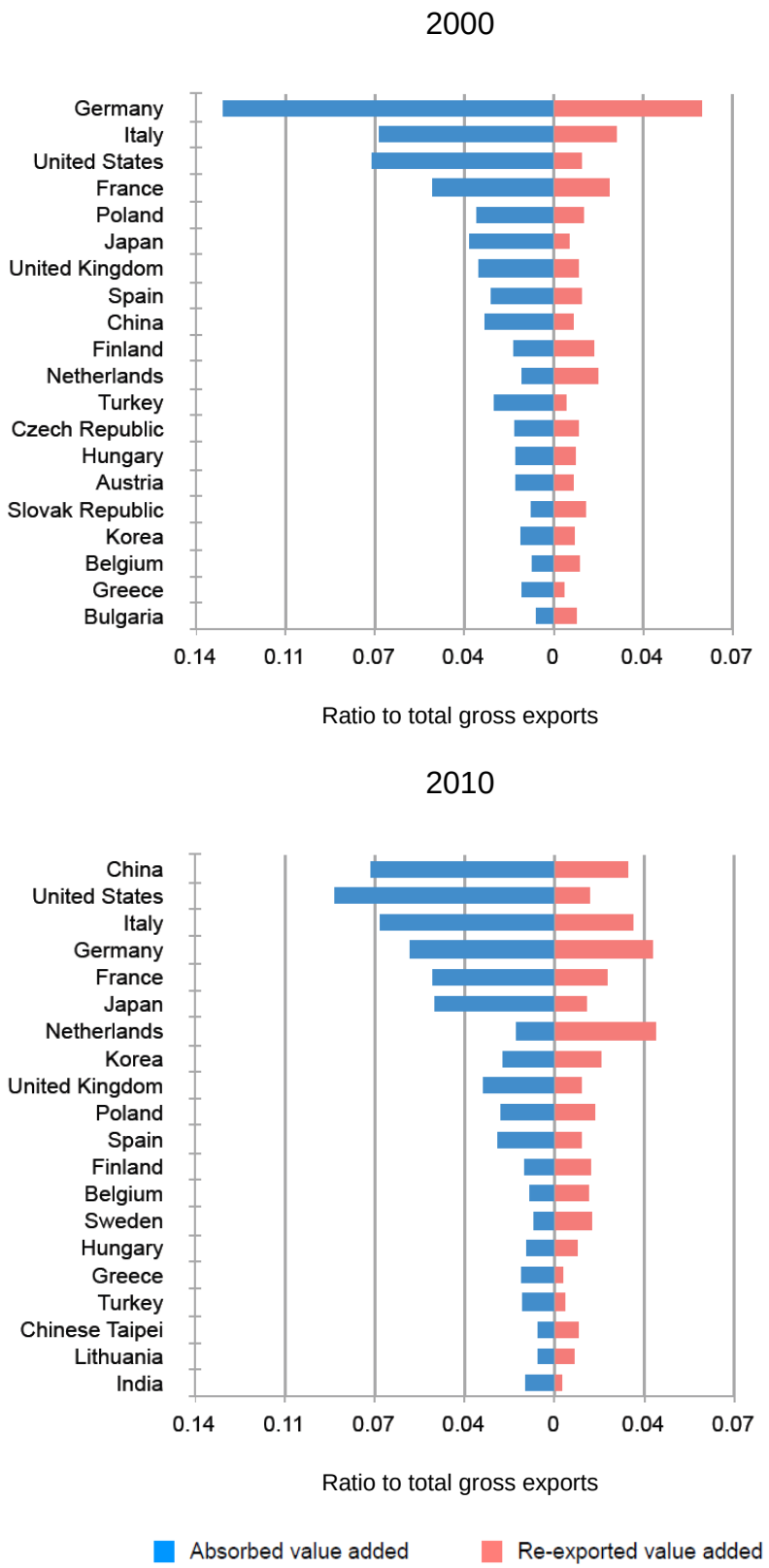

Figure 6. Russia's bilateral flows of value added, a basic decomposition

Note: the Rest of the World is dropped from the list of exporters.
Source: WIOD database, author's calculations.

Figure 6 visualises a similar decomposition of Russia's domestic value added in gross exports on a bilateral basis with 20 top export markets as bi-directional bar graphs. Export partners are ranked according to the cumulative (direct and indirect) Russia's value added received. All components are expressed as ratios to Russia's total gross exports.

The overall shift of the bars to the red zone from 2000 to 2010 in Figure 6 signifies the increasing importance of the re-exported value added. Although, this has been uneven across trading partners: the Netherlands, Finland, Belgium, Sweden predominantly use Russia's value added to produce their exports while the United States, United Kingdom and many others continue to use it for their final demand. Besides, apparent changes include the relative decline of Germany and the rise of China as the export markets where Russia's value added is destined.

4.3. What is the anatomy of Russia's value added in gross exports?

Equation (19) offers an itemised decomposition of the value added flows by country normalised to their total gross exports. This is particularly useful to see the indirect trade in a country's value added among its trade partners, that is not a direct component of gross exports and cannot be observed in simple versions of the value added accounting.

Figure 7 relates each component value added flow to gross exports of world 20 top exporters in 2000 and 2010 (bars for the itemised decompositions are unidirectional). It's now apparent that, as Russia rises up in this list, the importance of the foreign value added in its exports diminishes while that of the domestic value added in the form of intermediates processed and absorbed by partners increases. It is also clearly visible how partner/third countries get more involved in trading Russia's value added among themselves. Please note that, aggregated across all export destinations, the amount of Russia's value added that direct partners re-export to third countries equals Russia's value added that direct partners indirectly receive via third countries. These two components can be summed to cancel each other. This may be treated as partners' balance in trading Russia's value added that is zero at the aggregate country level. Also note that, given very few foreign inputs in Russia's gross exports, Figure 7 and the subsequent graphs focus on the itemised components of domestic value added. Although, as noted earlier, foreign value added can be easily split into detailed components, should such analytical requirement arise.

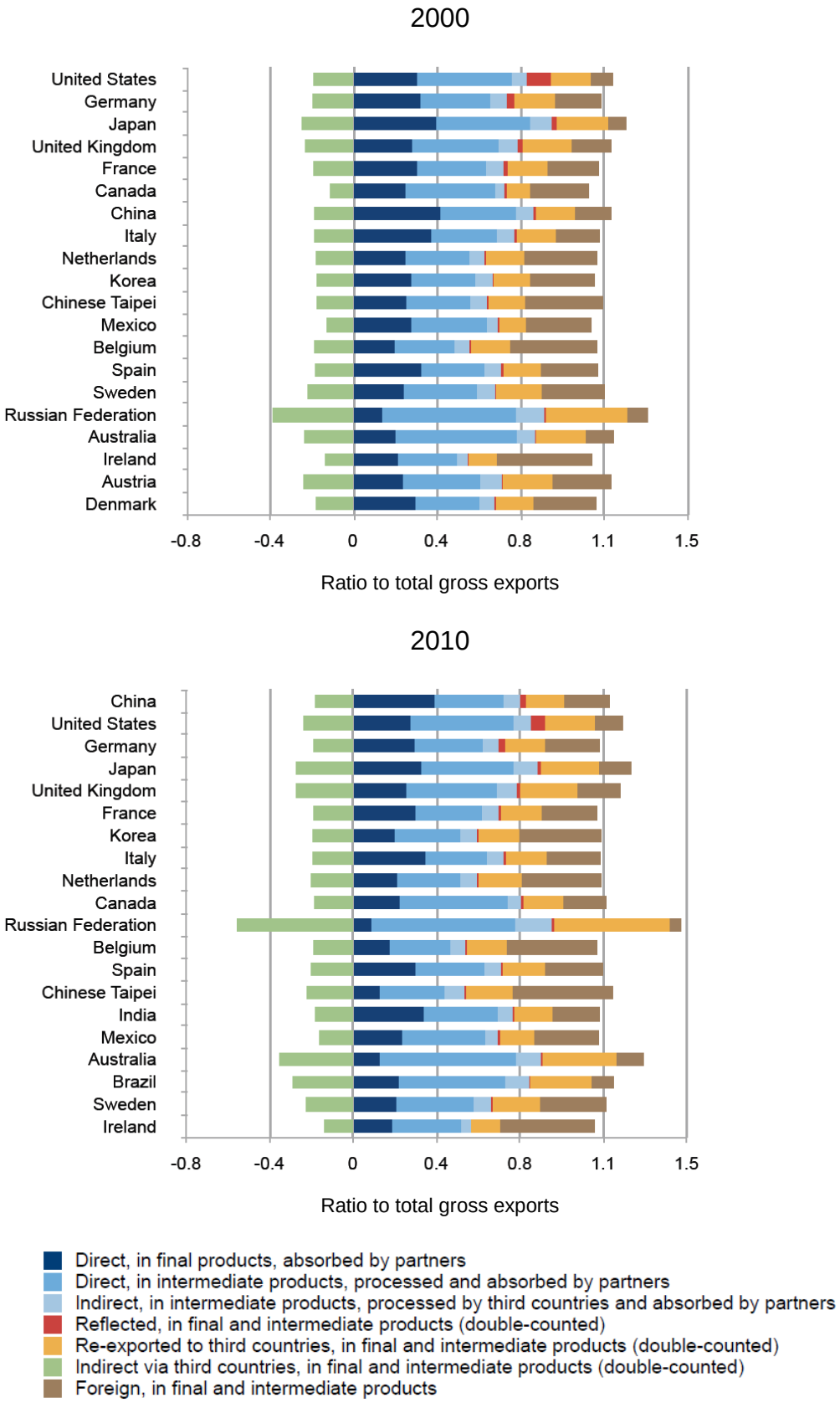

Figure 7. Domestic value added flows decomposed and normalised to gross exports of twenty largest global exporters, an itemised decomposition

Note: the Rest of the World is dropped from the list of exporters.
Source: WIOD database, author's calculations.

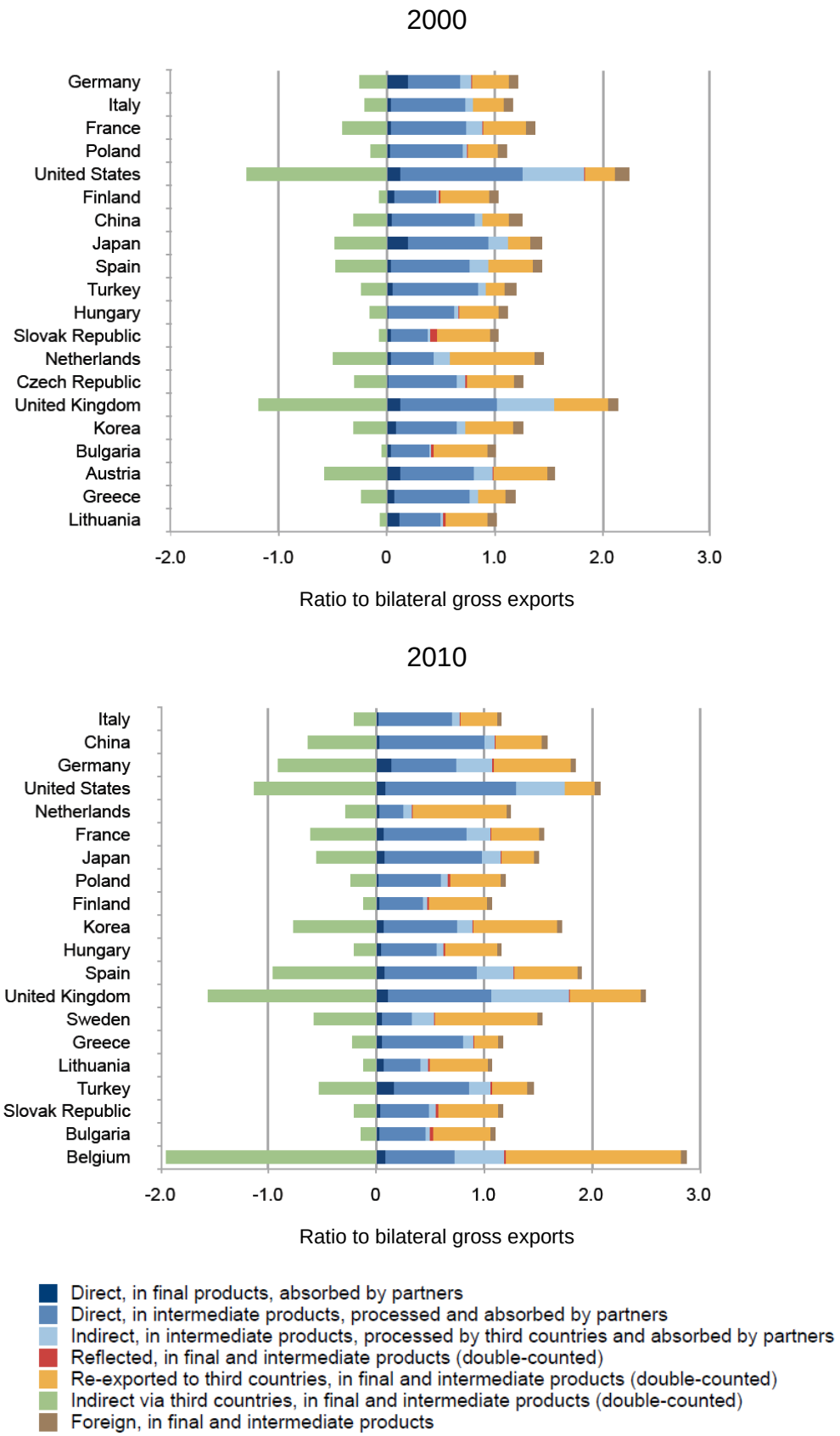

Figure 8. Russia's domestic value added bilateral flows decomposed and normalised to gross exports to twenty largest export destinations, an itemised decomposition

Note: the Rest of the World is dropped from the list of exporters.
Source: WIOD database, author's calculations.

Figure 8 presents a similar itemised decomposition of the value added flows normalised to Russia's gross exports at the bilateral level. Same trends can be observed: the dominant role of intermediates as carriers of Russia's value added used by trading partners and a "value added re-export boom". The country breakdown provides an interesting insight. In 2000, only two countries – the United States and United Kingdom – were heavily involved in trading value added that ultimately originated from Russia. In 2010, Germany and Spain joined, each indirectly receiving Russia's value added equivalent to more than 90% of the direct gross exports from Russia while Belgium received 195%! For China, France, Japan, Korea, Sweden, Turkey this measure exceeded 50%. In 2010, many of the largest Russia's export partners have negative balance of trade in Russia's value added with third countries which explains why they earn higher profile as destinations for Russia's value added when the indirect flows are explicitly accounted for. Italy and the Netherlands are notable exceptions: they indirectly re-export more Russia's value added than they receive.

4.4. Which economic sectors in Russia are responsible for the largest contribution to exports, directly and indirectly?

The basic accounting equation in the KN×K dimension $\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil}\right]=\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{F}\right]+\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{tot}\right]$ uncovers sector detail behind the aggregated results. Figure 9 shows the contribution of top 10 Russia's sectors to total domestic value added in global exports in 2000 and 2010 in the form of bi-directional bar graph. The sectors are ranked in accordance with the respective ratios of sectoral value added to total gross exports, and the breakdown discerns value added for final demand and for re-exports.

Both graphs for 2000 and 2010 show the prevalence in terms of value added in exports of a few sectors that generate trade and transport margins. However, it is "Mining and quarrying" that is clearly responsible for an expansion of Russia's value added flows to the downstream value chain through 2000s. And the re-exported component of the value added created in this sector was growing faster than the finally absorbed one. Manufacturing sectors other than fuel production had their relative value added in exports shrunk through this period.

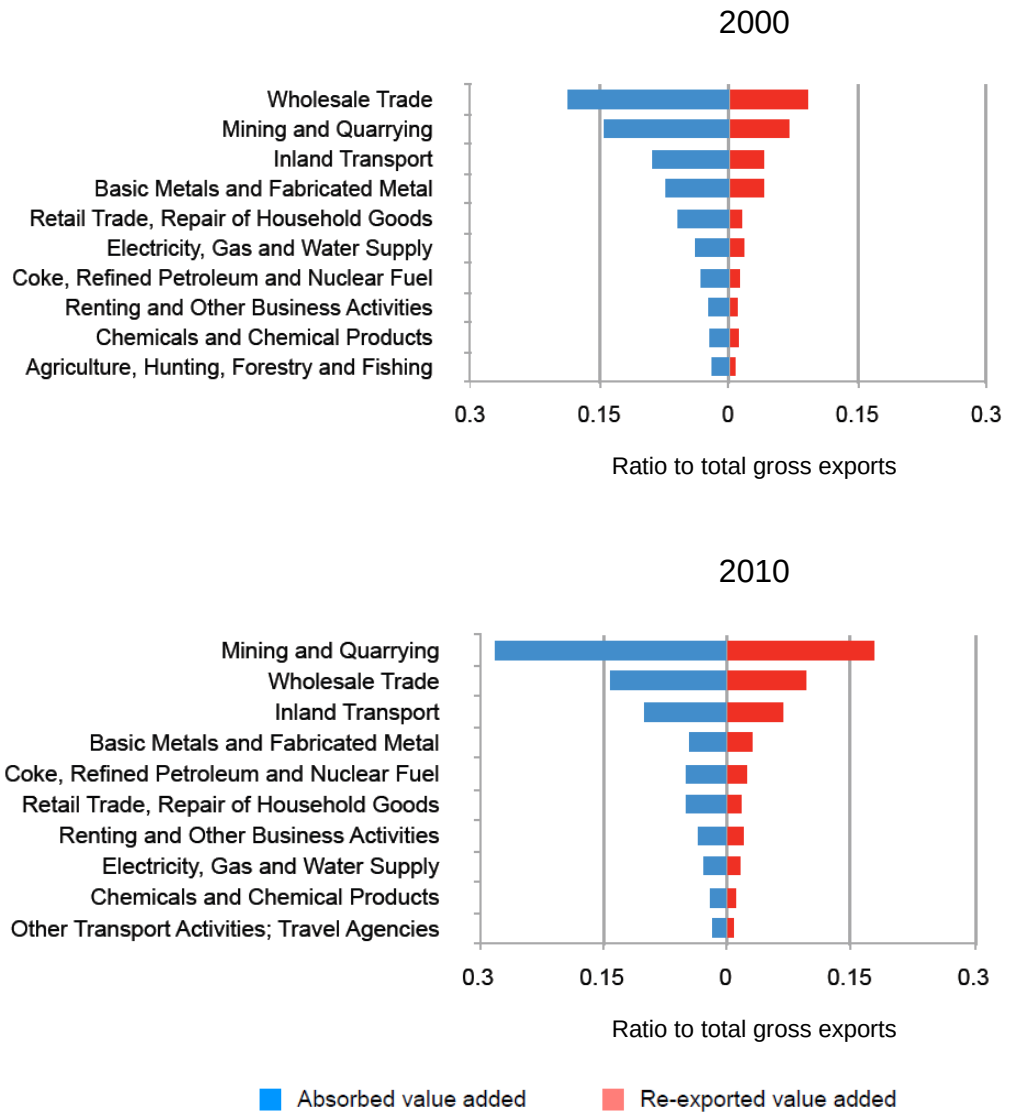

Figure 9. Russia's domestic value added global flows from ten largest exporting sectors, a basic decomposition

Source: WIOD database, author's calculations.

4.5. Given the importance of the re-exports of Russia's value added via downstream value chains, what is exactly the role of partners and third countries in trading Russia's value added?

Let us focus on the single sector that is critical for Russia's export performance, "Mining and quarrying". Application of the equation (14) results in an itemised decomposition of the value added originating in this sector in Russia rearranged to equal gross exports of that sector. The unidirectional bar graphs in Figure 10 correspond to the ratios of the respective bilateral component value added flows to the total gross exports of the "Mining and quarrying" sector in 2000 and 2010.

In 2000, the smoothness of the graph reflects the proportionality of hidden value added components to the observed gross exports flows. Sizable brown parts of the bars signify that

products of Russia's "Mining and quarrying" were carriers for other sectors' value added (presumably, first of all that of trade and transport sectors). A different pattern emerges from the 2010 graph. Many top partners are now actively engaged in the back-and-forth trade in intermediates containing the value added from Russia's "Mining and quarrying". The Netherlands is now the top indirect exporter followed by Italy and Germany, while Germany emerged as the top indirect importer followed by the United States and China. Note that the United States are now the second largest final destination for Russia's "Mining and quarrying". Besides, products of this sector now absorb less value added from other sectors on their way to foreign markets (the brown bars have shrunk).

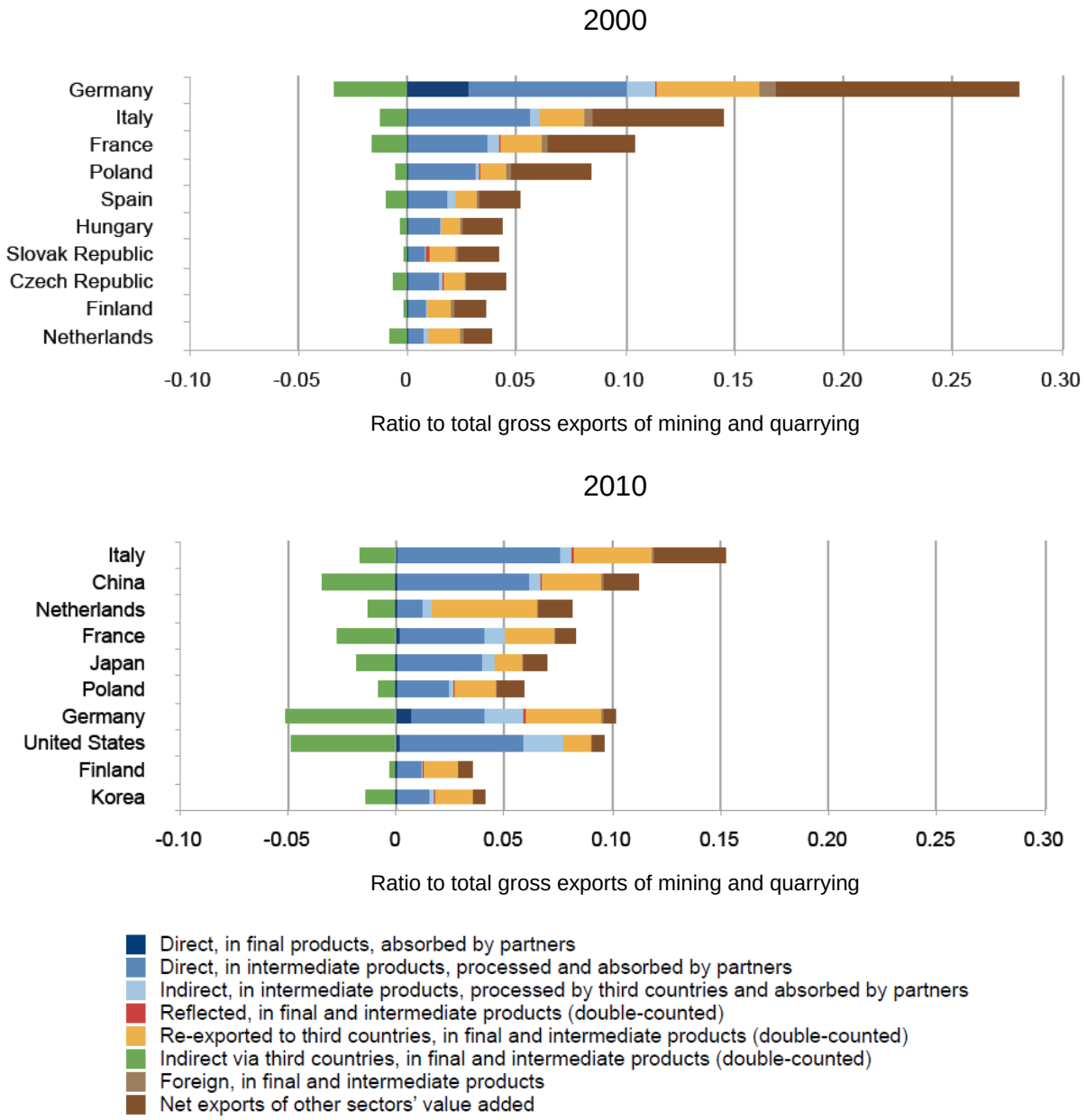

Figure 10. Value added exported from Russia's mining and quarrying sector to ten largest export markets, an itemised decomposition

Note: the Rest of the World is dropped from the list of exporters.
Source: WIOD database, author's calculations.

The matrix representation of the detailed value added components in this paper makes them highly customisable for specific analytical purposes. This is exemplified below as we make one step further in seeing how the value added from Russia's "Mining and quarrying" sector is embodied in the back-and forth trade among Russia's partners. First, the bilateral "foreign value added in trade" matrix in the KN×K dimension $\left[\hat{\mathbf{V}}_{\mathbf{c}(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil}$ is modified to include Russia's value added only. The result is a KN×K matrix of trade in Russia's sectoral value added among all K countries. Next, extracting the rows sector-wise gives N K×K matrices that depict the bilateral flows of value added originating in Russia's sector *n* in partners' trade. The matrix elements should be normalized, e.g. with respect to sectoral gross exports, for a sensible visualisation.

An example of such visualisation is shown in Figure 11. The most important flows of the value added originating in Russia's "Mining and quarrying" sector are identified and highlighted on a map that centers on Europe, which is the principal market for many Russian products as shown in previous figures. In 2000, many European countries appeared as net exporters of value added of the said origin and Germany was a trade hub. By 2010, more countries shifted to the net importer status while the picture of the value added flows became more complex. The directions of flows are now more diversified and no single hub may be discerned.

Note that the maps depict the circulation of the value added embodied in gross exports. Interested analyst may attempt at a deeper decomposition using again that $\mathbf{E}_{bil} = \check{\mathbf{F}} + \check{\mathbf{Z}}_{(KN\times K)}$ and thus account explicitly for the final and intermediate double-counted components.

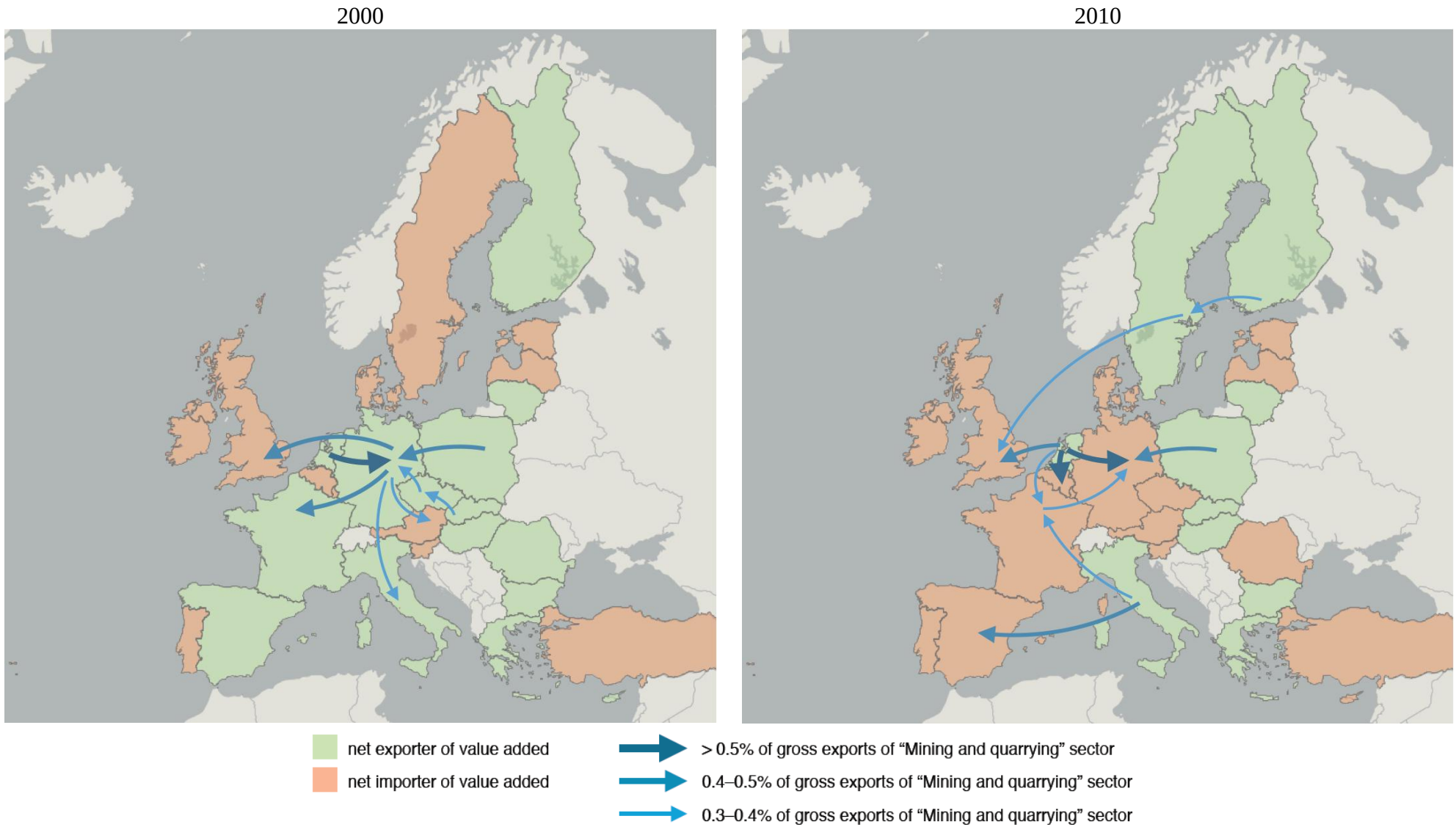

Figure 11. Circulation of the value added originating in Russia's "Mining and Quarrying" sector embodied in gross trade of European countries

Source: WIOD database, author's calculations.

4.6. How do countries source the value added embodied in domestically consumed and re-exported goods and services and what's the role of Russia?

Answering this question will in fact involve an extension into the KN×KN or K×KN dimension or applying the "value added at destination" concept, complementary to "value added at origin" that framed the previous results.

In previous subsections, the concern was to discover the actual origin of the value added embodied in products somehow used by Russia's trading partners. In this subsection, the concern is two-fold: (1) to detect sectors that deliver products where Russia's value added is embodied, and (2) to detect the use of those products – final domestic use or re-exports? The visualisation of results in the KN×KN, or [country/sector]×[country/sector] dimension is a more complex task, so in view of space limitation the tables below focus again on the "Mining and Quarrying" sector.

The row from equation (32) $\left[\mathbf{V_c L E}^{\vee}_{bil(KN\times KN)}\right]=\left[\mathbf{V_c L F}^{\vee}_{(KN\times KN)}\right]+\left[\mathbf{V_c L E}^{\vee}_{tot(KN\times KN)}\right]-\left[\mathbf{V_c L}^{\vee}\mathbf{Z}^{*}\right]$ that corresponds to our sector in focus gives the flows of the value added from "Mining and Quarrying" in Russia to sectors in other countries where it is embodied for further use. The elements in $\left[\mathbf{V_c L F}^{\vee}_{(KN\times KN)}\right]$ and $\left[\mathbf{V_c L E}^{\vee}_{tot(KN\times KN)}\right]$ are estimates of the use of the value added sourced from Russia's "Mining and Quarrying" in the products of partner countries' sectors and in sum are equal to $\left[\mathbf{V_c L E}^{\vee}_{bil(KN\times KN)}\right]$, or the total value added sourced from "Mining and Quarrying". These two terms may be treated as the demand factors for the generation of value added in the sector considered. Meanwhile, $\left[\mathbf{V_c L}^{\vee}\mathbf{Z}^{*}\right]$ explains the individual sectoral deviations between the amount of the value added that left the origin and the amount that was received at the destination. For example, in 2010, products delivered by "Construction" sector for final domestic use worldwide received 8.94% of the value added generated in Russia's "Mining and Quarrying" and those products for re-exports received 0.10%. But the decomposition of the gross exports ($\left[\mathbf{V_c L E}^{\vee}_{bil(KN\times KN)}\right]$) shows that only 0.09% of total Russia's value added from "Mining and Quarrying" went to the products of "Construction". Then the respective entry in $\left[\mathbf{V_c L}^{\vee}\mathbf{Z}^{*}\right]$, 8.95%, unequivocally attributes the difference to the indirect flows of value added through other sectors of the recipient countries. This means that the use of "Construction" products abroad is a significant factor not directly observed creating demand for the exports from Russia's "Mining and Quarrying" sector.

Results in Table 4 can be interpreted in a similar manner. Naturally, products of "Coke, Refined Petroleum and Nuclear Fuel" sector for both domestic use and exports appear as the principal absorbers of the value added created in our sector in focus. The increased significance of their use for exports largely corresponds to the trends discovered in previous figures. Other products largely responsible for demand for Russia's value added from "Mining and Quarrying" include those of "Construction" and "Electricity, Gas and Water Supply" for domestic use, "Basic Metals and Fabricated Metal", "Chemicals and Chemical Products" for exports. Interestingly, the top 10 list also includes such entries as products of "Food, Beverages and Tobacco" and "Public Administration and Defence, Compulsory Social Security". So in 2010, the domestic use of public administration and defence services worldwide worth a dollar helped generate nearly 3 cents of value added in Russia's mining and quarrying.

Table 4. Top twenty sectors where Russia's "Mining and Quarrying" sector's value added in global exports is embodied "at destination", by sector and type of use (percent)

Share of total, %	WIOD sector	Type of use by partner country (domestically used or exported)	Share in exporter's domestic value added in all exports	Inter-sectoral transfer of value added at destination
	2000			
16.76	Coke, Refined Petroleum and Nuclear Fuel	domestic final use	11.68	-16.08
11.00	Coke, Refined Petroleum and Nuclear Fuel	exports	11.68	-16.08
6.02	Construction	domestic final use	0.13	-6.04
5.58	Electricity, Gas and Water Supply	domestic final use	0.57	-5.51
3.75	Chemicals and Chemical Products	exports	3.95	-2.04
3.74	Mining and Quarrying	domestic final use	63.74	59.56
3.29	Food, Beverages and Tobacco	domestic final use	0.80	-3.31
2.87	Basic Metals and Fabricated Metal	exports	5.16	1.18
2.46	Transport Equipment	domestic final use	2.10	-2.44
2.45	Public Admin and Defence; Compulsory Social Security	domestic final use	0.02	-2.45
2.24	Chemicals and Chemical Products	domestic final use	3.95	-2.04
2.24	Inland Transport	domestic final use	1.25	-1.86
2.09	Electrical and Optical Equipment	exports	2.14	-1.52
2.08	Transport Equipment	exports	2.10	-2.44
2.06	Health and Social Work	domestic final use	0.01	-2.06
1.84	Machinery, Nec	domestic final use	1.64	-1.56
1.69	Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles	domestic final use	0.54	-1.44
1.56	Electrical and Optical Equipment	domestic final use	2.14	-1.52
1.50	Agriculture, Hunting, Forestry and Fishing	domestic final use	0.44	-1.47
1.44	Hotels and Restaurants	domestic final use	0.07	-1.44
23.34	Other	domestic final use and exports	n/a	n/a

		2010		
14.95	Coke, Refined Petroleum and Nuclear Fuel	exports	17.25	-10.40
12.70	Coke, Refined Petroleum and Nuclear Fuel	domestic final use	17.25	-10.40
8.94	Construction	domestic final use	0.09	-8.95
5.05	Chemicals and Chemical Products	exports	5.37	-2.02
4.78	Electricity, Gas and Water Supply	domestic final use	0.50	-4.67
4.22	Basic Metals and Fabricated Metal	exports	6.12	0.61
2.94	Public Admin and Defence; Compulsory Social Security	domestic final use	0.02	-2.94
2.67	Food, Beverages and Tobacco	domestic final use	0.83	-2.67
2.65	Electrical and Optical Equipment	exports	2.68	-1.81
2.46	Health and Social Work	domestic final use	0.01	-2.46
2.36	Transport Equipment	domestic final use	1.83	-2.39
2.34	Chemicals and Chemical Products	domestic final use	5.37	-2.02
2.04	Inland Transport	domestic final use	1.82	-0.75
1.98	Machinery, Nec	domestic final use	1.63	-1.86
1.86	Transport Equipment	exports	1.83	-2.39
1.84	Electrical and Optical Equipment	domestic final use	2.68	-1.81
1.51	Machinery, Nec	exports	1.63	-1.86
1.42	Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles	domestic final use	0.86	-0.82
1.28	Basic Metals and Fabricated Metal	domestic final use	6.12	0.61
1.28	Hotels and Restaurants	domestic final use	0.06	-1.27
20.73	Other	domestic final use and exports	n/a	n/a

Source: WIOD database, author's calculations.

In Table 5, the results from Table 4 are disaggregated across partner countries and ranked again by their contribution to the total value added of "Mining and Quarrying" in Russia's exports. This provides some interesting details. The top list in 2000 is almost exclusively featured by European economies that use Russia's value added to produce and export fuels. In 2010, China and the United States enter the picture while the Rest of the World raises to top positions. Contrary to European countries, China mostly uses value added of "Mining and Quarrying" of Russian origin for domestic construction and exported electrical and optical equipment. The United States use it for domestically consumed fuels but also for public administration and defence. In other words, every dollar spent in the United States for public administration and defence services generates 0.9 cents of value added in Russia's mining and quarrying sector. This is an intrinsic implication of global value chains that link two seemingly unrelated sectors in two distant countries.

Table 5. Top twenty countries/sectors where Russia's "Mining and Quarrying" sector's value added in global exports is embodied "at destination", by country, sector and type of use (percent)

Share of total, %	Partner country	WIOD sector	Type of use by partner country (domestically used or exported)	Share in exporter's domestic value added in all exports	Inter-sectoral transfer of value added at destination
		2000			
3.90	Germany	Coke, Refined Petroleum and Nuclear Fuel	domestic final use	1.52	-4.18
3.48	Germany	Mining and Quarrying	domestic final use	15.69	12.20
2.59	Italy	Coke, Refined Petroleum and Nuclear Fuel	domestic final use	0.41	-3.28
1.99	France	Coke, Refined Petroleum and Nuclear Fuel	domestic final use	0.82	-2.23
1.80	Germany	Coke, Refined Petroleum and Nuclear Fuel	exports	1.52	-4.18
1.17	Italy	Electricity, Gas and Water Supply	domestic final use	0.05	-1.15
1.10	Italy	Coke, Refined Petroleum and Nuclear Fuel	exports	0.41	-3.28
1.06	France	Coke, Refined Petroleum and Nuclear Fuel	exports	0.82	-2.23
1.02	Germany	Chemicals and Chemical Products	exports	0.37	-1.06
0.99	Spain	Coke, Refined Petroleum and Nuclear Fuel	domestic final use	0.60	-0.99
0.98	Poland	Coke, Refined Petroleum and Nuclear Fuel	domestic final use	0.31	-1.08
0.98	Netherlands	Coke, Refined Petroleum and Nuclear Fuel	exports	0.44	-0.82
0.83	Germany	Construction	domestic final use	0.04	-0.80
0.82	Finland	Coke, Refined Petroleum and Nuclear Fuel	exports	0.10	-1.04
0.79	Slovak Republic	Coke, Refined Petroleum and Nuclear Fuel	exports	0.05	-0.92
0.76	RoW	Construction	domestic final use	0.01	-0.76
0.72	Germany	Transport Equipment	exports	0.30	-0.85
0.60	Spain	Coke, Refined Petroleum and Nuclear Fuel	exports	0.60	-0.99
0.59	Czech Republic	Electricity, Gas and Water Supply	domestic final use	0.01	-0.65
0.59	Poland	Construction	domestic final use	0.01	-0.65
73.24	Other	Other	domestic final use and exports	n/a	n/a
		2010			
3.99	RoW	Coke, Refined Petroleum and Nuclear Fuel	exports	3.32	-2.08
2.80	RoW	Construction	domestic final use	0.01	-2.80
2.44	Netherlands	Coke, Refined Petroleum and Nuclear Fuel	exports	0.59	-2.14
2.04	China	Construction	domestic final use	0.01	-2.04
1.58	RoW	Chemicals and Chemical Products	exports	0.71	-1.15
1.51	RoW	Basic Metals and Fabricated Metal	exports	1.16	-0.72
1.47	Italy	Coke, Refined Petroleum and Nuclear Fuel	domestic final use	0.37	-2.40
1.47	France	Coke, Refined Petroleum and Nuclear Fuel	domestic final use	1.27	-0.92
1.41	RoW	Coke, Refined Petroleum and Nuclear Fuel	domestic final use	3.32	-2.08
1.37	Germany	Coke, Refined Petroleum and Nuclear Fuel	domestic final use	1.69	-0.12
1.30	Italy	Coke, Refined Petroleum and Nuclear Fuel	exports	0.37	-2.40
1.25	Italy	Electricity, Gas and Water Supply	domestic final use	0.02	-1.25
1.21	United States	Coke, Refined Petroleum and Nuclear Fuel	domestic final use	1.49	-0.01
1.06	Sweden	Coke, Refined Petroleum and Nuclear Fuel	exports	0.41	-0.82
0.87	United States	Public Admin and Defence; Compulsory Social Security	domestic final use	0.00	-0.88
0.80	Finland	Coke, Refined Petroleum and Nuclear Fuel	exports	0.16	-0.90

0.76	China	Electrical and Optical Equipment	exports	0.45	-0.67
0.72	France	Coke, Refined Petroleum and Nuclear Fuel	exports	1.27	-0.92
0.67	Japan	Coke, Refined Petroleum and Nuclear Fuel	domestic final use	0.53	-0.26
0.65	RoW	Inland Transport	domestic final use	0.83	0.10
70.63	Other	Other	domestic final use and exports	n/a	n/a

Source: WIOD database, author's calculations.

Finally, one may wish to change perspective and look into the dependence of either domestic final demand or exports of particular products of partner countries on Russia's supply of value added from the "Mining and Quarrying" sector. Note that this requires computing full matrices $\mathbf{V_c LF}_{(KN\times KN)}$ and $\mathbf{V_c LE}_{tot(KN\times KN)}$, i.e. without removing any block elements, to account for the domestic value added. Then the column sums will give, respectively, the total final demand and total exports of partner countries at the product level. Next, we extract the rows corresponding to our sector in focus and normalise the elements with respect to the column sums. The results for 2000 and 2010 are ranked and presented in Table 6.

The percentages should be treated with caution as these relate to different totals and do not reflect countries' importance in terms of gross or value added exports. The general rule appears to be: the closer to Russia and the smaller is the partner country, the more dependent it is on Russia's "Mining and Quarrying" sector. As might be expected, "Coke, Refined Petroleum and Nuclear Fuel" appears as the single most important sector products of which require Russia's value added. For example, in 2010, to satisfy each dollar of domestic final demand for coke and petroleum fuels, Finland required about 21 cents of value added of Russian origin. And to produce each dollar of coke and petroleum fuels for exports it required 31 cents of the said value added. However, such large (and in a sense more distant) Russia's trading partners as China, Japan and the United States experienced much lower dependence which didn't exceed 1.63% for domestic final use and 1.40% for exports in 2010.

Table 6. Contribution of value added generated in Russia's "Mining and Quarrying" sector to domestic final demand and exports of Russia's trade partners: top twenty partners by country and sector (percent)

Partner country	WIOD sector	Share in domestic final demand	Partner country	WIOD sector	Share in exports
		2000			
Latvia	Mining and Quarrying	46.48	Bulgaria	Coke, Refined Petroleum and Nuclear Fuel	20.55
Cyprus	Mining and Quarrying	41.73	Slovak Republic	Coke, Refined Petroleum and Nuclear Fuel	20.12
Slovak Republic	Coke, Refined Petroleum and Nuclear Fuel	17.40	Cyprus	Coke, Refined Petroleum and Nuclear Fuel	19.65
Slovenia	Mining and Quarrying	16.80	Lithuania	Coke, Refined Petroleum and Nuclear Fuel	17.56
Bulgaria	Coke, Refined Petroleum	16.76	Czech	Coke, Refined Petroleum	16.86

	and Nuclear Fuel		Republic	and Nuclear Fuel	
Czech Republic	Coke, Refined Petroleum and Nuclear Fuel	13.36	Lithuania	Chemicals and Chemical Products	14.34
Cyprus	Coke, Refined Petroleum and Nuclear Fuel	12.51	Hungary	Coke, Refined Petroleum and Nuclear Fuel	12.36
Hungary	Coke, Refined Petroleum and Nuclear Fuel	11.94	Finland	Coke, Refined Petroleum and Nuclear Fuel	11.62
Poland	Coke, Refined Petroleum and Nuclear Fuel	10.63	Poland	Coke, Refined Petroleum and Nuclear Fuel	10.84
Germany	Mining and Quarrying	10.63	Cyprus	Electricity, Gas and Water Supply	8.96
Lithuania	Coke, Refined Petroleum and Nuclear Fuel	9.73	Romania	Coke, Refined Petroleum and Nuclear Fuel	8.94
Lithuania	Chemicals and Chemical Products	8.97	Slovak Republic	Inland Transport	8.76
Cyprus	Electricity, Gas and Water Supply	8.87	Hungary	Electricity, Gas and Water Supply	8.67
Romania	Coke, Refined Petroleum and Nuclear Fuel	8.76	Slovenia	Coke, Refined Petroleum and Nuclear Fuel	6.68
Finland	Coke, Refined Petroleum and Nuclear Fuel	8.45	Bulgaria	Other Non-Metallic Mineral	6.47
Hungary	Electricity, Gas and Water Supply	8.41	Bulgaria	Chemicals and Chemical Products	6.18
Slovak Republic	Inland Transport	8.14	Czech Republic	Electricity, Gas and Water Supply	6.04
Estonia	Coke, Refined Petroleum and Nuclear Fuel	7.81	Bulgaria	Electricity, Gas and Water Supply	5.72
Latvia	Coke, Refined Petroleum and Nuclear Fuel	7.26	Germany	Coke, Refined Petroleum and Nuclear Fuel	5.66
Czech Republic	Electricity, Gas and Water Supply	5.99	Greece	Coke, Refined Petroleum and Nuclear Fuel	5.52
Memo:					
China	Coke, Refined Petroleum and Nuclear Fuel	0.34	China	Coke, Refined Petroleum and Nuclear Fuel	0.33
United States	Coke, Refined Petroleum and Nuclear Fuel	0.12	Japan	Coke, Refined Petroleum and Nuclear Fuel	0.04
Japan	Coke, Refined Petroleum and Nuclear Fuel	0.06	United States	Coke, Refined Petroleum and Nuclear Fuel	0.03
			2010		
Latvia	Mining and Quarrying	58.19	Lithuania	Coke, Refined Petroleum and Nuclear Fuel	32.44
Finland	Coke, Refined Petroleum and Nuclear Fuel	20.78	Finland	Coke, Refined Petroleum and Nuclear Fuel	30.95
Lithuania	Coke, Refined Petroleum and Nuclear Fuel	19.13	Lithuania	Chemicals and Chemical Products	25.09
Slovak Republic	Coke, Refined Petroleum and Nuclear Fuel	18.53	Bulgaria	Coke, Refined Petroleum and Nuclear Fuel	24.54
Latvia	Coke, Refined Petroleum and Nuclear Fuel	18.21	Slovak Republic	Coke, Refined Petroleum and Nuclear Fuel	24.42
Lithuania	Chemicals and Chemical Products	16.02	Sweden	Coke, Refined Petroleum and Nuclear Fuel	17.42
Estonia	Coke, Refined Petroleum and Nuclear Fuel	15.83	Hungary	Coke, Refined Petroleum and Nuclear Fuel	14.93
Bulgaria	Coke, Refined Petroleum and Nuclear Fuel	15.74	Poland	Coke, Refined Petroleum and Nuclear Fuel	14.87
Poland	Coke, Refined Petroleum and Nuclear Fuel	13.68	Greece	Coke, Refined Petroleum and Nuclear Fuel	14.46
Hungary	Coke, Refined Petroleum and Nuclear Fuel	13.01	Hungary	Electricity, Gas and Water Supply	13.14
Hungary	Electricity, Gas and Water Supply	12.60	Italy	Coke, Refined Petroleum and Nuclear Fuel	11.68
Greece	Coke, Refined Petroleum and Nuclear Fuel	12.45	Netherlands	Coke, Refined Petroleum and Nuclear Fuel	11.35
Italy	Coke, Refined Petroleum and Nuclear Fuel	10.17	Lithuania	Rubber and Plastics	9.05
Germany	Mining and Quarrying	9.72	Latvia	Electricity, Gas and Water Supply	7.88

Sweden	Coke, Refined Petroleum and Nuclear Fuel	9.29	Lithuania	Electricity, Gas and Water Supply	7.64
Malta	Coke, Refined Petroleum and Nuclear Fuel	9.21	France	Coke, Refined Petroleum and Nuclear Fuel	6.99
Czech Republic	Coke, Refined Petroleum and Nuclear Fuel	8.73	Romania	Coke, Refined Petroleum and Nuclear Fuel	6.91
Austria	Coke, Refined Petroleum and Nuclear Fuel	7.92	Czech Republic	Coke, Refined Petroleum and Nuclear Fuel	6.53
Latvia	Electricity, Gas and Water Supply	7.83	Estonia	Electricity, Gas and Water Supply	6.48
Lithuania	Electricity, Gas and Water Supply	7.57	Austria	Coke, Refined Petroleum and Nuclear Fuel	6.14
Memo:					
Japan	Coke, Refined Petroleum and Nuclear Fuel	1.63	Japan	Coke, Refined Petroleum and Nuclear Fuel c8	1.40
China	Coke, Refined Petroleum and Nuclear Fuel	1.63	China	Coke, Refined Petroleum and Nuclear Fuel	1.31
United States	Coke, Refined Petroleum and Nuclear Fuel	1.05	United States	Coke, Refined Petroleum and Nuclear Fuel	0.72

Source: WIOD database, author's calculations.

5. Conclusion

This paper has attempted to review and summarise the recently developed frameworks for the gross export accounting using value added trade concepts. A user of such frameworks, either a trade economist or policy analyst, needs a clear understanding which framework is best applicable to each specific purpose and what kind of estimates it can deliver. The framework itself will benefit the user if it is comprehensive, customisable and easy to implement. While a one-fit-all solution is unlikely to be found, it has been proposed that the available frameworks are classified into two types: the gross exports accounting and the cumulative value added accounting. The former decomposes direct exports into additive value added components and may be more useful for the trade policy analysis. The latter identifies direct and indirect flows of value added via gross exports and may be better applied to the global value chain analysis. The main contribution of this paper is then in an attempt to generalise and elaborate the framework of the second type.

At the core of the technical discussion is an elegant and simple way to derive a basic decomposition of cumulative value added flows that attribute each flow to the country and sector of origin and to the country and sector of destination. Two basic components include value added that "ends up", or is finally absorbed, in partner country and value added that only "lands" in partner country to be further re-exported. The clearly distinguished final component is in fact a part of a country's GDP that is absorbed (consumed) overseas. The corresponding measures can then be treated as the ultimate external demand factors that contribute to the GDP and that are not traceable in gross trade statistics or other type of decomposition frameworks.

There are various ways to split these basic components and obtain more detailed indicators or aggregate those across sectors or countries or both. The matrix representation appears to be

highly customisable and adaptable to matrix computation software. The formulations proposed yield results that are mostly identical to those of Stehrer (2013) who built his work on Koopman *et al.* (2012). The discussion has also led to the derivation of two matrices of the inter-sectoral transfer of value added for which, respectively, the exporting country and partner country is responsible. These matrices, to the author's knowledge, didn't explicitly feature previous studies. Excluding these two matrices or respective matrix elements from the value added accounting equations at the bilateral sectoral level will make them incomplete. The basic form of derived accounting relationship is also used to prove in a quick and efficient way that the total trade balances are equal in gross and value added terms.

Applied to real data from the WIOD database, the proposed formulations uncover a great deal of detail intrinsic to the expanding global value chains. Gross trade statistics hide a multitude of indirect linkages that shape countries' export performance. Indirect linkages show how production in one country responds to final or intermediate demand in another country while visible link may appear weak or may not exist at all. Russia seems to be a very good example to test the significance of such indirect links. Exported value added of Russian origin – primarily from the mining and quarrying sector – is then repeatedly used in the downstream value chain, and to a higher extent than for any other country in the WIOD database. This has little effect on total gross exports or total value added satisfying foreign final demand, but affects Russia's bilateral relations with trade partners. Indirect links shed light on the importance of some partners. For example, the flows of Russia's value added that end up in the United States' final domestic consumption are largely not governed by direct trade policies, as those are mostly indirect flows. This means that imposing restrictions on direct trade between the United States and Russia will unlikely result in a significant change in demand for products of Russia's exporting sectors unless most other trading partners do so.

For the convenience of a potential user, main formulae for the cumulative value added accounting are summarised in the Appendix A. In addition, Appendix B lists and explains the matrices featuring those formulations.

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Appendix A. Summary of formulae

Table A.1. Summary of the formulae obtained for the cumulative value added accounting

Dimension	Condensed from	Itemised form
Bilateral value added accounting equation		
KN×KN	$\left[\mathbf{V_c L E}^{\vee}_{bil(KN\times KN)}\right]=\left[\mathbf{V_c L F}^{\vee}_{(KN\times KN)}\right]+\left[\mathbf{V_c L E}^{\vee}_{tot(KN\times KN)}\right]-\left[\mathbf{V_c L}^{\vee}\mathbf{Z}^{*}\right]$	$\left[\mathbf{V_c L E}^{\vee}_{bil(KN\times KN)}\right]=\mathbf{V_c}\hat{\mathbf{L}}\hat{\mathbf{F}}_{(KN\times KN)}+\mathbf{V_c}\hat{\mathbf{L}}\hat{\mathbf{F}}_{(KN\times KN)}+\left[\mathbf{V_c}\hat{\mathbf{L}}\hat{\mathbf{F}}^{\vee}_{(KN\times KN)}\right]+$ $+\mathbf{V_c}\hat{\mathbf{L}}\circ\mathbf{E}'_{bil(KN\times KN)}+\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{tot(KN\times KN)}-\mathbf{V_c}\hat{\mathbf{L}}\circ\mathbf{E}'_{bil(KN\times KN)}\right]-\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{Z}^{*}\right]$
KN×K	$\left[\mathbf{V_c L}^{\vee}\mathbf{E}_{bil}\right]=\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{F}\right]+\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{tot}\right]$	$\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil}\right]=\mathbf{V_c}\hat{\mathbf{L}}\hat{\mathbf{F}}+\mathbf{V_c}\hat{\mathbf{L}}\hat{\mathbf{F}}+\left[\mathbf{V_c}\hat{\mathbf{L}}\hat{\mathbf{F}}\right]+\mathbf{V_c}\hat{\mathbf{L}}\circ\mathbf{E}'_{bil}+\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{tot}-\mathbf{V_c}\hat{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]$
K×KN	$\mathbf{S}'_n\left[\mathbf{V_c L E}^{\vee}_{bil(KN\times KN)}\right]=\mathbf{S}'_n\left[\mathbf{V_c L F}^{\vee}_{(KN\times KN)}\right]+\mathbf{S}'_n\left[\mathbf{V_c L E}^{\vee}_{tot(KN\times KN)}\right]-$ $-\mathbf{S}'_n\left[\mathbf{V_c L}^{\vee}\mathbf{Z}^{*}\right]$	$\mathbf{S}'_n\left[\mathbf{V_c L E}^{\vee}_{bil(KN\times KN)}\right]=\mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\hat{\mathbf{F}}_{(KN\times KN)}+\mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\hat{\mathbf{F}}_{(KN\times KN)}+\mathbf{S}'_n\left[\mathbf{V_c}\hat{\mathbf{L}}\hat{\mathbf{F}}^{\vee}_{(KN\times KN)}\right]+$ $+\mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\circ\mathbf{E}'_{bil(KN\times KN)}+\mathbf{S}'_n\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{tot(KN\times KN)}-\mathbf{V_c}\hat{\mathbf{L}}\circ\mathbf{E}'_{bil(KN\times KN)}\right]-\mathbf{S}'_n\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{Z}^{*}\right]$
K×K	$\mathbf{S}'_n\left[\mathbf{V_c L}^{\vee}\mathbf{E}_{bil}\right]=\mathbf{S}'_n\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{F}\right]+\mathbf{S}'_n\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{tot}\right]$	$\mathbf{S}'_n\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil}\right]=\mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\hat{\mathbf{F}}+\mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\hat{\mathbf{F}}+\mathbf{S}'_n\left[\mathbf{V_c}\hat{\mathbf{L}}\hat{\mathbf{F}}\right]+\mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\circ\mathbf{E}'_{bil}+$ $+\mathbf{S}'_n\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{tot}-\mathbf{V_c}\hat{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]$
Bilateral gross exports accounting equation		
KN×KN	The equation has no meaningful interpretation	The equation has no meaningful interpretation

KN×K	$\mathbf{E}_{bil} = \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right] + \mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{E}}_{bil}\right] + \left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\check{\mathbf{L}}\right]\mathbf{E}_{bil} +$ $+\left(\mathbf{I} - \left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\mathbf{L}\right]\right)\mathbf{E}_{bil}$	$\mathbf{E}_{bil} = \mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}} + \mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}} + \left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{F}}\right] + \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil} + \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right] -$ $-\left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{E}}_{bil}\right] + \left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\check{\mathbf{L}}\right]\mathbf{E}_{bil} + \left(\mathbf{I} - \left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\mathbf{L}\right]\right)\mathbf{E}_{bil}$
K×KN	$\mathbf{S}'_n\mathbf{E}_{bil(KN\times KN)} = \mathbf{S}'_n\left[\mathbf{V_c}\mathbf{L}\check{\mathbf{F}}_{(KN\times KN)}\right] + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot(KN\times KN)} -$ $-\mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{E}}_{bil(KN\times KN)}\right] + \mathbf{S}'_n\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\check{\mathbf{L}}\right]\mathbf{E}_{bil(KN\times KN)} - \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{Z}^*\right]$	$\mathbf{S}'_n\mathbf{E}_{bil(KN\times KN)} = \mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}}_{(KN\times KN)} + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}}_{(KN\times KN)} + \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{F}}_{(KN\times KN)}\right] +$ $+\mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil(KN\times KN)} + \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot(KN\times KN)} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil(KN\times KN)}\right] -$ $-\mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{E}}_{bil(KN\times KN)}\right] - \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{Z}^*\right]$
K×K	$\mathbf{S}'_n\mathbf{E}_{bil} = \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right] + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{E}}_{bil}\right] + \mathbf{S}'_n\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\check{\mathbf{L}}\right]\mathbf{E}_{bil}$	$\mathbf{S}'_n\mathbf{E}_{bil} = \mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}} + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}} + \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{F}}\right] + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil} +$ $+\mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right] - \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{E}}_{bil}\right] + \mathbf{S}'_n\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\check{\mathbf{L}}\right]\mathbf{E}_{bil}$
Total value added accounting equation: value added in total exports		
KN×1	$\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}\right]\mathbf{i} = \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right]\mathbf{i} + \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}\right]\mathbf{i}$	$\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}\right]\mathbf{i} = \mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}}\mathbf{i} + \mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}}\mathbf{i} + \left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{F}}\right]\mathbf{i} + \left[\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i} +$ $+\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i}$
K×1	$\mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}\right]\mathbf{i} = \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right]\mathbf{i} + \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}\right]\mathbf{i}$	$\mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}\right]\mathbf{i} = \mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}}\mathbf{i} + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}}\mathbf{i} + \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{F}}\right]\mathbf{i} +$ $+\mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i} + \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i}$

Total gross exports accounting equation		
KN×1	$\mathbf{E}_{bil}\mathbf{i} = \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right]\mathbf{i} + \mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}\mathbf{i} - \left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i} +$ $+\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\check{\mathbf{L}}}\right]\mathbf{E}_{bil}\mathbf{i} + \left(\mathbf{I} - \left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\right]\right)\mathbf{E}_{bil}\mathbf{i} =$ $= \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right]\mathbf{i} + \left[\mathbf{V_c}\check{\hat{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i} + \left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\check{\mathbf{L}}}\right]\mathbf{E}_{bil}\mathbf{i} + \left(\mathbf{I} - \left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\right]\right)\mathbf{E}_{bil}\mathbf{i}$	$\mathbf{E}_{bil}\mathbf{i} = \mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}}\mathbf{i} + \mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}}\mathbf{i} + \left[\mathbf{V_c}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right]\mathbf{i} + \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\mathbf{i} + \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}\mathbf{i} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\mathbf{i}\right] -$ $-\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i} + \left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\check{\mathbf{L}}}\right]\mathbf{E}_{bil}\mathbf{i} + \left(\mathbf{I} - \left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\right]\right)\mathbf{E}_{bil}\mathbf{i} =$ $= \mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}}\mathbf{i} + \mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}}\mathbf{i} + \left[\mathbf{V_c}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right]\mathbf{i} + \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\mathbf{i} +$ $+\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\check{\mathbf{L}}}\right]\mathbf{E}_{bil}\mathbf{i} + \left(\mathbf{I} - \left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\right]\right)\mathbf{E}_{bil}\mathbf{i}$
K×1	$\mathbf{S}'_n\mathbf{E}_{bil}\mathbf{i} = \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right]\mathbf{i} + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}\mathbf{i} - \mathbf{S}'_n\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i} +$ $+\mathbf{S}'_n\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\check{\mathbf{L}}}\right]\mathbf{E}_{bil}\mathbf{i} =$ $= \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right]\mathbf{i} + \mathbf{S}'_n\left[\mathbf{V_c}\check{\hat{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i} + \mathbf{S}'_n\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\check{\mathbf{L}}}\right]\mathbf{E}_{bil}\mathbf{i}$	$\mathbf{S}'_n\mathbf{E}_{bil}\mathbf{i} = \mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}}\mathbf{i} + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}}\mathbf{i} + \mathbf{S}'_n\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right]\mathbf{i} + \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i} +$ $+\mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i} - \mathbf{S}'_n\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i} + \mathbf{S}'_n\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\check{\mathbf{L}}}\right]\mathbf{E}_{bil}\mathbf{i} =$ $= \mathbf{S}'_n\mathbf{E}_{bil}\mathbf{i} = \mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}}\mathbf{i} + \mathbf{S}'_n\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}}\mathbf{i} + \mathbf{S}'_n\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right]\mathbf{i} +$ $+\mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i} + \mathbf{S}'_n\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\check{\mathbf{L}}}\right]\mathbf{E}_{bil}\mathbf{i}$
Total imports (value added and gross) accounting equation		

1×KN	$\mathbf{i}'\mathbf{V_c}\mathbf{L}\mathbf{E}_{bil\,(KN\times KN)} = \mathbf{i}'\mathbf{E}_{bil\,(KN\times KN)} = \mathbf{i}'\left[\mathbf{V_c}\mathbf{L}\check{\mathbf{F}}_{(KN\times KN)}\right] + \mathbf{i}'\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot\,(KN\times KN)} +$ $+\mathbf{i}'\left[\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{E}}_{bil\,(KN\times KN)}\right] - \mathbf{i}'\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{Z}^*\right]$	$\mathbf{i}'\mathbf{V_c}\mathbf{L}\mathbf{E}_{bil\,(KN\times KN)} = \mathbf{i}'\mathbf{E}_{bil\,(KN\times KN)} = \mathbf{i}'\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}}_{(KN\times KN)} + \mathbf{i}'\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}}_{(KN\times KN)} +$ $+\mathbf{i}'\left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{F}}_{(KN\times KN)}\right] + \mathbf{i}'\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil\,(KN\times KN)} +$ $+\mathbf{i}'\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot\,(KN\times KN)} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil\,(KN\times KN)}\right] + \mathbf{i}'\left[\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{E}}_{bil\,(KN\times KN)}\right] - \mathbf{i}'\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{Z}^*\right]$
1×K	$\mathbf{i}'\mathbf{V_c}\mathbf{L}\mathbf{E}_{bil} = \mathbf{i}'\mathbf{E}_{bil} = \mathbf{i}'\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right] + \mathbf{i}'\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} + \mathbf{i}'\left[\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{E}}_{bil}\right]$	$\mathbf{i}'\mathbf{V_c}\mathbf{L}\mathbf{E}_{bil} = \mathbf{i}'\mathbf{E}_{bil} = \mathbf{i}'\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}} + \mathbf{i}'\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}} + \mathbf{i}'\left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{F}}\right] + \mathbf{i}'\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil} +$ $+\mathbf{i}'\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right] + \mathbf{i}'\left[\mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{E}}_{bil}\right]$

Notes:
KN×KN = [country/sector]×[country/sector]
KN×K = [country/sector]×country
K×KN = country×[country/sector]
K×K = country×country
KN×1 = [country/sector]×total
K×1 = country×total
1×KN = total×[country/sector]
1×K = total×country

Appendix B. Description and interpretation of matrices

Table B.1. Description and interpretation of matrices in the cumulative value added accounting framework (default dimension is KN×KN = [country/sector]×[country/sector])

Compact matrix representation	Compact matrix interpretation	Zoom in view on matrix	Interpretation of elements *ij, rs**
$\left[\mathbf{V_c}\mathbf{L}\check{\mathbf{E}}_{bil\,(KN\times KN)}\right]$	bilateral "domestic value added in trade" matrix	$\begin{bmatrix} \mathbf{0} & \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\hat{\mathbf{e}}_{t2} & \cdots & \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\hat{\mathbf{e}}_{tk} \\ \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\hat{\mathbf{e}}_{t1} & \mathbf{0} & \cdots & \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\hat{\mathbf{e}}_{tk} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\hat{\mathbf{e}}_{t1} & \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\hat{\mathbf{e}}_{t2} & \cdots & \mathbf{0} \end{bmatrix}$	value added that originates in sector *i* of country *r* and via both direct and indirect trade flows is embodied in products of sector *j* used in country *s* to satisfy aggregate (intermediate plus final) demand in country *s*
$\left[\mathbf{V_c}\mathbf{L}\check{\mathbf{F}}_{(KN\times KN)}\right]$	bilateral "domestic trade in value added" matrix	$\begin{bmatrix} \mathbf{0} & \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\hat{\mathbf{f}}_{t2} & \cdots & \mathbf{V}_{c,1}\sum_{t=1}^{K}\mathbf{L}_{1t}\hat{\mathbf{f}}_{tk} \\ \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\hat{\mathbf{f}}_{t1} & \mathbf{0} & \cdots & \mathbf{V}_{c,2}\sum_{t=1}^{K}\mathbf{L}_{2t}\hat{\mathbf{f}}_{tk} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\hat{\mathbf{f}}_{t1} & \mathbf{V}_{c,k}\sum_{t=1}^{K}\mathbf{L}_{kt}\hat{\mathbf{f}}_{t2} & \cdots & \mathbf{0} \end{bmatrix}$	value added that originates in sector *i* of country *r* and via both direct and indirect trade flows is embodied in products of sector *j* used in country *s* to satisfy final demand in country *s*
$\left[\mathbf{V_c}\mathbf{L}\check{\mathbf{E}}_{tot\,(KN\times KN)}\right]=\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot\,(KN\times KN)}$	"domestic value added in total trade" matrix	$\begin{bmatrix} 0 & \mathbf{V}_{c,1}\mathbf{L}_{12}\hat{\mathbf{e}}_2 & \cdots & \mathbf{V}_{c,1}\mathbf{L}_{1k}\hat{\mathbf{e}}_k \\ \mathbf{V}_{c,2}\mathbf{L}_{21}\hat{\mathbf{e}}_1 & 0 & \cdots & \mathbf{V}_{c,2}\mathbf{L}_{2k}\hat{\mathbf{e}}_k \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\mathbf{L}_{k1}\hat{\mathbf{e}}_1 & \mathbf{V}_{c,k}\mathbf{L}_{k2}\hat{\mathbf{e}}_2 & \cdots & 0 \end{bmatrix}$	value added that originates in sector *i* of country *r* and via both direct and indirect trade flows is embodied in products of sector *j* used by country *s* for its global exports of products of sector *j*

$\mathbf{V}_c \hat{\mathbf{L}} \check{\mathbf{F}}_{(KN\times KN)}$	direct bilateral "domestic trade in value added" in final products matrix	$\begin{bmatrix} \mathbf{0} & \mathbf{V}_{c,1}\mathbf{L}_{11}\hat{\mathbf{f}}_{12} & \cdots & \mathbf{V}_{c,1}\mathbf{L}_{11}\hat{\mathbf{f}}_{1k} \\ \mathbf{V}_{c,2}\mathbf{L}_{22}\hat{\mathbf{f}}_{21} & \mathbf{0} & \cdots & \mathbf{V}_{c,2}\mathbf{L}_{22}\hat{\mathbf{f}}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\mathbf{L}_{kk}\hat{\mathbf{f}}_{k1} & \mathbf{V}_{c,k}\mathbf{L}_{kk}\hat{\mathbf{f}}_{k2} & \cdots & \mathbf{0} \end{bmatrix}$	value added that originates in sector *i* of country *r* and via direct trade flows is embodied in products of sector *j* made in country *r* for final demand in country *s*
$\mathbf{V}_c \check{\mathbf{L}} \hat{\mathbf{F}}_{(KN\times KN)}$	bilateral "domestic trade in value added" in intermediate products matrix	$\begin{bmatrix} \mathbf{0} & \mathbf{V}_{c,1}\mathbf{L}_{12}\hat{\mathbf{f}}_{22} & \cdots & \mathbf{V}_{c,1}\mathbf{L}_{1k}\hat{\mathbf{f}}_{kk} \\ \mathbf{V}_{c,2}\mathbf{L}_{21}\hat{\mathbf{f}}_{11} & \mathbf{0} & \cdots & \mathbf{V}_{c,2}\mathbf{L}_{2k}\hat{\mathbf{f}}_{kk} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\mathbf{L}_{k1}\hat{\mathbf{f}}_{11} & \mathbf{V}_{c,k}\mathbf{L}_{k2}\hat{\mathbf{f}}_{22} & \cdots & \mathbf{0} \end{bmatrix}$	value added that originates in sector *i* of country *r* and via both direct and indirect trade flows is embodied in products of sector *j* made in country *s* for final demand in country *s*
$\left[\mathbf{V}_c \check{\mathbf{L}} \check{\mathbf{F}}_{(KN\times KN)}\right]$	indirect bilateral "domestic trade in value added" matrix	$\begin{bmatrix} \mathbf{0} & \mathbf{V}_{c,1}\sum_{t\neq 1,2}^{K}\mathbf{L}_{1t}\hat{\mathbf{f}}_{t2} & \cdots & \mathbf{V}_{c,1}\sum_{t\neq 1,k}^{K}\mathbf{L}_{1t}\hat{\mathbf{f}}_{tk} \\ \mathbf{V}_{c,2}\sum_{t\neq 1,2}^{K}\mathbf{L}_{2t}\hat{\mathbf{f}}_{t1} & \mathbf{0} & \cdots & \mathbf{V}_{c,2}\sum_{t\neq 2,k}^{K}\mathbf{L}_{2t}\hat{\mathbf{f}}_{tk} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\sum_{t\neq 1,k}^{K}\mathbf{L}_{kt}\hat{\mathbf{f}}_{t1} & \mathbf{V}_{c,k}\sum_{t\neq 2,k}^{K}\mathbf{L}_{kt}\hat{\mathbf{f}}_{t2} & \cdots & \mathbf{0} \end{bmatrix}$	value added that originates in sector *i* of country *r* and via indirect trade flows is embodied in products of sector *j* made in third countries for final demand in country *s*
$\mathbf{V}_c \check{\mathbf{L}} \circ \mathbf{E}'_{bil\,(KN\times KN)}$	bilateral "domestic reflected value added in trade" matrix	$\begin{bmatrix} \mathbf{0} & \mathbf{V}_{c,1}\mathbf{L}_{12}\hat{\mathbf{e}}_{21} & \cdots & \mathbf{V}_{c,1}\mathbf{L}_{1k}\hat{\mathbf{e}}_{k1} \\ \mathbf{V}_{c,2}\mathbf{L}_{21}\hat{\mathbf{e}}_{12} & \mathbf{0} & \cdots & \mathbf{V}_{c,2}\mathbf{L}_{2k}\hat{\mathbf{e}}_{k2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\mathbf{L}_{k1}\hat{\mathbf{e}}_{1k} & \mathbf{V}_{c,k}\mathbf{L}_{k2}\hat{\mathbf{e}}_{2k} & \cdots & \mathbf{0} \end{bmatrix}$	value added that originates in sector *i* of country *r* and via both direct and indirect trade flows is embodied in products of sector *j* used by country *s* for its exports of products of sector *j* back to country *r* for both final and intermediate use

$\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot(KN\times KN)} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil(KN\times KN)}\right]$	bilateral "domestic redirected value added in trade" matrix	$\begin{bmatrix} \mathbf{0} & \mathbf{V}_{c,1}\mathbf{L}_{12}\sum_{t\neq 1}^{K}\hat{\mathbf{e}}_{2t} & \cdots & \mathbf{V}_{c,1}\mathbf{L}_{1k}\sum_{t\neq 1}^{K}\hat{\mathbf{e}}_{kt} \\ \mathbf{V}_{c,2}\mathbf{L}_{21}\sum_{t\neq 2}^{K}\hat{\mathbf{e}}_{1t} & \mathbf{0} & \cdots & \mathbf{V}_{c,2}\mathbf{L}_{2k}\sum_{t\neq 2}^{K}\hat{\mathbf{e}}_{kt} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\mathbf{L}_{k1}\sum_{t\neq k}^{K}\hat{\mathbf{e}}_{1t} & \mathbf{V}_{c,k}\mathbf{L}_{k2}\sum_{t\neq k}^{K}\hat{\mathbf{e}}_{2t} & \cdots & \mathbf{0} \end{bmatrix}$	value added that originates in sector *i* of country *r* and via both direct and indirect trade flows is embodied in products of sector *j* used by country *s* for its exports of products of sector *j* to third countries for both final and intermediate use
$\left[\mathbf{V_c}\check{\mathbf{L}}\check{\mathbf{E}}_{bil(KN\times KN)}\right]$	indirect bilateral "domestic value added in trade" matrix	$\begin{bmatrix} \mathbf{0} & \mathbf{V}_{c,1}\sum_{t\neq 1}^{K}\mathbf{L}_{1t}\hat{\mathbf{e}}_{t2} & \cdots & \mathbf{V}_{c,1}\sum_{t\neq 1}^{K}\mathbf{L}_{1t}\hat{\mathbf{e}}_{tk} \\ \mathbf{V}_{c,2}\sum_{t\neq 2}^{K}\mathbf{L}_{2t}\hat{\mathbf{e}}_{t1} & \mathbf{0} & \cdots & \mathbf{V}_{c,2}\sum_{t\neq 2}^{K}\mathbf{L}_{2t}\hat{\mathbf{e}}_{tk} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\sum_{t\neq k}^{K}\mathbf{L}_{kt}\hat{\mathbf{e}}_{t1} & \mathbf{V}_{c,k}\sum_{t\neq k}^{K}\mathbf{L}_{kt}\hat{\mathbf{e}}_{t2} & \cdots & \mathbf{0} \end{bmatrix}$	value added that originates in sector *i* of country *r* and via indirect trade flows is embodied in products of sector *j* used in country *s* to satisfy aggregate (intermediate plus final) demand in country *s*
$\left[\mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{E}}_{bil(KN\times KN)}\right] = \mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil(KN\times KN)}$	direct bilateral "domestic value added in trade" matrix	$\begin{bmatrix} \mathbf{0} & \mathbf{V}_{c,1}\mathbf{L}_{11}\hat{\mathbf{e}}_{12} & \cdots & \mathbf{V}_{c,1}\mathbf{L}_{11}\hat{\mathbf{e}}_{1k} \\ \mathbf{V}_{c,2}\mathbf{L}_{22}\hat{\mathbf{e}}_{21} & \mathbf{0} & \cdots & \mathbf{V}_{c,2}\mathbf{L}_{22}\hat{\mathbf{e}}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{V}_{c,k}\mathbf{L}_{kk}\hat{\mathbf{e}}_{k1} & \mathbf{V}_{c,k}\mathbf{L}_{kk}\hat{\mathbf{e}}_{k2} & \cdots & \mathbf{0} \end{bmatrix}$	value added that originates in sector *i* of country *r* and via direct trade flows is embodied in products of sector *j* used in country *s* to satisfy aggregate (intermediate plus final) demand in country *s*
$\left[\hat{\mathbf{V}}_{c(N\times KN)}\check{\mathbf{L}}\right]\mathbf{E}_{bil(KN\times KN)}$	bilateral "foreign value added in trade" matrix	$\begin{bmatrix} \mathbf{0} & \left[\sum_{t\neq 1}^{K}\mathbf{V}_{c,t}\mathbf{L}_{t1}\right]\hat{\mathbf{e}}_{12} & \cdots & \left[\sum_{t\neq 1}^{K}\mathbf{V}_{c,t}\mathbf{L}_{t1}\right]\hat{\mathbf{e}}_{13} \\ \left[\sum_{t\neq 2}^{K}\mathbf{V}_{c,t}\mathbf{L}_{t2}\right]\hat{\mathbf{e}}_{21} & \mathbf{0} & \cdots & \left[\sum_{t\neq 2}^{K}\mathbf{V}_{c,t}\mathbf{L}_{t2}\right]\hat{\mathbf{e}}_{23} \\ \vdots & \vdots & \ddots & \vdots \\ \left[\sum_{t\neq k}^{K}\mathbf{V}_{c,t}\mathbf{L}_{tk}\right]\hat{\mathbf{e}}_{k1} & \left[\sum_{t\neq k}^{K}\mathbf{V}_{c,t}\mathbf{L}_{tk}\right]\hat{\mathbf{e}}_{k2} & \cdots & \mathbf{0} \end{bmatrix}$	value added that originates in sector *i* of countries other than *r* and via both direct and indirect trade flows is embodied in products of sector *j* used in country *s* to satisfy aggregate (intermediate plus final) demand in country *s*

$[\mathbf{V_c}\check{\mathbf{L}}\mathbf{Z}^*]$	matrix of inter-sectoral transfer of value added on the partner coutnry's side	see Appendix D	see Appendix D

Note: * *i* – sector of origin in the exporting country, *j* – sector of destination in the partner country (where the value added flows from *i* are embodied), *r* – exporting country, *s* – partner country.

Table B.2. Description and interpretation of matrices and vectors of other dimensions (other than the default dimension is KN×KN = [country/sector]×[country/sector])

Compact matrix representation	Compact matrix interpretation	Zoom in view on matrix	Interpretation of elements *ij, rs**
$\left(\mathbf{I}-\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\right]\right)\mathbf{E}_{bil}$	matrix of inter-sectoral transfer of value added on the exporting country's side	see Appendix C	see Appendix C
$\left[\mathbf{V_c}\hat{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i}$	vector of total "reflected value added in exports"	results from $\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}\mathbf{i}-\left[\mathbf{V_c}\hat{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i}$, see respective matrices above	value added that originates in sector *i* of country *r* and via both direct and indirect trade flows is embodied in products of sector *j* used by all partner countries for their exports of products of sector *j* back to country *r* for both final and intermediate use

Note: * *i* – sector of origin in the exporting country, *j* – sector of destination in the partner country (where the value added flows from *i* are embodied), *r* – exporting country, *s* – partner country.

Appendix C. Matrix of inter-sectoral transfer of value added on the exporting countries' side

As explained in subsection 3.2, the matrix of inter-sectoral transfer of value added on the exporting countries' side is required to account for the deviation of the sectoral value added in gross exports from the sectoral gross exports. For example, the exports of manufactured products may embody value added created in agriculture or services sectors. So the value added originating in manufacturing tends to be less than the gross exports of manufacturing, and the value added originating in services tends to exceed the observed gross exports of services.

Below is a zoom in view on this matrix and its computation:

$$\left(\mathbf{I}-\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\right]\right)\mathbf{E}_{bil}=\left(\begin{bmatrix}\mathbf{I} & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & \mathbf{I} & \cdots & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I}\end{bmatrix}-\begin{bmatrix}\left[\sum_{\mathbf{t=1}}^{\mathbf{K}}\mathbf{V}_{\mathbf{c,t}}\mathbf{L}_{\mathbf{t1}}\right] & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & \left[\sum_{\mathbf{t=1}}^{\mathbf{K}}\mathbf{V}_{\mathbf{c,t}}\mathbf{L}_{\mathbf{t2}}\right] & \cdots & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{0} & \mathbf{0} & \cdots & \left[\sum_{\mathbf{t=1}}^{\mathbf{K}}\mathbf{V}_{\mathbf{c,t}}\mathbf{L}_{\mathbf{tk}}\right]\end{bmatrix}\right)\times\begin{bmatrix}\mathbf{0} & \mathbf{e}_{\mathbf{12}} & \cdots & \mathbf{e}_{\mathbf{1k}}\\ \mathbf{e}_{\mathbf{21}} & \mathbf{0} & \cdots & \mathbf{e}_{\mathbf{2k}}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{e}_{\mathbf{k1}} & \mathbf{e}_{\mathbf{k2}} & \cdots & \mathbf{0}\end{bmatrix}=$$

$$=\begin{bmatrix}\mathbf{0} & \left(\mathbf{I}-\sum_{\mathbf{t=1}}^{\mathbf{K}}\mathbf{V}_{\mathbf{c,t}}\mathbf{L}_{\mathbf{t1}}\right)\mathbf{e}_{\mathbf{12}} & \cdots & \left(\mathbf{I}-\sum_{\mathbf{t=1}}^{\mathbf{K}}\mathbf{V}_{\mathbf{c,t}}\mathbf{L}_{\mathbf{t1}}\right)\mathbf{e}_{\mathbf{13}}\\ \left(\mathbf{I}-\sum_{\mathbf{t=1}}^{\mathbf{K}}\mathbf{V}_{\mathbf{c,t}}\mathbf{L}_{\mathbf{t2}}\right)\mathbf{e}_{\mathbf{21}} & \mathbf{0} & \cdots & \left(\mathbf{I}-\sum_{\mathbf{t=1}}^{\mathbf{K}}\mathbf{V}_{\mathbf{c,t}}\mathbf{L}_{\mathbf{t2}}\right)\mathbf{e}_{\mathbf{23}}\\ \vdots & \vdots & \ddots & \vdots\\ \left(\mathbf{I}-\sum_{\mathbf{t=1}}^{\mathbf{K}}\mathbf{V}_{\mathbf{c,t}}\mathbf{L}_{\mathbf{tk}}\right)\mathbf{e}_{\mathbf{k1}} & \left(\mathbf{I}-\sum_{\mathbf{t=1}}^{\mathbf{K}}\mathbf{V}_{\mathbf{c,t}}\mathbf{L}_{\mathbf{tk}}\right)\mathbf{e}_{\mathbf{k2}} & \cdots & \mathbf{0}\end{bmatrix}$$

In the following views on the structure of individual block elements, lower indices *r* and *s* denote, respectively, exporting country and partner country. Upper indices denote inter-industry direction of flow, from sector *i* to sector *j*. The matrix $\mathbf{I}-\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\right]$ has only diagonal block elements as below:

$$\left[\mathbf{I}-\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\right]\right]_{rr}=\begin{bmatrix}1-\sum_{t=1}^{\mathbf{K}}v_{c,t}^{1}l_{tr}^{11} & -\sum_{t=1}^{\mathbf{K}}v_{c,t}^{1}l_{tr}^{12} & \cdots & -\sum_{t=1}^{\mathbf{K}}v_{c,t}^{1}l_{tr}^{1n}\\ -\sum_{t=1}^{\mathbf{K}}v_{c,t}^{2}l_{tr}^{21} & 1-\sum_{t=1}^{\mathbf{K}}v_{c,t}^{2}l_{tr}^{22} & \cdots & -\sum_{t=1}^{\mathbf{K}}v_{c,t}^{2}l_{tr}^{2n}\\ \vdots & \vdots & \ddots & \vdots\\ -\sum_{t=1}^{\mathbf{K}}v_{c,t}^{n}l_{tr}^{n1} & -\sum_{t=1}^{\mathbf{K}}v_{c,t}^{n}l_{tr}^{n1} & \cdots & 1-\sum_{t=1}^{\mathbf{K}}v_{c,t}^{n}l_{tr}^{nn}\end{bmatrix}$$

It is apparent that the summation of the columns of each block element in $\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\right]$ is equal to the summation of the columns in $\mathbf{V}_{\mathbf{c}}\mathbf{L}$ and will therefore yield a vector of ones (again, provided that

$\mathbf{v} = \mathbf{x'} - \mathbf{i'Z}$). This confirms that the aggregation across exporting country's sectors will make $\mathbf{I} - \left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\right]$ equal to zero: $\mathbf{S}'_n\left(\mathbf{I} - \left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\right]\right) = 0$.

A block in the resulting $\left(\mathbf{I} - \left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\right]\right)\mathbf{E}_{bil}$ matrix is diagonalised row-wise before summation for clarity:

$$\left[\left(\mathbf{I} - \left[\mathbf{V}_{\mathbf{c}(K\times KN)}\hat{\mathbf{L}}\right]\right)\mathbf{E}_{bil}\right]_{rs} = \begin{bmatrix} \left(1-\sum_{t=1}^{\mathbf{K}} v_{c,t}^{1} l_{tr}^{11}\right)e_{rs}^{1} & \left(-\sum_{t=1}^{\mathbf{K}} v_{c,t}^{1} l_{tr}^{12}\right)e_{rs}^{2} & \cdots & \left(-\sum_{t=1}^{\mathbf{K}} v_{c,t}^{1} l_{tr}^{1n}\right)e_{rs}^{n} \\ \left(-\sum_{t=1}^{\mathbf{K}} v_{c,t}^{2} l_{tr}^{21}\right)e_{rs}^{1} & \left(1-\sum_{t=1}^{\mathbf{K}} v_{c,t}^{2} l_{tr}^{22}\right)e_{rs}^{2} & \cdots & \left(-\sum_{t=1}^{\mathbf{K}} v_{c,t}^{2} l_{tr}^{2n}\right)e_{rs}^{n} \\ \vdots & \vdots & \ddots & \vdots \\ \left(-\sum_{t=1}^{\mathbf{K}} v_{c,t}^{n} l_{tr}^{n1}\right)e_{rs}^{1} & \left(-\sum_{t=1}^{\mathbf{K}} v_{c,t}^{n} l_{tr}^{n1}\right)e_{rs}^{2} & \cdots & \left(1-\sum_{t=1}^{\mathbf{K}} v_{c,t}^{n} l_{tr}^{nn}\right)e_{rs}^{n} \end{bmatrix} \times \mathbf{i}$$

In the above representation, a diagonal element, e.g. $1-\sum_{t=1}^{\mathbf{K}} v_{c,t}^{1} l_{tr}^{11}$ corresponding to sector 1, is equal to the value added from all sectors other than sector 1 of all countries embodied in the exports of products of sector 1 from country *r* to country *s*. An off-diagonal element in the same row, e.g. $-\sum_{t=1}^{\mathbf{K}} v_{c,t}^{1} l_{tr}^{12}$, is equal to the negative flow of value added from sector 1 of all countries embodied in the exports of products of sector 2 from country *r* to country *s*, and similarly for the remaining off-diagonal elements. So each row sum for sector *i* gives other sectors' value added embodied in the exports of sector *i*'s products less sector *i*'s value added in exports of other sectors' products. The positive values signify that sector *i* is a net importer of the sectoral value added, and the negative values signify that it is a net exporter of the sectoral value added.

The identity matrix **I** in the formulae above may be replaced with $\left[\mathbf{i}'\hat{\mathbf{V}}_{\mathbf{c}}\mathbf{L}\right]$ as in equation (7) to account for the fact that **v** is usually not equal to $\mathbf{x'} - \mathbf{i'Z}$ in real world datasets.

Appendix D. Matrix of inter-sectoral transfer of value added on the partner countries' side

In subsection 3.7, the matrices $\mathbf{Z}^*$ and $\mathbf{V_cLZ}^*$ have been constructed to allow for the extension of the framework to the KN×KN and K×KN dimensions. Recall that $\mathbf{V_cLZ}^*$ accounts for the inter-sectoral transfer of value added embodied in intermediate products on their way from the sector of origin *i* to the sector of destination *j*. In fact, this is a matrix of the inter-sectoral transfer of value added for which the partner country is responsible:

$$\mathbf{V_cLZ^*} = \mathbf{V_cL} \times \left(\begin{bmatrix} \mathbf{Z_{11}} & \mathbf{Z_{12}} & \cdots & \mathbf{Z_{1k}} \\ \mathbf{Z_{21}} & \mathbf{Z_{22}} & \cdots & \mathbf{Z_{2k}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Z_{k1}} & \mathbf{Z_{k2}} & \cdots & \mathbf{Z_{kk}} \end{bmatrix} - \begin{bmatrix} [\hat{\mathbf{Z}}_{\mathbf{11}}\mathbf{i}] & [\hat{\mathbf{Z}}_{\mathbf{12}}\mathbf{i}] & \cdots & [\hat{\mathbf{Z}}_{\mathbf{1k}}\mathbf{i}] \\ [\hat{\mathbf{Z}}_{\mathbf{21}}\mathbf{i}] & [\hat{\mathbf{Z}}_{\mathbf{22}}\mathbf{i}] & \cdots & [\hat{\mathbf{Z}}_{\mathbf{2k}}\mathbf{i}] \\ \vdots & \vdots & \ddots & \vdots \\ [\hat{\mathbf{Z}}_{\mathbf{k1}}\mathbf{i}] & [\hat{\mathbf{Z}}_{\mathbf{k2}}\mathbf{i}] & \cdots & [\hat{\mathbf{Z}}_{\mathbf{kk}}\mathbf{i}] \end{bmatrix} \right)$$

To secure conformance with other matrices in equation (30), diagonal block elements need to be removed. Then, a zoom in view on an off-diagonal block in the $\mathbf{Z}^*$ matrix is given below. Lower indices *r* and *s* denote, respectively, exporting country and partner country. Upper indices denote inter-industry direction of flow, from sector *i* to sector *j* (so index *u* counts purchasing sectors).

$$\mathbf{Z}^*_{rs} = \begin{bmatrix} z_{rs}^{11} - \sum_{u=1}^{\mathbf{N}} z_{rs}^{1u} & z_{rs}^{12} & \cdots & z_{rs}^{1n} \\ z_{rs}^{21} & z_{rs}^{22} - \sum_{u=1}^{\mathbf{N}} z_{rs}^{2u} & \cdots & z_{rs}^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ z_{rs}^{n1} & z_{rs}^{n2} & \cdots & z_{rs}^{nn} - \sum_{u=1}^{\mathbf{N}} z_{rs}^{nu} \end{bmatrix}$$

Each block $\mathbf{Z}_{rs}$ then has negative entries in the diagonal elements and positive entries elsewhere.

A block in the resulting matrix $\mathbf{V_cLZ}^*$ is then as follows:

$$[\mathbf{V_cLZ^*}]_{rs} = \begin{bmatrix} v_{c,r}^1 \sum_{t=1}^{\mathbf{K}} \sum_{u=1}^{\mathbf{N}} \left(l_{rt}^{1u} z_{ts}^{u1} - l_{rt}^{11} z_{ts}^{1u} \right) & v_{c,r}^1 \sum_{t=1}^{\mathbf{K}} \sum_{u=1}^{\mathbf{N}} \left(l_{rt}^{1u} z_{ts}^{u2} - l_{rt}^{12} z_{ts}^{2u} \right) & \cdots & v_{c,r}^1 \sum_{t=1}^{\mathbf{K}} \sum_{u=1}^{\mathbf{N}} \left(l_{rt}^{1u} z_{ts}^{un} - l_{rt}^{1n} z_{ts}^{nu} \right) \\ v_{c,r}^2 \sum_{t=1}^{\mathbf{K}} \sum_{u=1}^{\mathbf{N}} \left(l_{rt}^{2u} z_{ts}^{u1} - l_{rt}^{21} z_{ts}^{1u} \right) & v_{c,r}^2 \sum_{t=1}^{\mathbf{K}} \sum_{u=1}^{\mathbf{N}} \left(l_{rt}^{2u} z_{ts}^{u2} - l_{rt}^{22} z_{ts}^{2u} \right) & \cdots & v_{c,r}^2 \sum_{t=1}^{\mathbf{K}} \sum_{u=1}^{\mathbf{N}} \left(l_{rt}^{2u} z_{ts}^{un} - l_{rt}^{2n} z_{ts}^{nu} \right) \\ \vdots & \vdots & \ddots & \vdots \\ v_{c,r}^n \sum_{t=1}^{\mathbf{K}} \sum_{u=1}^{\mathbf{N}} \left(l_{rt}^{nu} z_{ts}^{u1} - l_{rt}^{n1} z_{ts}^{1u} \right) & v_{c,r}^n \sum_{t=1}^{\mathbf{K}} \sum_{u=1}^{\mathbf{N}} \left(l_{rt}^{nu} z_{ts}^{u2} - l_{rt}^{n2} z_{ts}^{2u} \right) & \cdots & v_{c,r}^n \sum_{t=1}^{\mathbf{K}} \sum_{u=1}^{\mathbf{N}} l_{rt}^{nu} z_{ts}^{un} - l_{rt}^{nn} z_{ts}^{nu} \end{bmatrix}$$

One should carefully interpret this matrix looking at the elements in each block $[\mathbf{V_cLZ^*}]_{rs}$. From the exporting country *r*'s perspective, each element represents the net value added from sector *i* of country *r* indirectly embodied in intermediate inputs of sector *j* in country *s* supplied to other sectors in country *s*. Unlike in other matrices in the KN×KN dimension – $[\mathbf{V_cLE}^{\vee}_{bil(KN\times KN)}]$, $[\mathbf{V_cLF}^{\vee}_{(KN\times KN)}]$,

$\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot(KN\times KN)}$ – where due to the original dimension of $\mathbf{E}_{\text{bil}}$, $\mathbf{F}$ and $\mathbf{E}_{\text{tot}}$ index *j* relates to "products of sector *j*", in $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{Z}^*\right]$ *j* can be interpreted as "sector *j*".

Diagonal elements in each block $\left[\mathbf{V_c}\mathbf{L}\mathbf{Z}^*\right]_{rs}$ are often negative, e.g. $v_{c,r}^{1}\sum_{t=1}^{\mathbf{K}}\sum_{u=1}^{\mathbf{N}}\left(l_{rt}^{1u}z_{ts}^{u1}-l_{rt}^{11}z_{ts}^{1u}\right)$, meaning that sector 1 in partner country *s* is a net supplier of value added created in sector 1 of exporting country *r* to other sectors in country *s*. Off-diagonal elements are often positive, e.g. $v_{c,r}^{1}\sum_{t=1}^{\mathbf{K}}\sum_{u=1}^{\mathbf{N}}\left(l_{rt}^{1u}z_{ts}^{u2}-l_{rt}^{12}z_{ts}^{2u}\right)$, meaning that sector 2 in partner country *s* is a net recipient of value added created in sector 1 of exporting country *r* with respect to other sectors in country *s*. For off-diagonal elements in a block of $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{Z}^*\right]$, it may appear that the sum of respective *i,j*th elements in $\left[\mathbf{V_c}\mathbf{L}\check{\mathbf{F}}_{(KN\times KN)}\right]$ and $\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot(KN\times KN)}$ are considerably larger than the *i,j*th element in $\left[\mathbf{V_c}\mathbf{L}\check{\mathbf{E}}_{bil(KN\times KN)}\right]$, i.e. value added of sector *i* in gross exports. Then the respective *i,j*th element in $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{Z}^*\right]$ shows that the products of sector *j* receive value added of sector *i* indirectly via other sectors in partner country.

Matrix $\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{Z}^*\right]$ therefore re-allocates value added of certain origin among partner country's sectors via trade in intermediate inputs. Recall that $\left(\mathbf{I}-\left[\mathbf{V}_{\mathbf{c}(N\times KN)}\hat{\mathbf{L}}\right]\right)\mathbf{E}_{bil}$ reallocates value added across exporting country's sectors.

Owing to the structure of the elements in $\mathbf{V_c}\mathbf{L}\mathbf{Z}^*$, the aggregation across partner country's sectors, i.e. row sums of each block of $\mathbf{V_c}\mathbf{L}\mathbf{Z}^*$, will produce a vector of zeros: $\left[\mathbf{V_c}\mathbf{L}\mathbf{Z}^*\right]\mathbf{S}_n=0$.

Appendix E. Equality of exporting country's value added re-exported by partners to third countries and exporting country's value added indirectly exported via third countries to partners: the proof

Start with equation (17):

$$\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{bil}\right]\mathbf{i} = \mathbf{V_c}\hat{\mathbf{L}}\check{\mathbf{F}}\mathbf{i} + \mathbf{V_c}\check{\mathbf{L}}\hat{\mathbf{F}}\mathbf{i} + \left[\mathbf{V_c}\check{\check{\mathbf{L}}}\check{\mathbf{F}}\right]\mathbf{i} + \left[\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i} + \left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i}$$

Split the term on the left side using (4), condense "trade in value added" terms on the right side using (10) and obtain an equation for the direct bilateral domestic value added in exports, aggregated across partner countries:

$$\mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil}\mathbf{i} = \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right]\mathbf{i} + \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i} + \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i} - \mathbf{S}'_n\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i} \qquad \text{(E.1)}$$

Next, take the diagonal block elements of the basic accounting relationship (2):

$$\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil}\right] = \left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{F}\right] + \left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{tot}\right] - \left[\mathbf{V_c}\hat{\hat{\mathbf{x}}}_{(KN\times K)}\right] = \left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{F}\right] + \mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\hat{\mathbf{x}}_{(KN\times K)}$$

Using that $\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil}\right] = \left[\mathbf{V_c}\hat{\hat{\mathbf{L}}}\mathbf{E}_{bil}\right] + \left[\mathbf{V_c}\hat{\check{\mathbf{L}}}\mathbf{E}_{bil}\right] = \left[\mathbf{V_c}\hat{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]$, rewrite the previous equation as:

$$\left[\mathbf{V_c}\hat{\check{\mathbf{L}}}\mathbf{E}_{bil}\right] = \left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{F}\right] + \mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{tot} - \mathbf{V_c}\hat{\mathbf{x}}_{(KN\times K)}$$

Aggregate across exporting country's sectors and across trading partners and plug-in $\mathbf{V_c}\hat{\mathbf{x}}_{(KN\times K)}\mathbf{i} = \mathbf{V_c}\mathbf{x} = \mathbf{V_c}\mathbf{L}\mathbf{F}\mathbf{i}$:

$$\mathbf{S}'_n\left[\mathbf{V_c}\hat{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i} = \mathbf{S}'_n\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{F}\right]\mathbf{i} + \mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{tot}\mathbf{i} - \mathbf{S}'_n\mathbf{V_c}\mathbf{L}\mathbf{F}\mathbf{i}$$

Using that $\mathbf{S}'_n\mathbf{V_c}\mathbf{L}\mathbf{F}\mathbf{i} - \mathbf{S}'_n\left[\mathbf{V_c}\hat{\mathbf{L}}\mathbf{F}\right]\mathbf{i} = \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right]\mathbf{i}$, rearrange as follows:

$$\mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{tot}\mathbf{i} = \mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{F}\right]\mathbf{i} + \mathbf{S}'_n\left[\mathbf{V_c}\hat{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i} \qquad \text{(E.2)}$$

Now compare (E.2) to (E.1) and note that $\mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{tot}\mathbf{i}$ is equal to $\mathbf{S}'_n\mathbf{V_c}\hat{\mathbf{L}}\mathbf{E}_{bil}\mathbf{i}$, so the left sides are equal. $\mathbf{S}'_n\left[\mathbf{V_c}\hat{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i}$ is equal to the sum of all reflected value added in exports

$\mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i}$. Then it follows that the difference between the two terms $\mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}-\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i}-\mathbf{S}'_n\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i}$ in (E.1) must be zero. Exporting country's value added re-exported by partners to third countries $\mathbf{S}'_n\left[\mathbf{V_c}\check{\mathbf{L}}\mathbf{E}_{tot}-\mathbf{V_c}\check{\mathbf{L}}\circ\mathbf{E}'_{bil}\right]\mathbf{i}$ must therefore equal exporting country's value added indirectly exported via third countries to partners $\mathbf{S}'_n\left[\mathbf{V_c}\check{\check{\mathbf{L}}}\mathbf{E}_{bil}\right]\mathbf{i}$. This is the end of the proof.

Мурадов, К. Ю. В поисках всеобъемлющей и гибкой системы учета добавленной стоимости в международной торговле [Электронный ресурс] : препринт WP2/2014/03 / К. Ю. Мурадов ; Нац. исслед. ун-т «Высшая школа экономики». – Электрон. текст. дан. (2 МБ). – М. : Изд. дом Высшей школы экономики, 2014. – (Серия WP2 «Количественный анализ в экономике»). – 71 с.

Статистическая декомпозиция экспортной торговли – новое направление исследований, в которых определяются страновое и отраслевое происхождение и назначение добавленной стоимости в совокупных торговых потоках. Для осуществления такой декомпозиции недавно был предложен ряд аналитических конструкций, которые рассматриваются и классифицируются в настоящей работе в рамках двух обобщенных понятий: статистическая декомпозиция экспортной торговли и статистический учет совокупной добавленной стоимости в торговле. Если первая из указанных тем была исчерпывающе изучена в работе Ч. Вана, Ш.-Ч. Вэя и К. Чжу (2013), то настоящее исследование нацелено на совершенствование и обобщение второй. Результаты в целом соответствуют тем, которые были получены Р. Купманом, Ч. Ваном и Ш.-Ч. Вэем (2012) и Р. Штерером (2013), однако обобщенная аналитическая конструкция обладает достаточно высокой вычислительной эффективностью и гибкостью для приспособления под конкретные цели анализа глобальных производственных цепочек. На основе усовершенствованной методологии описывается положение России как экспортера в системе глобальных производственных цепочек с использованием базы данных глобальных таблиц «затраты – выпуск» (WIOD) за 2000, 2005 и 2010 годы.

Классификация JEL: D57, F15

Ключевые слова: статистическая декомпозиция экспортной торговли, добавленная стоимость в торговле, глобальные производственные цепочки, межстрановые таблицы «затраты – выпуск»

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Препринт WP2/2014/03
Серия WP2
Количественный анализ в экономике

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В поисках всеобъемлющей и гибкой системы учета добавленной стоимости в международной торговле