An analysis of several novel frameworks and models in the consensus reaching process

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Outline

- Background: consensus reaching process
- Model I: consensus with minimum adjustments
- Model II: consensus based on consistency and consensus measures
- Model III: direct consensus framework
- Future research
Background: consensus reaching process

Fig. 1. A common consensus framework
Consensus with minimum adjustments

Motivation: How to minimize the adjustments?
Consensus with minimum adjustments

- Minimizing the distance between the original and adjusted preferences

- Minimizing the number of adjusted preference values
Minimizing the distance between the original and adjusted preferences

Basic model

\[
\min_{\overline{o}_k} \sum_{k=1}^{m} \left| \overline{o}_k - o_k \right|
\]

\[
\begin{cases}
\overline{o} = Ag(\overline{o}_1, \overline{o}_2, \ldots, \overline{o}_m) \\
\left| \overline{o}_k - \overline{o} \right| \leq \alpha, \; k = 1, 2, \ldots, m
\end{cases}
\]  

(1)


Minimizing the number of adjusted preference values

- Basic model

\[
\min \sum_{k=1}^{m} c_{O_k}
\]

\[
\bar{o} = Ag(\bar{o}_1, \bar{o}_2, ..., \bar{o}_m)
\]

s.t. \[
\sum_{k=1}^{m} (| \bar{o}_k - \bar{o} |) \leq \alpha
\]

\[
c_{O_k} = \begin{cases} 1, & o_k \neq \bar{o}_k \\ 0, & o_k = \bar{o}_k \end{cases}
\]

Consensus with minimum adjustments

Advantages

- Minimize adjustment amounts.
- Provide new references for experts to modify their preferences.
Consensus based on consistency and consensus measures

Motivation: How to preserve/improve individual consistency in consensus reaching process?
Consensus based on consistency and consensus measures

- **Iteration-based consensus model**
  
  (Take multiplicative preference relations as example)

- **Optimization-based consensus model**
  
  (Take additive preference relations as example)
Iteration-based consensus model

Consistency and consensus measures

• The *consistency index* of $A^k$ is defined by

$$\overline{CI}(A^k) = \frac{2}{(n-1)(n-2)} \sum_{i<j} (\log(a^k_{ij}) - \log(p^k_i) + \log(p^k_j))^2$$

• The *consensus index* of $A^k$ is defined by

$$CI(A^k) = \frac{2}{(n-1)(n-2)} \sum_{i<j} (\log(a^k_{ij}) - \log(p^c_i) + \log(p^c_j))^2$$
Iteration-based consensus model

Feedback adjustment

When constructing \( \bar{A}^k = [\bar{a}^k_{ij}]_{n \times n} \), we suggest that
\[
\bar{a}^k_{ij} = (\bar{a}^k_{ij})^\theta \left( \frac{p_i^c}{p_j^c} \right)^{(1-\theta)},
\]
where \( 0 < \theta < 1 \).
Iteration-based consensus model

Advantages

• The consistency level of multiplicative preference relations are improved.
• The consensus level is improved.
• The Pareto principle is satisfied.

Optimization-based consensus model

Basic model

\[
\min_{\overline{F}^k} \sum_{k=1}^{m} d(F^k, \overline{F}^k)
\]

\[s.t. \begin{cases}
CI(\overline{F}^k) \leq \beta, & k = 1, 2, \ldots, m \\
CI\{\overline{F}^1, \ldots, \overline{F}^m\} \leq \alpha
\end{cases}
\]  

Optimization-based consensus model

Advantages

- The consistency and consensus are improved in one linear programming model.
- Individuals’ original preferences are preserved as much as possible.
Direct consensus framework

Motivation: How to obtain the consensus solution?

Experts → Individual opinions with different preference representation structures → Consensus solution
Experts

Individual opinions with different preference representation structures

Individual preference vectors

Normalized individual preference vectors

Temporal collective preference vector

Temporal normalized collective preference vector

Preference information

Individual selection methods

Feedback adjustment

Consensus measure

Normalization

Aggregation

Normalization

Reached consensus?

Selection process

Consensus process

Fig. 2. Direct consensus framework
Direct consensus framework

- Advantages

  - The internal inconsistency issue is avoided.
  
  - The Pareto principle is satisfied.

Future research

- Incorporate the behaviours of experts into the consensus reaching process.

- Investigate the consensus reaching process in dynamic situations.

- Design a general comparison framework to compare different consensus models.
Thank you!

Any questions?