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Studying the standard monopolistic competition model with unspecified utility/cost functions, we find necessary and sufficient conditions on the function elasticities, when an expanding market or trade incur welfare losses. Two numerical examples explain why: either excessive or insufficient entry of firms is aggravated by market growth. The variable marginal cost enforces the harmful effect. Still harm looks practically improbable.

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Introduction

Gains from trade or large markets remain important in New Trade theory: see Arkolakis et al. (2012), Arkolakis et al. (2013), an overview Melitz and Redding (2012), and recent paper Mrázová and Neary (2014) developing an approach like ours. By contrast, possibility of *harm* from trade is less popular. The rare examples include Dixit and Stiglitz (1977) and Epifani and Gancia (2011), both treating inter-sectoral distortion more extensively than intra-sectoral. Possible harm is perceived as an anomaly rather than a normal outcome in New Trade. We generally support this view of monopolistic competition (leaving aside neoclassical and oligopoly theories) but we are interested in finding *conditions for the preferences and costs* of such abnormal outcome. Such conditions could suggest which industries are likely or unlikely to be harmed by globalization--this goal requires considering variable elasticity of substitution (VES) and general technologies.

Our setting closely follows the VES Dixit-Stiglitz-Krugman framework with homogeneous firms and one sector (Krugman (1979)). Adding heterogeneity (see Dhingra and Morrow (2013)) would not abolish the competition effects that we study, only adding selection effects. Instead, we build upon Zhelobodko et al. (2012) in our homogeneous model with unspecified additive utility/cost functions. However, to demonstrate both excessive and/or insufficient entry, we depart from Zhelobodko et al. (2012) in allowing not only convex but also *concave* total cost (see Bykadorov et al. (2013)). This non-traditional assumption is motivated by an R&D-dependent endogenous technology, because in this case higher output should foster investment in marginal cost reduction. More traditionally, the opening of trade is modelled as “economic integration versus autarky” i.e., as a population increase.

To explain the results, we interpret any losses from trade as a kind of market distortion or social inefficiency–aggravated by a larger market. Focusing on intra-sectoral distortion, we find two kinds of such distortion, highlighted also in Dixit and Stiglitz (1977) and Dhingra and Morrow (2013): insufficient entry or excessive entry. Under linear costs, the latter paper formulates a *sufficient* condition on preferences for trade gains as “alligned elasticities of revenue and utility.” Our contribution is the proposition that this condition on demand is also *necessary*, in the sense that its violation enables trade losses under some cost function. Thereby, the question of intra-sectoral trade gains in the Dixit-Stiglitz-Krugman model obtains the final clarification. Additionally, our numerical examples demonstrate the *existence* of both distortion directions; show how distortion works and why this effect is *fragile*.

The paper concludes with an optimistic moral from the pessimistic examples of trade losses: this effect looks weak and exotic, i.e., rare among all possible preferences/costs. Thereby, we indirectly support the common wisdom of economists about probable *trade gains rather than losses*, under most typical utility/cost elasticities.

1. Model and preliminaries

To skip some model details keeping them easily available to the reader, our exposition exactly follows Zhelobodko et al. (2012). We consider a closed economy with general monopolistic competition. “General” means *unspecified* additive functions of utility and non-linear costs. The utility may display variable elasticity of substitution; costs may indirectly express endogenous technology. The only production factor is labor, supplied inelastically by L identical consumers/workers. A single sector involves an endogenous interval $[0, N]$ of identical firms producing varieties of some differentiated good, one variety by each firm. (An extension can involve several sectors; if they attract fixed budget shares each–the same results apply directly, otherwise some modification is needed.)

Each consumer displays additive preferences, expressed through some strictly concave sub-utility u. Utility maximization takes the form

$$U = \int_0^N u(x_i)di \to \max{}_{X \geq 0}, \quad s.t. \quad \int_0^N p_i x_i di \leq w. \tag{1.1}$$

Here $X = (x_j)_{j<N}$ is a function, scalar $x_i \equiv x(i)$ denotes consumer's consumption of i-th variety, p_i is the price, constant $w \equiv 1$ denotes consumer's expenditure equal to wage, index $_i$ everywhere replaces parentheses (i) (only for concise notations).

We need only minimal restrictions on utility. Here, as in Zhelobodko et al. (2012), market classification is based on an elasticity operator $\varepsilon_g(z) \equiv \frac{zg'(z)}{g(z)}$ defined for any function g, and on Arrow-Pratt concavity operator $r_g(z) \equiv -\frac{zg''(z)}{g'(z)}$. Following Zhelobodko et al. (2012), Mrázová and Neary (2013), for the existence of an equilibrium, uniqueness, symmetry and positivity we assume that, at some zone $[0, \breve{z})$ of possible equilibria $(\breve{z} \leq \infty)$, the elementary utility function $u(\cdot)$ is thrice differentiable, increasing $(u'(z) > 0)$, strictly concave $(u''(z) < 0)$, normalized $(u(0) = 0)$ and its main characteristics behave as $r_u(z) \in [0,1), \quad r_{u'}(z) < 2 \quad \forall z \in [0, \breve{z})$.

Using these assumptions and the first-order condition (FOC) with a Lagrange multiplier λ, we find the inverse demand function p for any variety i as

$$\mathrm{p}(x_i, \lambda) = \frac{u'(x_i)}{\lambda}. \tag{1.2}$$

Therefore, the marginal utility of expenditures λ becomes the single aggregate market statistic important for producers, like the price index in the CES case of this model.

Each producer i perceives the inverse demand function p and the marginal utility of income λ as given. Her non-linear total cost function $C(q)$ depends upon the output $q \equiv Lx$, profit maximization taking the form

$$\pi(x, \lambda) \equiv \frac{u'(x)}{\lambda} xL - C(Lx) \to \max{}_{x \geq 0}$$

(here choice of maximizers x, q or p brings an equivalent result). The firm's identity i is dropped because of symmetry among firms. Denoting revenue

$$R(x, \lambda, L) \equiv \frac{u'(x)xL}{\lambda},$$

we write the usual FOC for marginal revenue and marginal cost:

$$\frac{d}{dx} R(x, \lambda, L) - \frac{d}{dx} C(Lx) = 0.$$

Further, the usual second-order condition (SOC) is

$$\frac{d^2}{dx^2} R(x, \lambda, L) - \frac{d^2}{dx^2} C(Lx) < 0,$$

this assumption supports symmetry. We assume that firms enter the market until their profits vanish, and the labor balance is equivalent to the budget constraint.

Symmetric (free-entry) equilibrium is a bundle $(\bar{x}, \bar{p}, \bar{\lambda}, \bar{N})$ satisfying: the utility maximization condition (1.2); profit maximization FOC and SOC; free entry and labor market clearing conditions:

$$R(\bar{x}, \bar{\lambda}, L) - C(L\bar{x}) = 0, \tag{1.3}$$

$$\bar{N} C(\bar{x}L) = L. \tag{1.4}$$

Now we can divide each producer's FOC by the free entry condition to express such equilibrium in elasticities:

$$\varepsilon_R(\bar{x}) \equiv 1 - r_u(\bar{x}) = \varepsilon_C(L\bar{x}). \tag{1.5}$$

Here ε_R is the elasticity of revenue and $\varepsilon_C(q) \equiv \frac{q}{C(q)} \cdot \frac{\partial C(q)}{\partial q}$ is the cost elasticity. Using consumption $\bar{x}$ determined here, one can find equilibrium prices $\bar{p}$ and masses $\bar{N}$ of firms from the remaining equations. Thus, each consumer's equilibrium welfare $\bar{U} = \bar{N}u(\bar{x})$ indirectly depends on market size L through the equilibrium magnitudes $\bar{x}(L), \bar{N}(L)$ (bar accent henceforth denotes equilibria).

Using the total differentiation of the equilibrium equation (1.5) w.r.t. consumer population L and (1.4), we express total utility elasticity $E_{\bar{U}/L}$ at equilibrium through other total elasticities E and partial elasticities ε as follows.

Lemma (welfare elasticity). *The effect of market size L on equilibrium welfare $\bar{U}$ can be decomposed into other equilibrium elasticities as follows:*

$$E_{\bar{U}/L} \equiv \frac{L}{\bar{U}} \cdot \frac{d\bar{U}}{dL} = E_{\bar{N}/L} + \varepsilon_u \cdot E_{\bar{x}/L}, \tag{1.6}$$

where $E_{\bar{N}/L} \equiv \frac{L}{\bar{N}} \cdot \frac{d\bar{N}}{dL}$, $E_{\bar{x}/L} \equiv \frac{L}{\bar{x}} \cdot \frac{d\bar{x}}{dL}$ *denote total equilibrium elasticities,* $\varepsilon_u \equiv \varepsilon_u(\bar{x}) \equiv \frac{z}{u(z)} \cdot \frac{\partial u(z)}{\partial z}$ *is the elasticity of this function at* $z = \bar{x}$*. The SOC for profit maximization at equilibrium is*

$$SOC \equiv r_u'(\bar{x}) \cdot \bar{x} + \varepsilon_C'(L\bar{x}) \cdot L\bar{x} > 0. \tag{1.7}$$

Proof. Formula (1.6) reformulates the elasticity of $\bar{U} = \bar{N}u(\bar{x})$. In Zhelobodko et al. (2012), online Appendix, SOC is formulated as $r_u' \cdot x + (r_u - r_C) \cdot (1 - r_u) > 0$. Using equilibrium condition (1.5) and $r_C \equiv -\varepsilon_{C'}$ the second term here becomes $(r_u - r_C) \cdot \varepsilon_C = (1 - \varepsilon_C + \varepsilon_{C'}) \cdot \varepsilon_C$. Using identity $(1 - \varepsilon_g(z) + \varepsilon_{g'}(z)) \cdot \varepsilon_g(z) \equiv \varepsilon_g'(z) \cdot z$ valid for any function g, we get expression (1.7). 〈

2. Harm from market size

We focus here on intra-sectoral distortion. To create an intuition of such distortion, we note that diversity $\bar{N}(L)$ and per-variety consumption $\bar{x}(L)$ in a growing market should change oppositely $(E_{\bar{N}/L} \cdot E_{\bar{x}/L} < 0)$, because of the labor balance $N = \frac{L}{C(Lx)}$ and cost elasticity $\varepsilon_C = \varepsilon_R < 1$ is restricted by our assumptions. So, observing the utility elasticity $E_{\bar{U}/L} = E_{\bar{N}/L} + \varepsilon_u \cdot E_{\bar{x}/L}$, it becomes clear that "harmful trade" $(E_{U/L} < 0)$ can occur either when the welfare gain from the additional variety $(E_{\bar{N}/L} > 0)$ is outweighed by decreasing per-variety consumption $(E_{\bar{x}/L} < 0)$, or when increasing consumption $(E_{\bar{x}/L} > 0)$ is outweighed by decreasing variety $(E_{\bar{N}/L} < 0)$. Extending this reasoning about the two possible cases, the proposition below establishes the necessary and sufficient conditions for "harmful trade". There are four point-wise properties of functions: decreasingly elastic utility (DEU): $\varepsilon_u'(x) < 0$, increasingly elastic utility (IEU): $\varepsilon_u'(x) > 0$, decreasingly elastic demand (DED) also called strictly super-convex: $r_u'(x) < 0$, increasingly elastic demand (IED) $(r_u'(x) > 0)$, also called strictly sub-convex (see Mrázová and Neary (2013)).

Proposition. *Consider an equilibrium $\bar{x}$ under market size L.*

(i) The local effect of a growing market on welfare can be represented in elasticities (taken at the equilibrium values) as follows:

$$E_{\overline{U}/L} = (1-\varepsilon_u) - \frac{\overline{x}^2}{\varepsilon_u} \cdot \frac{\varepsilon_u' \cdot r_u'}{SOC} = r_u + \frac{L\overline{x}^2}{\varepsilon_u} \cdot \frac{\varepsilon_u' \cdot \varepsilon_C'}{SOC}, \quad (2.1)$$

$$E_{\overline{U}/L} < 0 \Leftrightarrow -r_u'(\overline{x}) < \varepsilon_C'(L\overline{x}) \cdot L < -r_u'(\overline{x}) \cdot \frac{r_u(\overline{x})}{1-\varepsilon_u(\overline{x})}. \quad (2.2)$$

Thereby, condition [DED&DEU or IED&IEU], meaning $\varepsilon_u' \cdot r' > 0$*, is necessary for the welfare decrease, as well as the opposite monotonicity of utility and cost elasticities in the sense* $\varepsilon_u' \cdot \varepsilon_C' < 0$*. In particular, the welfare decrease is impossible under a linear or convex cost supplemented with IED or/and IEU preferences.*

(ii) Sufficiency: for any utility demonstrating the property (DED&DEU or IED&IEU) at some point $\overline{x}$ *under given* $\overline{L}$*, one can find a cost function* C *such that* $\overline{x}$ *is an equilibrium, and welfare locally decreases w.r.t.* L *at* $\overline{L}$*. Alternatively, welfare can be made locally increasing under some other cost function* $\tilde{C}$*.*

Proof. (i) Using our Lemma, elasticity expressions borrowed from Zhelobodko et al. (2012) Appendix are reformulated as follows:

$$\varepsilon_{\overline{x}/L} = -\frac{(r_u - r_C)\cdot(1-r_u)}{r_u' \cdot \overline{x} + (r_u - r_C)(1-r_u)} = -\frac{L\overline{x}}{SOC} \cdot \varepsilon_C'(L\overline{x}),$$

$$\varepsilon_{\overline{N}/L} = 1 - \varepsilon_C \cdot \frac{r_u' \cdot \overline{x}}{r_u' \cdot \overline{x} + (r_u - r_C)\cdot(1-r_u)} = \frac{r_u(\overline{x}) \cdot r_u'(\overline{x}) \cdot \overline{x} + \varepsilon_C'(L\overline{x}) \cdot L\overline{x}}{SOC}.$$

Plugging these into (1.6) we get

$$E_{\overline{U}/L} = \frac{r_u \cdot r_u' \cdot \overline{x} + (1-\varepsilon_u) \cdot \varepsilon_C' \cdot L\overline{x}}{r_u' \cdot \overline{x} + \varepsilon_C' \cdot L\overline{x}}. \quad (2.3)$$

Reformulating (2.3) with SOC and FOC (1.5), we get the needed equality (2.1). Double inequality (2.2) is just a reformulation of SOC and (2.3). Further, properties necessary for welfare decrease---following from (2.1) and $r_u > 0, 1-\varepsilon_u > 0$. To ensure the impossibility of a decrease in welfare under a linear or convex cost ($C'' \geq 0$) and IED or IEU, we express the cost elasticity derivative as

$$\varepsilon_C' = \frac{C'}{C} \cdot \left(\frac{q \cdot C''}{C'} + 1 - \varepsilon_C \right) = \frac{C'}{C} \cdot \left(\frac{q \cdot C''}{C'} + r_u \right) > 0,$$

which is positive under our assumptions. Such positivity contradicts the inequality (2.2) needed for losses under IED ($r_u' > 0$), and yields welfare gain in (2.1) under IEU ($\varepsilon_u > 0 \Rightarrow E_{\overline{U}/L} > 0$).

(ii) Proving sufficiency consists of adjusting our cost function so that SOC in (2.1) becomes close to zero. Under (2.2), we need to construct a cost function C going through given point $\overline{x}$, but first construct its elasticity. For this we take a slight modification of the revenue elasticity, namely $\varepsilon_C(q) \equiv \varepsilon_{C_\delta}(q) \equiv 1 - r_u(q/L) + \delta \cdot (q - L\overline{x})$ with some $\delta > 0$ chosen to be small enough to fit our double inequality (2.2). This admissible interval (2.2) for $\varepsilon_{C_\delta}(q)$ is non-empty only when $r_u'(\overline{x}) - r_u'(\overline{x}) \cdot \frac{r_u(\overline{x})}{1-\varepsilon_u(\overline{x})} > 0$. This under DED ($r_u' < 0$) requires DEU because of the identity $1 - \varepsilon_u - r_u \equiv \varepsilon_{\varepsilon_u}$, and under IED requires IEU. Now we *can* solve the differential equation $\frac{dC}{C} = \frac{\varepsilon_C(q)}{q} \cdot dq$ to find the needed cost function $C(q) \equiv C_\delta(q) \equiv \exp\left(\int_0^q \frac{1 - r_u(z/L) + \delta \cdot (z - L\overline{x})}{z} dz \right)$ which generates our elasticity ε_{C_δ}. By this, the equilibrium condition (1.5) and SOC are satisfied at $\bar{x}$, this cost function is positive and increasing, as needed. To prove the last proposition statement (increasing welfare under another function $\tilde{C}$), it suffices to note that our δ can be chosen big enough to violate the double inequality (2.2). $\langle$

When the required demand properties hold globally, this proposition can be extended from point-wise changes in population and welfare onto global ones (see Fig.1).

As to the literature on this question and its interpretations, Dixit and Stiglitz (1977) find under linear costs $C \equiv f + cq$ that entry into a separate sector is socially excessive under IEU but insufficient under DEU, only the borderline CES case yielding the social optimum. When the market grows, such a distortion should typically soften. Namely, under IED and linear cost function, Krugman (1979) argues that a welfare gain from a larger market---stems from the "double advantage" for the consumer: variety grows and prices go down (see also Zhelobodko et al. (2012) generalizing this claim to convex cost). Our IED&DEU requirement for trade gains is replaced in these papers by IED & convex cost ($r_u' > 0, \varepsilon_C' > 0$) but there is no contradiction, as one can see from our proof. This line of reasoning is extended under linear cost onto firm heterogeneity in Dhingra and Morrow (2013). Here the demand characteristic $r_u(x)$ is reasonably called "markup", and function $1-\varepsilon_u(x)$ is called "social markup." These two markups are called "aligned" when both increase (for IED&DEU), or both decrease (for DED&IEU). In Dhingra and Morrow (2013) the DED&IEU case is added to the previously known sufficient conditions for welfare gains---by Proposition 9: "market expansion increases welfare when preferences are aligned." Our formula (2.1) yields the same conclusion because "aligned" means $\varepsilon_u' \cdot r_u' < 0$ and $1-\varepsilon_u > 0$. The interpretation in Dhingra and Morrow (2013) says that the market maximizes markup, whereas the social planner pursues a social markup; which is why welfare must grow when these goals are aligned. Our proposition extends this sufficient condition to non-linear costs (eg, endogenous technology) and adds that it is also a necessary condition on demand for welfare gains, when we mean "gains under any possible cost functions." The general economic conclusion is that these are the "demand-side elasticities that determine how resources are misallocated and when increased competition from market expansion provides welfare gains" Dhingra and Morrow (2013). Supporting this idea, both our examples in Fig.1 show that for welfare loss, the curves of utility elasticity $\varepsilon_u(x)$ and revenue elasticity $1-r_u(x)$, for welfare loss must be "misaligned", i.e., display the opposite monotonicity. Finally, we should mention the extensions of sufficient conditions for trade gains onto costly trade. IED condition is confirmed in Mrázová and Neary (2014) under linear costs, general utilities and symmetric countries, and also in Behrens and Murata (2012) under CARA utility. The latter approach is developed into an empirical estimation of intra- and inter-sectoral market distortions in the presentations of these authors, reporting about 5% of GDP (HSE-St.Petersburg, January 2014).

To demonstrate the non-empty set of cases discussed, we turn to harmful trade examples. They are built on our criterion (2.2) and explain our proposition. We are unaware of such examples of harmful trade in the literature, except for Peter Neary's example, presented during his lectures (HSE-Moscow, October 2013), which is similar to our Example 3 below.

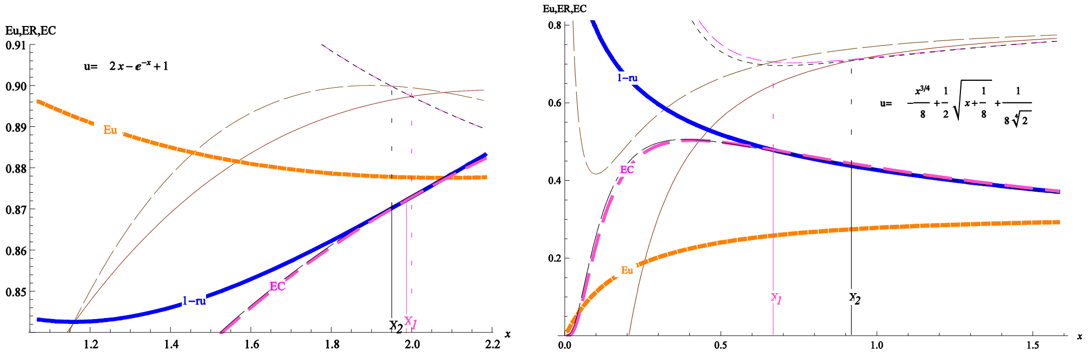

Figure 1: Elasticities of utility, revenue and cost: 2 examples.

Example 1: DED&DEU (excessive entry aggravated, Fig. 1-left)

To construct a utility function that fits the inequality (2.2), we combine CARA utility with a linear or polynomial term:

$$u(x)=\begin{cases}1-\exp(-x)+2x, & if \quad x<2;\\ -\exp(-x)+3x-0.25x^2, & if \quad x\geq 2.\end{cases}$$

This function brings “anti-competitive” price-increasing effect under $x<2$, whereas interval $x>2$ is needed only to formally guarantee SOC everywhere; all our equilibria lie at $x<2$. Here the utility elasticity is $\varepsilon_u(x)=\dfrac{(2+\exp(-x))\cdot x}{1-\exp(-x)+2x}$, being plotted by software as the thick orange dotted curve in Fig.1. Revenue elasticity $\varepsilon_R(x)=1-r_u(x)=\dfrac{\exp(-x)\cdot x}{2+\exp(-x)}$ is thick blue. It is challenging to confine ourselves with *linear* costs $C(q)=f+cq=1+q$. The specially adjusted market size is $L_1=3.4447$, related cost elasticity $\varepsilon_C(L_1x)=\dfrac{L_1cx}{f+L_1cx}$ is dashed thick magenta line. The main equilibrium equation (1.5) in this picture is represented by the lower of two (almost indistinguishable) intersections between $1-r_u(x)$ and $\varepsilon_C(L_1x)$, whereas the upper intersection violates SOC. This (pink) equilibrium point is $x_1\approx 1.98683$, $\mathrm{U}_1\approx 2.12396$. Increasing the market size to $\mathrm{L}_2=3.446$, we get another (black) equilibrium point with decreased consumption and *smaller* utility: $\mathrm{x}_2\approx 1.96165$, $\mathrm{U}_2\approx 2.12389$, which was our goal. Though the new, thin dashed, curve $\varepsilon_C(L_2x)$ almost coincides with old curve $\varepsilon_C(L_1x)$, it still brings a slight difference, and the variety increases from $\mathrm{N}_1=0.439149$ to $\mathrm{N}_2=0.444081$. With a higher population $\mathrm{L}_3=3.447$, we get $\mathrm{x}_3=1.95041$, $\mathrm{U}_3=2.12389$, observing no further welfare decrease. Therefore we cannot get a stronger welfare change than $\Delta U\approx$ -0.00007, the strongest decrease we managed to construct under linear cost.

Why is this effect so fragile, so *small* is the change ΔU? For explanation, look at the additional (thin) curves representing our double inequality (2.). The thin pale solid curve is $\underline{E'_C}\equiv -r'_u(x)-\Delta$ (solid) and the thin pale dashed curve is $\overline{E'_C}\equiv -r'_u(x)\cdot\dfrac{r_u(x)}{1-\varepsilon_u(x)}-\Delta$. Both borders of the double inequality (2.2) are shifted down with special offset $\Delta=0.842$, which helps to see how their lower intersection $\hat{\mathrm{x}}\approx 1.1572$ coincides with the minimum of curve $1-r_u(x)$ and another intersection abscissa $\breve{\mathrm{x}}\approx 2$ coincides with the maximum of $\varepsilon_{\mathrm{u}}(\mathrm{x})$ (one can understand the coincidence from our formulae). The interval $[1.1572,2]$ between these two crucial intersections is the maximal domain for constructing various equilibria bringing a utility decrease, because only here can the inequality (2.2) hold. This domain is not so small. However, keeping to only the linear cost, we are additionally restricted by the constraint represented by the region where the dashing magenta curve $\mathrm{E}'_{\mathrm{C}_1} \equiv \varepsilon'_C(\mathrm{L}_1 x)\mathrm{L}_1 - \Delta$ goes between $\underline{E'_C}$ and $\overline{E'_C}$ as required by the inequality (2.2). (The new dotted curve $\mathrm{E}'_{\mathrm{C}_2} \equiv \varepsilon'_C(\mathrm{L}_2 x)\mathrm{L}_2 - \Delta$ almost coincides with initial $\mathrm{E}'_{\mathrm{C}_1}$.) Thus, under cost linearity, welfare decrease is feasible only in a small domain $[1.96165, 2.0]$, marked by long-dashing vertical lines. It lies approximately between our equilibria x_2 and x_1, specially adjusted to cover this domain, i.e., to find *the maximal possible welfare decrease.*

To make the decrease more noticeable under same utility function but some non-linear cost, we have constructed a non-linear function $\mathrm{C} \approx \exp\left(\int_0^q \frac{1 - \mathrm{r_u}(z/L) + \cdot(z - L\bar{x})}{z} dz\right)$ as in the proof of proposition, so that almost any point $\mathrm{x} \in [1.1572, 2]$ satisfying the double inequality turns into an equilibrium. Then the maximal possible decrease that we achieved goes from $\breve{\mathrm{x}} \approx 2$ to $\hat{\mathrm{x}} \approx 1.1572$. Using labor balance $\mathrm{NC}(\mathrm{Lx}) = \mathrm{L}$ we express welfare as $\mathrm{U} = N\mathrm{u}(\mathrm{x}) = \frac{\mathrm{u}(\mathrm{x})\mathrm{L}}{\mathrm{C}(\mathrm{Lx})}$, and plugging in these two consumption points we get the strongest possible welfare decrease under this utility and for any costs: $\Delta\,\mathrm{U} = \mathrm{U}_1 - \mathrm{U}_2 = 2.07256 - 2.12423 = -0.0516737$, i.e., -2.5%. It is bigger than under linear cost but still very small.

Let us explain why in our proof and examples, a welfare decrease requires curves $\varepsilon_{\mathrm{C}}(\mathrm{Lx})$ and $1 - \mathrm{r_u}(\mathrm{x})$ being almost tangent to each other. We have mentioned that formally it means $SOC \approx 0$ in (2.1), is needed to outweigh the positive magnitudes $1 - \varepsilon_{\mathrm{u}} > 0$ or $r_u > 0$. In other words, the utility formula $\mathrm{U} = N\mathrm{u}(\mathrm{x}) = \frac{\mathrm{u}(\mathrm{x})\mathrm{L}}{\mathrm{C}(\mathrm{Lx})}$ says that a decrease in consumption x brings harm when insufficiently compensated for by an increase in variety N. The utility elasticity w.r.t. x is $\varepsilon_{U/x} = \varepsilon_{u/x} - \varepsilon_{\mathrm{C/x}}$, which at equilibrium becomes $\varepsilon_{\mathrm{U/x}} = \varepsilon_u - 1 + \mathrm{r_u} \equiv \varepsilon_{\varepsilon_{\mathrm{u}}}$. We see that the negative influence ($\varepsilon_{U/x} = \varepsilon_{\varepsilon_u} < 0$) occurs under DEU, whereas market size pushes x down under DED (see Zhelobodko et al. (2012)). When curves $\varepsilon_{\mathrm{C}}(\mathrm{Lx})$ and $1 - \mathrm{r_u}(\mathrm{x})$, determining the equilibrium, are almost tangent to each other (as in our example and proof), then a negligible increase in L brings large decrease in x. Thus, the total utility effect reduces to approximately $\varepsilon_{U/L} \approx \varepsilon_{U/x} = \varepsilon_{u/x} - \varepsilon_{C/x} \approx \varepsilon_{u/x} - \varepsilon_{R/x}$ and “insufficient compensation” takes place. We conclude also that DEU may generate social distortion in the form of an *excessive* variety N, which means inefficiently *high* average costs. In examples like this one, in response to a small increase in L, increasing variety *pulls the average cost up, driving the economy further away from optimum. So, excessive entry must be aggravated by market expansion in DED&DEU,* combined with a cost elasticity almost tangent to revenue elasticity. This is the mechanism of utility decrease.

Example 2: IED&IEU (insufficient entry aggravated, Fig.1-right)

Now we take

$$u(x) = \begin{cases} 0.5\sqrt{0.125 + x} - 0.125x^{3/4} + 0.125 \cdot 2^{-1/4}, & if \quad x \geq 0.1; \\ 6.04076x - 28.4021x^2, & if \quad x < 0.1. \end{cases}$$

All our equilibria are among $\mathrm{x} \geq 0.1$; the quadratic function on the initial interval $\mathrm{x} \in [0, 0.1]$ is needed only for formal normalization $\mathrm{u}(0) = 0$. The non-linear cost function is $\mathrm{C}(\mathrm{q}) = 0.2 + 1.5 \cdot \exp\left(-1/\sqrt{\mathrm{q}/7}\right)$. The related equilibrium is displayed in Fig.1, right panel, which keeps the line coloring and legend of the left panel. Namely, under $\mathrm{L}_1 = 4.25$ the first (pink) equilibrium point is $x_1 \approx 0.669948$, $U_1 \approx 3.7994$. Increasing the market size to $L_2 = 4.47$ we get another (black) equilibrium point with smaller consumption $x_2 \approx 0.921192$ and *smaller* utility $U_2 \approx 3.6732$, as we needed to show. The strongest possible welfare decrease under this utility is thereby $\Delta U = -0.1262$, i.e., -3.3%. Though the new dashing curve $\varepsilon_C(L_2 x)$ almost coincides with the old curve $\varepsilon_C(L_1 x)$, the mass of firms decreases from $N_1 = 8.28935$ to $N_2 = 7.36122$.

The economic explanation mirrors the previous one. From Dhingra and Morrow (2013) we know that IEU indicates *a socially insufficient* mass N of firms, which remains true under non-linear costs. Here the social distortion takes the form of inefficiently low average costs and variety. Moreover, in response to a small increase in L, *the variety further decreases, being insufficiently compensated by the consumption increase*, that drives the economy further away from optimum. In other words, *insufficient entry is aggravated by market expansion* under IED&IEU, combined with cost elasticity almost tangent to revenue elasticity.

Example 3: absent normalization

To show other possible sources of intra-sectoral trade loss, we consider linear cost $C = f + cLx$ and non-normalized utility $u(x) = \sqrt{x} - a \quad (a > 0)$. Essentially, it violates our assumption in non-normalization $u(0) < 0$ and the discontinuity of ε_u at $x = a^2$ (so, our proposition is not applicable). Such utilities bring harm from market size like in our two examples but on other grounds: here utility behavior at $x < a^2$ is crucial. Indeed, the calculation shows that the same prices $p = 2c$ operate under any growing population L, equilibrium variety $N(L) = 0.5L/f$ increases linearly, and per-variety consumption $x_i(L) = f/(cL)$ tends to zero. So, at some stage (near $x = a^2$ and below a^2) total utility $N \cdot (\sqrt{x} - a)$ *decreases* with variety though prices do not change and the previous consumption vector remains available.

A doubtful interpretation of this harm from variety per se---is *envy* growing with the broader consumer choice. However, it looks artificial and contradicts general idea of monopolistic competition which implies a love of variety. The same doubt goes for the alternative assumption $u(0) > 0$, e.g., $u(x) = \sqrt{x} + a$, which means positive welfare from zero consumption. Here we see artificial utility growth with variety, even without changing the consumption vector. We observe that an *arbitrary* constant added to the sub-utility can change all the welfare conclusions from plus to minus---without any changes in the demand function and market outcomes. Thus, *an ordinal approach to sub-utilities looks inappropriate* for a welfare analysis of monopolistic competition; normalization is important.

3. Concluding remarks

Summarizing, the general form of Dixit-Stiglitz-Krugman monopolistic competition allows for harmful trade or harmful market growth. We provide examples, and the necessary and sufficient conditions for this effect: DED&DEU or IED&IEU of demand, combined with almost tangent cost and revenue elasticities. However, the harmful effect in these examples is so weak and the combination of conditions looks so exotic, that our possible result should be perceived as an impossibility: it is hard to imagine a real-life sector where all these conditions are satisfied. We believe this is the final word on homogenous intra-sectoral trade distortion from the Dixit-Stiglitz monopolistic competition theory. For the inter-sectoral distortion and empirical estimates, they allow for further study along the lines of Behrens and Murata (2012).

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