The loyalty-competence tradeoff in dictatorships and outside options for subordinates.

Alexei V. Zakharov∗

National Research University - Higher School of Economics, Moscow, Russia

September 4, 2014

Abstract

Dictators rely on loyalty of their subordinates to remain in power, and being loyal often involves taking costly actions on behalf of the dictator. In turn, a subordinate’s decision is affected by his payoff in case the dictator is removed from power. This provides the incentive for a dictator to hire a subordinate who has a small value of outside option. It is especially true if the dictator cares little about the subordinate’s competence — that is, performance at other tasks, such as carrying out economic policy. Starting with these assumptions, I propose a theory of subordinate recruitment by dictators. In a dynamic setting, I endogenize the value of the outside option for a subordinate as his subsequent payoff in case he is hired by the dictator’s successor. I show that, as long as dictators differ in how they value the competence of their subordinates, a less competent subordinate will be more loyal solely because his probability of being hired by the dictator’s successor

∗al.v.zakharov@gmail.com. The author thanks Maxim Ananiev, Georgy Egorov, Andrei Gomberg, Sergei Guriev, Philip Keefer, Tommaso Nannicini, and Konstantin Sonin for their comments.
will be smaller. As a result, dictators who value economic performance will hire more competent subordinates and will have shorter tenures. Incompetent subordinates will be hired more often (and will be less loyal) if the dictators discount future payoffs less heavily. At the same time, the discount rate of the subordinates has a nonmonotonic effect on the probability that an incompetent subordinate is hired.

“Those people think that the Soviet authority will not dare to touch them because of their past achievements. [they] think that they are irreplaceable, and thus can violate the directions of the upper-level organs with impunity. What should we do about them? Such people should be dismissed immediately from high-level positions regardless of their past achievements... This is necessary in order to break down the pride of such arrogant bureaucrats and show them their place.”

- Joseph Stalin, spoken at XVII Communist Party Congress, 20.01-10.02.1934

1 Introduction

Dictators do not rule alone. Each dictator must rely on a team of lieutenants to accomplish a number of tasks both to ensure his survival in office, and to promote the efficient functioning of the government. Such tasks may include drafting and carrying out economic policy, collecting taxes, punishing dissent, or brainwashing the public.

Lieutenants differ in their ability, and the truly capable ones may be few and far between. A gifted administrator can expect a demand for his services from his patron’s potential successor. Than, in turn, will make him want to exert less effort to keep his current patron in power. One such example is Charles Maurice Talleyrand of France, who was famous for his diplomatic prowess, for his political longevity, and for his lack of loyalty to the ones whom he served. He has held senior offices (including foreign minister, prime minister, and ambassador...
to the United Kingdom) under four different political regimes, repeatedly falling out of favor and shifting allegiances.

Talleyrand started his administrative career during the *ancien régime* and became the minister of foreign affairs in 1797 under the French Directory, after a period of exile. In 1799 he conspired with Napoleon in the coup of 18 Brumaire when the Directory was overthrown. Talleyrand once again became the foreign minister.

He resigned his position in 1807, but continued to play a significant role in Napoleon’s court, perhaps being one of the most influential men in the Empire (Cooper, 2001). At the same time, he started secret negotiations with Austria and Russia, accepting bribes for information on Napoleon’s plans. Napoleon seemed to be aware of Talleyrand’s lack of loyalty and limited his influence, at one time subjecting him to public humiliation. However, Napoleon stopped short of using violence against Talleyrand, believing him to be indispensable. When the armies of Russia, Austria and Prussia marched on Paris in the spring of 1814, Napoleon regretted that his old foreign minister was not there to help: “If only Talleyrand were here — he would get me out of it”.¹

After the first defeat of Napoleon, Talleyrand was instrumental in restoring the Bourbon dynasty to the throne, becoming the prime minister in 1814 and once again in 1815 after Napoleon’s final defeat at Waterloo. Resigning shortly after, he spend the next 15 years in political semi-exile. After the 1830 July revolution he was summoned once again to become the ambassador to the United Kingdom — a position he held for four years, before retiring from politics for good due to his old age.

Highly capable senior officials are sometimes recruited into conspiracies because of their value to any future government. Field marshal Erwin Rommel was one of the most decorated military commanders in Nazi Germany. In 1944 he was approached by leaders of the anti-Hitler

¹Quoted in Cooper (2001).
plot who planned an overthrow of the Nazi-led German government. They needed a general who was of a senior rank, on active duty, and highly popular with the military. Rommel was one of the few Wermacht officers who fit this description (Shirer, 1990), and agreed to join the conspiracy. In their government, the conspirators also planned to include a number of senior Nazi officials who were not part of the plot. One such person was Albert Speer, who was to retain his position as the Minister of Armament and War Production (Speer, 1970), but was not aware of these plans, or even of the conspiracy itself.

Talleyrand and Rommel are examples of highly capable subordinates who served (or were intending to serve) more than one national leader. On the other hand, countless incompetent loyalists, cronies, and leader’s relatives were promoted to top positions over more qualified candidates; their political fortunes usually rose and fell with those of their patrons. Such loyalty-based promotions are a pervasive feature of authoritarian regimes. In personalist dictatorships in particular competence may be an undesirable trait to such an extent that state capacity — and, sometimes, even the leader’s own political survival – are seriously undermined by subordinates (often, the leader’s own relatives) who are too unfit for their positions (Chehabi and Linz, 1998; Ezrow and Frantz, 2011; Bueno de Mesquita and Smith, 2012). Tsar Nicholas I of Russia was one of the many leaders who prized loyalty above competence and integrity, even at the cost of loyal subordinates being very corrupt. When Nicholas learned about an extremely egregious case of theft by one of his closest confidants, he blurted: “Ryleyev and his conspirators

---

2After the coup’s failure, Rommel was forced to commit suicide, but his involvement with the plot was not revealed to the general public because of his popularity. Instead, it was announced that Rommel died from combat wounds sustained in an earlier Allied bombing, and was buried with full military honors.

3A number of such cases for both contemporaty and historic dictatorships are discussed in Egorov and Sonin (2011), but the loyalty-competence tradeoffs is not limited to dictatorial regimes. It is also faced by both democratically elected leaders (Edwards, 2001) and owners of private firms (Glazer, 2002; Burkart, Panunzi, and Shleifer, 2003; Prendergast and Topel, 1996).
would never have done this to me”⁴. Kondratii Ryleyev was one of the executed leaders of the failed 1825 Decembrist revolt that sought the abolition of serfdom and other political reforms. The Decembrists belonged to a subculture within Russian nobility that emphasised honesty, integrity, patriotism, and learning (Lotman, 1984).

Lieutenants are often well rewarded for their services, but also may face considerable costs when attending to their duties. There are several reasons, beside effort itself, why working to increase the dictator’s tenure may be costly. First, a subordinate’s duties may involve the use of violence against regime’s opposition or performing other tasks that bear the risk of retribution once the current leader is out of power (Kim and Sikkink, 2010). Regime officials who are in charge of dispensing violence or promoting controversial policies often face travel and banking restrictions while abroad, such as the 18 Russian officials under the 2012 US Magnitsky Act, or other officials targeted by the US and UE in the wake of the 2014 Ukrainian conflict. Trials and lustrations punishing former state officials have followed, to varying degrees, a significant share of repression and political violence episodes over the past four decades (Olsen, Payne, and Reiter, 2010).

Second, high ranking government executives appointed to lead government agencies may face political costs of their own when following the needs of their appointers. This is true regardless of whether the country’s leadership is dictatorial or democratic. Wilson (1989, p. 199) describes the dilemma facing politically appointed heads of government agencies: continuing to serve his appointer’s political needs, versus “going native” — trying to build relationships with their bureaucratic environment and interest groups.

Finally, remaining loyal implies foregoing opportunities to participate in plots to overthrow the country’s leadership. Dictators face threats from both outside as well as inside their countries. The latter type of threats materialize when a sufficiently powerful alternative coalition is

⁴Quoted in Tarle (1950).
The theoretical argument of this work relies on dictators being different in their preferences toward achieving economic performance versus staying in power, and consequently, in their demand for competent officials. This is contrary to the usual assumption made in economic literature that all agents are the same (Haber, 2005). However, the economic outcomes that are produced by autocratic regimes (especially, repressive autocracies) vary to a much greater extent than the economic outcomes of democracies (Weede, 1996), and number of factors and institutional constraints were implicated in affecting the autocrat’s choice between economic development and economic ruin.

Zaire’s Mobutu Sese Seko boasted that he “never built one road” during his 31 years in office (Robinson, 1999) — a period over which his country’s real per capita GDP was reduced by almost a factor of 3, making Zaire one of the world’s poorest countries (Maddison, 2006). At the other extreme, per capita GDP of Taiwan increased nearly fivefold in the 1949-1975 period under the dictatorial (and repressive) leadership of General Chiang Kai-Shek.

Wintrobe (1998) in an influential book distinguishes between tinpot and totalitarian types of dictators. A tinpot dictator uses the office to maximize personal wealth and consumption, while the totalitarian type is interested in exercising pervasive control over economic, social and intellectual life of his subjects. In Windrobe’s world, the sorting of dictators into types depends on economic constraints (e.g. how well does money convert into the loyalty of the subjects), as well as the dictator’s intrinsic preferences for power and consumption. A similar argument is used by Guttman and Reuveny (2014). In their setting, dictators may choose either to implement expansionary economic policies to placate the population and prevent it from supporting rebellion, and/or repress the rebels directly. The choice of the dictator depends on the amount of capital available to the regime, as well as technological constraints of building infrastructure.
In Overland, Simons, and Spagat (2005) model of economic growth it is assumed that a dictator must expend resources to satisfy certain interest groups to remain in power. His equilibrium choice is either to plunder the country’s economy if the initial capital stock is below a certain threshold, or to choose an above-optimal investment if the initial capital is high enough. Grossman and Noh (1994) assume that countries are heterogeneous in the relationship between public utility and regime survival; as a result, some dictators produce more benevolent economic policies than others.

Prices of main export goods are a major source of uncertainty that affects the amount of rent that is available to a dictator, and his decision to pursue the course of economic development\(^5\).

In Ghana the onset of political stability in the mid-1980s coincided with a fall in the price of cocoa, a principal export commodity. According to McBride (2005), a fall in the cocoa prices led the government to introduce economic reforms that strategically lowered the capacity of the regime to extract rents from the economy, deterring potential challengers to the regime. Another source of heterogeneity in preferences for economic development is the penalty that a dictator is expected to receive upon leaving office.

Finally, there is evidence that the impact of institutions and external factors is not deterministic, and that personality of political leaders does matter for economic growth. Jones and Olken (2005) find that deaths of national leaders cause sharp changes in economic growth rates and also in the how monetary policy is conducted. The effect of individual leaders is stronger in autocratic countries, as there are fewer constraints on the actions of the national leaders (these findings are disputed by Easterly and Pennings, 2014). In a related study, Besley, Montalvo, and Reynal-Querol (2011) show that education of national leaders has a positive and

\(^5\)There is a large literature investigating the “natural resource curse” — the proposition that the abundance of natural resources can be adverse to economic and institutional development (Egorov, Guriev, and Sonin, 2009, Ploeg, 2011).
significant effect on economic growth. Gandhi (2008) argues that a dictator’s perception of political development is important, along with the existence of nominal democratic institutions. In particular, it matters whether the dictator believes that an eventual democratic transition is an ultimate goal.

In this work I model the loyalty-competence tradeoff assuming exogenous uncertainty with regard to the preferences of the dictator’s future (and unintended) successor. There are two types of players — dictators and subordinates. The subordinates are infinitely lived and can be either competent or incompetent, and the dictators are heterogeneous in how much they value competent versus incompetent subordinates. At any moment in time, there is one dictator who employs one subordinate (who might be competent or not). The remaining subordinates are unemployed and take no action, waiting for their opportunity to be hired when the dictator is removed from power and a new dictator enters the game. The employed subordinate, in turn, chooses the level of effort which determines the probability with which the dictator is to survive into the next period. Dictator’s payoffs each period depend on whether the subordinate is competent or not, and on the dictator’s preference for competence.

When a dictator is removed from power (exiting the game), his subordinate becomes unemployed. The dictator’s successor decides whether he should hire a competent or an incompetent subordinate. This decision depends on the new dictator’s type (which is randomly drawn once he enters office, and remains unchanged until he is ousted), and on the level of effort that he expects both types of subordinates to exert. If a new dictator chooses to hire a subordinate of either type, he chooses one at random from the pool of unemployed subordinates of that type (possibly choosing the subordinate who has just served the dictator’s predecessor).

In a subgame-perfect equilibrium, an incompetent subordinate always exerts a higher level of loyalty effort than a competent one. Otherwise, if the competent subordinates are also more loyal, every dictator type would prefer hiring a competent subordinate. This means that an
incompetent subordinate has a zero value of outside option, versus an above-zero outside option of competent subordinate who has a positive probability of being hired each period. But this means that an incompetent subordinate should put forth higher effort, as the marginal cost of loyalty effort is assumed to be increasing. So, incompetent subordinates are more loyal, and are preferred by the dictators who have a preference for competence below some threshold level.

This paper is structured as follows. Section 2 discusses the related literature. Section 3 contains the model. I first establish the existence of equilibrium and derive comparative statics for the special case of two subordinates (one of each type). The general case is then analyzed using numeric methods. Section 4 concludes.

2 Related literature

Modeling dictator behavior usually implies that dictators either maximize the probability of staying in office, or trade that probability off against other objectives (Wintrobe, 1998). Recent research looks at such aspects of decision-making in a dictatorial setting as concessions to opposition and sharing of rents (Gandhi and Przeworski, 2003), punishment and redistribution strategies (Acemoglu, Robinson, and Verdier, 2004), leader replacement by the elite (Besley and Kudamatsu, 2008, Bueno de Mesquita et. al., 2003), credibility of dictatorial commitment to power sharing agreements and rent redistribution (Boix and Svolik, 2013, Gehlbach and Keefer, 2011, Magaloni, 2010, Myerson, 2008), stability of governments and institutions given forward-looking behavior of stakeholders (Acemoglu, Egorov, and Sonin, 2010), involvement in international conflicts (Debs and Goemans, 2010), information manipulation (Edmond, 2013), or the choice of technology for monitoring subordinates (Egorov, Guriev, and Sonin, 2009). My framework of analysis is most closely related to the works of Debs (2006), Egorov and Sonin (2011), Glazer (2002), and Wagner (2006).
In Debs (2006) dynamic game, a dictator faces a possibility of revolt from the population that would replace him with his subordinate, who is responsible for the production of public goods in the economy. The type of the subordinate (capable or not) is assumed to be private information, while the dictator’s type is assumed to be observable. If the dictator is not capable, it is in his interest to replace a subordinate who was too successful in implementing economic policy, lest the public realizes that this subordinate is of a higher quality than the dictator. As a result, the subordinate’s incentives for the implementation of economic policy are depressed, and we should observe worse subordinate performance under less competent dictators.

A similar logic is applied to a dynamic setting by Francois, Rainer, and Trebbi (2013), who assume that subordinates have political capital that is accumulated at a constant rate. That capital is used both to produce output, and to contest for the spoils of office with the leader. Moreover, the subordinates also differ in their coup capacity, which grows with time and also depends on the subordinate’s rank. The leader’s decision to fire high-ranking subordinates before they obtain too much power results in a low level of subordinate competence. The authors then estimate hazard rates for the posts of cabinet ministers in sub-Saharan countries; in accordance with their theory, the hazard rates for the most powerful positions are higher.

Glazer (2002) looks at the employer’s decision on the competence (or quality) of the subordinate (or employee) as a tradeoff between internal and external rent seeking. A high-quality subordinate can help increase the firm’s profits, but he is also has a greater ability at rent-seeking within the firm — such as stealing from the owner, claiming credit for the firm’s achievements, or (in the political framework) acquiring too much political power and possibly unseating his superior. The quality of the owner is assumed to be exogenous and increases the firm’s success at external rent seeking, while decreasing the subordinate’s success at internal rent-seeking\(^6\). As a result, low-quality subordinates will be selected by either very low-quality

---

\(^6\)A different contest setting is analyzed by Kräkel (2012) who looks at the promotion process as a multi-stage
employers (who are posed to lose a lot due to internal rent seeking), or by very high-quality employers (who are sufficiently successful at external rent seeking); higher-quality subordinates will also be selected if the external rent-seeking task is more difficult, such as when a dictator is facing a war. The actions of the subordinates are assumed to be exogenous — in fact, only the employer (or dictator) decision is modeled.

In Egorov and Sonin (2011), a dictator chooses a single subordinate (a vizier), who may be either competent or not. After he has been chosen, the subordinate receives a noisy signal that tells him whether or not it is the appropriate moment to revolt and betray the dictator. For a competent subordinate the signal is less noisy; as the subordinate may expect to receive a punishment in the case of an unsuccessful revolt, the ex ante probability of subordinate betraying the dictator is higher if the latter is competent. For this reason, employing competent subordinates is risky, and the dictator may choose an incompetent subordinate. The incentive to hire an incompetent subordinate is greater if the dictator’s successor is likely to be undesirable. The subordinate’s payoffs due to choosing a disloyal action are exogenously determined by the subordinate’s type (and hence the quality of information that he receives) and by a random utility shock. In my paper, on the contrary, the subordinate is infinitely-lived, and their payoffs after they are dismissed together with the dictator are a function of the hiring strategies of the future dictators.

Wagner (2006) assumes both employer and the subordinate to be infinitely-lived. As in this work, the subordinate’s output is assumed to depend solely on competence, but not on his

7In Matozzi and Merlo (2010), politicians selected by parties engage in intra-party contests that are assumed to be beneficial to the parties; winners of these contests compete in the election. It thus might not be the best strategy for a party to recruit a star politician because that may depress competition for nomination within the party (although the winner of that contest will have a greater chance of winning the election).
effort. Every period, the subordinate may have an opportunity to partake in an activity that is beneficial to him and detrimental to the employer; the probability that such opportunity arises is increasing in the employee’s competence. The actions of the worker, however, have no consequence for the survival of the manager; more importantly, the outside employment opportunities are not affected by the strategies of any of the players. Hence, loyalty is a result of a self-enforcing contract between the employer and the subordinate. Several theoretical predictions that arise were tested in Wagner (2011) using a survey of experts from 35 countries.

Zudenkova (2014) analyzes a model of politicians recruiting teams of subordinates which may be of two types — cronies and experts. There are two types of tasks that a subordinate can accomplish: produce public goods and generate rent for the politician (which is assumed to be a club good for politician and his cronies, but not for the experts). Public goods increase the chances that the politician is reelected, and are more efficiently produced by the experts. The actions of the politician, as in my paper, are limited to choosing subordinates (he can select several subordinates of different types). However, the two-stage game ends after the subordinates choose their effort levels at the two tasks, and the politician is either reelected (based on the amount of public goods produced) or not; unlike in my model, the subordinates of both types receive identical payoffs if the politician is reelected. In equilibrium, politicians appoint just enough experts to reassure their reelection, and fill the remainder of the positions with cronies.

A number of recent empirical studies focus on loyalty-or-competence decisions of leaders in non-democratic countries. Xi (2013) look at the patterns of government promotions in China during the Qing period. The emperors of that period could choose from either Manchus — a small ethnic group to which the emperors themselves belonged — and Han, the majority group. Han officials were, on average, more efficient, due to stricter selection and better understanding of local issues; however, they were perceived as less loyal and were less likely to comply with
policies preferred by the ruler. The authors show that provincial governors were appointed from the majority group more often during the period of greater social unrest, when their knowledge of local politics was in greater demand.

In a similar analysis, Reuter and Robertson (2012) analyze the pattern of gubernatorial appointments in Russia during 2005-2010. They find that the economic performance of a region had very little explanatory effect on whether its governor was reappointed; far more important was his ability to deliver political support in the form of votes in favor of the pro-government United Russia party. The related study of Reuter and Buckley (2014) shows that both personal connections and administrative performance are important to the retention of vice-governors in the more authoritative of the Russian regions.

The work of Bai and Zhou (2014) focuses on the early period of Chinese Cultural Revolution (1966-69). The authors argue that the Chinese leader Mao Zedong followed an explicitly anti-competence policy when selecting members of the Communist Party’s Central Committee. During that period, most of the old members of the Central Committee were purged, while the Committee itself expanded in size from 180 to 279 members. The likelihood of being selected into the Committee decreased with one’s education or military rank, even controlling for age, year of joining the Party, or factional ties. Using Bayesian techniques, Shih, Adolph, and Liu (2012) analyze promotions to and within the Central Committee during 1982-2002. While higher education became a prerequisite for career advancement in this later period, factional ties and political importance of one’s home region were found to be far more important for promotions than home region’s economic performance (which, for some periods, had a negative effect on one’s Central Committee ranking). This finding is contrary to some earlier studies that found economic performance in one’s locality to have an effect on career advancement (Li and Zhou, 2002).

Finally, the loyalty-competence tradeoff has also been observed in experimental setting.
The design of Montinari, Nicolo, and Oexl (2012) is very similar to the assumptions made in my model. There is a single principal who must choose one of the two agents to carry out a task; one of the agents has a lower productivity level. The payoff of the agent is a fixed wage, while the payoff of the principal depends on the agent’s observable productivity, and on the noncontractible effort exerted by the agent (which additively contributes to the agent’s ability). After the agent was selected by the principal, but before the effort level was chosen, the principal could send a short message to the agent. About 29% of the principals chose the agent with the lower productivity, while the low-productivity agents contributed higher effort, which resulted in better overall performance. The authors attribute the better experimental performance of mediocre agents to a reciprocity norm that was induced by the principal’s messages.

3 The model

3.1 One subordinate of each type.

At every stage of an infinitely repeated game there are three players: the dictator and two infinitely-lived subordinates. One subordinate is competent, the other one is incompetent. The state of the game is characterized by two variables: the type of the dictator \( r \in [1, R] \) and the identity \( t \in \{H, L\} \) of the subordinate employed by the dictator, where \( H \) denotes the competent subordinate, and \( L \) denotes the incompetent one. At each stage the following sequence of events takes place:

1. The subordinate who is employed by the dictator chooses the probability \( p \in [0, 1] \) with which the dictator is to survive into the next period. The unemployed subordinate takes no action.
2. The dictator is ousted with probability $1 - p$. If a dictator is ousted, he leaves the game permanently.\footnote{Dragu and Polborn (2013) also assume that the survival of a political leader depends on a noncontractable, costly action of his subordinate. However, they study a different phenomenon — whether institutional constraints that punish the subordinate for implementing certain polities can influence the policy that the ruler chooses to implement.}

3. If the dictator is ousted, nature determines the type of the new dictator $r$, drawing it from a uniform distribution on $[1, R]$.

4. If there was a change of dictator this period, the new dictator chooses which of the two subordinates $t \in \{H, L\}$ to recruit. If the dictator did not change, he retains his current subordinate into the next period.

At the beginning of every period in office, the dictator receives a payoff of 1 if his subordinate is incompetent, and $r$ if the subordinate is competent. Each subordinate receives a rent of 1 every period that he is employed by a dictator, and a payoff of 0 every period that he is unemployed. The employed subordinate also incurs cost $c(p)$ due to his efforts to prolong the dictator’s term in office. Let $c'(0) = 0$, $c(0) = 0$, $c(1) = 1$, $c' > 0$, and $c'' > 0$.

A stationary strategy profile is a 3-tuple $(d(\cdot), p_H, p_L)$, where $p_H$ is the loyalty effort of the competent subordinate, $p_L$ - loyalty effort of the incompetent subordinate, and $d(\cdot) : [1, R] \rightarrow \{H, L\}$ is the dictator’s choice of subordinate depending on the dictator’s type. A Markov perfect equilibrium is a stationary strategy profile that is a subgame-perfect equilibrium of the infinitely repeated game. The definition of a stationary strategy in this work is in fact more restrictive than the standard definition (Myerson, 1997). I not only require each player to choose a strategy that depends only on the state of the game. Given an infinite number of dictator players, I also assume that all players of a given type play identical strategies. However, the
equilibrium still corresponds to the standard definition, as the discounted payoff of each player is maximized by his strategy at any stage of the game and in every state.

The following result can be established.

**Theorem 1** Suppose that \( c'(1) \) is sufficiently large. Then a Markov perfect equilibrium exists.

Let \( \delta < 1 \). Then in any such equilibrium we have \( 0 < p_H < p_L < 1 \). All dictators with \( r \in [1, \bar{r}] \) choose the incompetent subordinate, and all dictators with \( r \in [\bar{r}, R] \) choose the competent subordinate, where

\[
\bar{r} = \frac{1 - \delta_d p_H}{1 - \delta_d p_L} \tag{1}
\]

lies in \((1, R)\). The probability that the competent subordinate is chosen is given by

\[
\pi = 1 - \frac{\delta_d (p_L - p_H)}{(R - 1)(1 - \delta_d p_L)} , \tag{2}
\]

where we will always have \( \pi > \frac{1}{2} \).

If \( \delta = 1 \), then there exists a Markov perfect equilibrium with \( p_H = p_L = \pi = 1 \).

So, in any equilibrium we will have the dictators who value the performance of subordinates relatively low compared to office benefits choose the incompetent subordinate. The dictators who value performance relatively high compared to office rents choose the competent subordinate. The competent subordinate always exerts less effort to prolong the dictator’s term in office than his incompetent counterpart. This is true because otherwise every dictator type will prefer the competent subordinate, who in that case will not be induced to exert any effort at all, knowing that he will remain employed next period, no matter what happens to the incumbent dictator. As a result, those dictators who value the performance of their subordinates more highly will be short-lived compared to their counterparts who are willing to hire incompetent subordinates due to their greater loyalty. Moreover, a capable subordinate will be chosen more often (here, I assume that the type of each dictator is uniformly distributed on \([1, R]\); however, one can show that this result holds for any smooth distribution of \( r \) on \([1, R]\)).
Comparative statics can be evaluated for some limiting cases of the parameters.

**Theorem 2** Let \((p_H, p_L, \pi)\) be a Markov perfect equilibrium.

1. Suppose that \(\delta\) is sufficiently close to 0. Then we have \(\frac{\partial p_H}{\partial \delta} > 0\), \(\frac{\partial p_L}{\partial \delta} < 0\), \(\frac{\partial \pi}{\partial \delta} < 0\), \(\frac{\partial p_H}{\partial \delta} < 0\), \(\frac{\partial p_L}{\partial \delta} < 0\), and \(\frac{\partial \pi}{\partial \delta} > 0\). We have \(p_H \to 0\), \(p_L \to 0\), and \(\pi \to 1\) as \(\delta \to 0\). If \(\delta = 1\), \(p_H = 1\), \(p_L = 1\), and \(\pi = 1\), then \(\frac{\partial p_H}{\partial \delta} > 0\), \(\frac{\partial p_L}{\partial \delta} > 0\), and \(\frac{\partial \pi}{\partial \delta} > 0\).

2. Suppose that \(\delta_d\) is sufficiently close to 0, or \(R\) is sufficiently large. Then we have \(\frac{\partial p_H}{\partial \delta_d} > 0\), \(\frac{\partial p_L}{\partial \delta_d} < 0\), \(\frac{\partial \pi}{\partial \delta_d} < 0\), \(\frac{\partial p_H}{\partial R} < 0\), \(\frac{\partial p_L}{\partial R} > 0\), \(\frac{\partial \pi}{\partial R} > 0\), and \(\frac{\partial p_H}{\partial \delta} > 0\). We have \(p_H \to 0\), \(\pi \to 1\), and \(p_L \to \bar{p}\) as \(\delta_d \to 0\) or \(R \to \infty\), where \(\bar{p}\) is a solution to

\[
(1 - \delta \bar{p})c'(\bar{p}) - \delta (1 - c(\bar{p})) = 0. 
\]

3. Let \(R\) be sufficiently close to 1. Then we have \(\frac{\partial p_H}{\partial \delta_d} > 0\), \(\frac{\partial p_L}{\partial \delta_d} < 0\), \(\frac{\partial p_H}{\partial R} < 0\), and \(\frac{\partial p_L}{\partial R} > 0\).

Let \(\tilde{p}\) be the solution to

\[
(1 - \delta \tilde{p})c'(\tilde{p}) = \frac{\delta}{2}(1 - c(\tilde{p})). 
\]

Then the sign of \(\frac{\partial p_H}{\partial \delta_d}\), \(\frac{\partial p_L}{\partial \delta_d}\), and \(\frac{\partial \pi}{\partial R}\) is equal to the sign of \(2c''(\tilde{p})(1 - \delta \tilde{p}) - \delta c'(\tilde{p})\), while \(\frac{\partial \pi}{\partial \delta_d}\) has the opposite sign. We have \(p_H \to \tilde{p}\), \(p_L \to \tilde{p}\), and \(\pi \to \frac{1}{2}\) as \(R \to 1\).

The subordinate discount rate \(\delta\) has a non-monotonic effect on the fraction of dictators who hire competent subordinates (see Figure 1(b)\(^9\)). If the subordinates are not forward-looking, their efforts are small, and dictators prefer to hire the competent subordinate. As \(\delta\) increases, the efforts of the subordinates increase, as well as the difference between the efforts of the incompetent and competent subordinates. As a result, some of the dictators begin to hire the incompetent subordinate. Finally, as \(\delta\) approaches unity, the competent subordinate is also

\(^9\)The equilibrium was found using a gradient search algorithm implemented in Matlab. I assumed that the cost function is \(c(p) = 1 - \sqrt{1 - p^2}\).
forced to exert high levels of effort; that makes him once again attractive to dictators with low \( r \).

Figure 1: Markov perfect equilibria depending on parameter values.

The time preference of the dictators \( \delta_d \) and the expected value that the dictators place on competence (which is a linear function of \( R \)) have countervailing effects on all three endogenous variables: efforts of the competent and incompetent subordinates, as well as on the probability that the competent subordinate is hired. If the discount rate of a dictator is low, he prefers to hire the competent subordinate (because he does not worry whether he will survive into the next period); the competent subordinate, as a result, exert little effort to prolong the dictator’s term in office. The incompetent subordinate, in the rare event that he is hired, will show considerable loyalty to the dictator. If the discount rate of the dictators increases, more of them will prefer to hire the incompetent subordinate because of his loyalty. As a result, the competent subordinate will be induced to be more loyal, while the incompetent one will become less loyal because the value of his outside option has increased. We will see the opposite effect as \( R \) increases (Figure 1(a)). On the other hand, the efforts of both types of subordinates (as well as the probabilities that either one will be hired) will be the same if competence is not valued and \( R \) is close to 1.
I find that for any subordinate discount rate $\delta \in (0, 1)$, any dictator discount rate $\delta_d \in (0, 1)$, and any $R > 1$ there will be some dictator types $r \in [1, \bar{r})$ that will hire the incompetent subordinate. This is because I assume that, for some dictator types, the payoff of dictator $r$ from hiring the competent subordinate will be close enough to 1 — which is the dictator’s payoff if the incompetent subordinate is hired.

Now suppose that dictator type $r$ is distributed on $[\underline{r}, R]$ with $\underline{r} > 1$. Suppose that the dictators always prefer the competent subordinate. In that case we should have $\pi = 1$, $p_H = 0$, and $p_L = \bar{p}$ given by (3). That will be an equilibrium if and only if $\underline{r}$ is large enough. A dictator of type $r$ who hires the incompetent subordinate will get a payoff of

$$\tilde{U}_L = \frac{1}{1 - \delta_d \bar{p}}.$$

(5)

If he hires the competent subordinate, his payoff will be

$$\tilde{U}_H = r.$$

(6)

Therefore, we will have an equilibrium in which the incompetent subordinate is never hired if and only if

$$\underline{r} \geq \frac{1}{1 - \delta_d \bar{p}}.$$

(7)

Moreover, that will be the only equilibrium in the game, because this is a sufficient condition for dictators of all types hiring the competent subordinate.

Inequality (7) will hold under two conditions. First, this is true if the discount factor of dictators $\delta_d$ is small enough. If the dictators are not sufficiently forward-looking, they will prefer to hire the competent type who is not loyal but is good at other tasks such as carrying out economic policy. Second, the competent type is always hired if $\bar{p}$ — the loyalty effort of a subordinate who expects that he will never be hired if his patron’s tenure ends — is low enough. That, in turn, will happen if the discount factor of the subordinates $\delta$ is low enough.
It can be shown that \( \bar{p} \) increases with \( \delta \), while for small \( \delta \), \( \bar{p} \) is close to 0. Hence, if \( r > 1 \) and the subordinates are sufficiently not forward-looking, then all dictator types hire the competent subordinate.

### 3.2 Several subordinates of each type.

So far I have looked at the case when there are two subordinates, one of the capable and the other one of the incapable type. We can analyze a more general case, when there are \( N_H \geq 1 \) capable subordinates, and \( N_L \geq 1 \) incapable subordinates. As before, I assume that only one subordinate is employed at any time. When a new dictator assumes office, he (depending on his \( r \)) either chooses a capable subordinate at random from the pool of \( N_H \) candidates, or chooses an incapable subordinate in a similar manner. I assume that in either case, all candidates of the chosen type have the same probability of being hired by the dictator.

Equilibrium comparative statics results similar to Theorem 2 can only be obtained through numeric means. However, several results can still be derived analytically.

**Theorem 3** Let \((p_H, p_L, \pi)\) be a Markov perfect equilibrium. Then the following is true.

1. We have \( 0 < p_H < p_L < 1 \), with (1) and (2) holding.

2. As \( N_H \to \infty \), we have \( p_H \to \bar{p}, p_L \to \bar{p}, \) and \( \pi \to 1 \),

3. As \( N_L \to \infty \), we have \( p_L \to \bar{p} \),

where \( \bar{p} \) is given by (3).

Figures 2(a), 2(b) illustrate the comparative statics with respect to \( N_L \) and \( N_H \).

If the number of potential competent subordinates increases, with the number of incompetent subordinates remaining fixed, each competent subordinate will exert a greater loyalty.
effort while employed. As the number of competent subordinates becomes large, the effort levels of competent and incompetent subordinates converge, with almost all dictator types hiring competent subordinates.

As the number of potential incompetent subordinates increases, each incompetent subordinate faces an increasingly smaller probability of being hired, and is forced to exert greater effort to prolong the dictator’s term in office. Than makes incompetent subordinates more attractive to dictators. As a result, the probability \( \pi \) that a competent subordinate is hired by the dictator’s successor may decrease, with the effort of the competent subordinate(s) increasing for the same reason.

Now suppose again that dictator type \( r \) is distributed on \([r, R]\), with \( r > 1 \). When do we have an equilibrium with \( \pi = 1 \)? In that case we must have \( p_L = \bar{p} \). However, it can be shown that \( p_H \) will be given by \( \bar{p}_N \) the solution to

\[
c'(\bar{p}_N)(1 - \delta \bar{p}_N) = \delta (1 - c(p_N)) \frac{N-1}{N}.
\] (8)

For \( N = 1 \) we have \( p_N = 0 \), with \( p_N \) increasing with \( N \). Moreover, \( \bar{p}_N \to \bar{p} \) as \( N \to \infty \). Dictator’s payoff from hiring an incompetent subordinate will be given by (5). If a competent subordinate is hired, it will be

\[
\tilde{U}_H = \frac{r}{1 - \delta \bar{p}_N}.
\] (9)
It follows that an equilibrium in which an incompetent subordinate is never hired will be possible if and only if

$$\frac{r}{1 - \delta \bar{p}_N} \geq \frac{1}{1 - \delta_d \bar{p}}$$

(10)

If competent subordinates are in short supply, it may be possible that some dictator types prefer to hire incompetent subordinates because of their loyalty. However, if $r > 1$ and the pool of competent subordinates is large enough, then competent subordinates will be loyal, and all dictator types will prefer competent subordinates.

4 Discussion

Explaining the economic performance of autocracies is a theoretical challenge. Many such regimes are characterized by poor governance and suboptimal selection of officials, as creating competent and merit-based bureaucracies is politically costly for non-democratic leaders (Geddes, 1994) and conflicts with other goals, including immediate political survival. Long-lived personalist dictators in particular tend to be predatory and produce inferior economic outcomes (Haber, 2005), contrary to the well-known argument (such as in McGuire and Olson, 1996) that they will act as “stationary bandits” and will therefore be more interested in improving economic performance.

An autocrat faces a number of threats, such as elite schisms, betrayal, or public unrest (Bueno de Mesquita et. al., 2003, Egorov and Sonin, 2011, Svolik 2012); offsetting these threats takes effort on behalf of his lieutenants. In this work I build a dynamic model of an autocracy, where the dictator’s decision is to hire a competent or an incompetent subordinate, and the subordinate’s decision is how much loyalty to supply to the dictator. A subordinate acts as an agent of a dictator, weighing loyalty, that comes at a cost and is non-contractable, against his payoff in case the dictator loses his job. The central assumptions made in this work
are that a subordinate may be hired by the dictator’s successor, and that dictators differ in how they value the competence of their subordinates.

The model explains why dictators may prefer subordinates who are low quality (that is, less competent at other tasks, such as carrying out economic policy). I demonstrate that low-quality subordinates will always provide a higher level of loyalty than their high-quality counterparts. That will make them attractive to those dictators who for some reasons (institutional or, perhaps, personal) put less value at competence than others. Any other arrangement is untenable: if a subordinate is expected to be better both at ensuring the political survival of dictators and at providing public goods, then he should expect to be hired more often than someone who is worse at both tasks. However, for that very reason the incompetent subordinate should put forward more effort to keep his patron in power, as he is not likely to be employed by the dictator’s successor. As a result, those dictators who care less about economic performance (and, therefore, about competence) select less capable subordinates who, in turn, are more loyal because of their lower outside options.

The framework of analysis used in this work can be extended in several ways. For example, one can make a prediction that a dictator will be less likely to give important government posts to his relatives if he expects that his successor will also engage in nepotism. This is because expectations of nepotism from future dictators are likely to reduce outside options, and induce greater loyalty, from all kinds of current subordinates. That will make highly competent outsiders relatively more attractive to the less competent relatives of the current dictator.

I assumed that both subordinates have identical cost functions of their loyalty efforts. It can be argued that the cost of loyalty may be either higher or lower for the capable subordinate. It will be higher if the cost of loyalty reflects the bygone opportunity of outside employment or (in the manner of Egorov and Sonin, 2011) the bygone opportunity of joining a successful
revolt against the leader. On the other hand, the capable subordinate may be more efficient at prolonging the dictator’s term in office; in that case, his costs will be lower.

Another assumption made in this work is that the actions of the subordinate are non-contractible. Otherwise, if the dictator was able to offer the subordinate a contract that specified a transfer to the subordinate depending on his effort, we should observe a result that is opposite from the one obtained here. A dictator who receives a higher payoff from the services of a competent subordinate would be willing to pay more to prolong his term in office. Therefore, absent the agency problem between dictators and subordinates, we should expect all dictators to employ capable subordinates, and those with higher office rents to have longer tenures\textsuperscript{10}. In my case, however, the effort of a subordinate is not contractible, and is thus determined entirely by the value of his outside option.

\section*{Appendix}

\textbf{Proof of Theorem 1.}

Denote by $U_H$ and $U_h$ the continuation values of the competent subordinate if he is employed and if he is unemployed. Similarly define $U_L$ and $U_l$.

Let $\pi$ be the probability that a dictator whose type is unknown chooses a competent subordinate.

\textsuperscript{10}Stability of political regimes and subordinate competence are positively related in Lagerlöf (2012) dynamic model of power transfers in dictatorships where both loyalty and competence are assumed to be fixed qualities of subordinates.
We can then write out the continuation values:

\[ U_H = 1 - c(p_H) + \delta(p_H U_H + (1 - p_H)(\pi U_H + (1 - \pi)U_h)) \quad (11) \]

\[ U_h = \delta(p_L U_h + (1 - p_L)(\pi U_h + (1 - \pi)U_h)), \quad (12) \]

\[ U_L = 1 - c(p_L) + \delta(p_L U_L + (1 - p_L)((1 - \pi)U_L + \pi U_l)), \quad (13) \]

\[ U_l = \delta(p_H U_l + (1 - p_H)((1 - \pi)U_L + \pi U_l)). \quad (14) \]

The first-order condition for utility maximization gives us

\[ \frac{\partial U_H}{\partial p_H} = -c'(p_H) + \delta(1 - \pi)(U_H - U_h) = 0, \quad (15) \]

\[ \frac{\partial U_L}{\partial p_L} = -c'(p_L) + \delta\pi(U_L - U_i) = 0. \quad (16) \]

The second-order conditions for utility maximization are always satisfied:

\[ \frac{\partial^2 U_H}{\partial p_H^2} = -c''(p_H) < 0, \quad (17) \]

\[ \frac{\partial^2 U_L}{\partial p_L^2} = -c''(p_L) < 0. \quad (18) \]

Rewriting (12), we get

\[ U_h = \delta(p_L + (1 - \pi)(1 - p_L))U_h + \delta\pi(1 - p_L)U_H = \frac{\delta\pi(1 - p_L)U_H}{1 - \delta(1 - \pi + \pi p_L)} \quad (19) \]

and

\[ U_H - U_h = \frac{(1 - \delta)U_H}{1 - \delta(1 - \pi + \pi p_L)}. \quad (20) \]

Substituting (19) into (11) we get

\[ U_H = 1 - c(p_H) + \delta(p_H + \pi - \pi p_H)U_H + \delta(1 - \pi)(1 - p_H)U_h = \]

\[ = 1 - c(p_H) + \delta(p_H + \pi - \pi p_H)U_H + \frac{\delta^2\pi(1 - \pi)(1 - p_H)(1 - p_L)U_H}{1 - \delta(1 - \pi + \pi p_L)} = \quad (21) \]

\[ = \frac{1 - c(p_H)}{1 - B - C}, \quad (22) \]

where \( B = \delta(p_H + \pi - \pi p_H), C = \delta^2\pi(1 - \pi)(1 - p_H)(1 - p_L), D = 1 - \delta(1 - \pi + \pi p_L). \) We have

\[ U_H - U_h = \frac{U_H(1 - \delta)}{D} = \frac{(1 - c(p_H))(1 - \delta)}{D - DB - C} = \frac{1 - c(p_H)}{1 - \delta p_H - \delta\pi(p_L - p_H)}. \quad (23) \]
That allows us to rearrange the first-order condition (15) as

\[ H1 = (1 - \delta p_H - \delta \pi (p_L - p_H))c'(p_H) - \delta (1 - \pi)(1 - c(p_H)) = 0. \]  

(24)

The other first-order condition (16) becomes

\[ H2 = (1 - \delta p_L + \delta (1 - \pi)(p_L - p_H))c'(p_L) - \delta \pi (1 - c(p_L)) = 0. \]  

(25)

Let \( p_H \) and \( p_L \) satisfy (24) and (25). The lifetime utility of a dictator with type \( r \) will be

\[ \tilde{U}_H = \frac{r}{1 - \delta_d p_H} \]  

(26)

if he employs the competent subordinate, and

\[ \tilde{U}_L = \frac{1}{1 - \delta_d p_H} \]  

(27)

if the subordinate is incompetent. It follows that a competent subordinate is always chosen when \( p_H \geq p_L \), and is chosen whenever \( r \geq \tilde{r} \). In equilibrium we must have \( p_H < p_L \). Otherwise we will get \( \pi = 1 \) and, following (25), \( p_L = 0 \); however, substituting \( \pi = 1 \) into (24) we obtain \( p_H > 0 \), reaching a contradiction. We also cannot have \( \tilde{r} > R \). If so, we will have \( \pi = 0 \) from (25) and \( p_L = 0 \) but \( p_H > 0 \) from (24). It follows that in any equilibrium, we must have \( p_H < p_L \), with some dictator types \( r \in [1, \tilde{r}) \) employing the incompetent subordinate, and dictator types \( r \in [\tilde{r}, R] \) employing the competent subordinate. Rewriting (1), we get

\[ H_3 = \pi - 1 + \frac{\delta_d (p_L - p_H)}{(R - 1)(1 - \delta_d p_L)} = 0. \]  

(28)

This condition, together with (24) and (25), describes the Markov perfect equilibrium.
We have
\[
\frac{\partial H_1}{\partial p_H} = c''(p_H)(1 - \delta p_H - \delta \pi (p_L - p_H)),
\]
(29)
\[
\frac{\partial H_1}{\partial p_L} = -\delta \pi c'(p_H),
\]
(30)
\[
\frac{\partial H_1}{\partial \pi} = -\delta (p_L - p_H)c'(p_H) + \delta (1 - c(p_H)),
\]
(31)
\[
\frac{\partial H_2}{\partial p_H} = -\delta (1 - \pi)c'(p_L),
\]
(32)
\[
\frac{\partial H_2}{\partial p_L} = c''(p_L)(1 - \delta p_L + \delta (1 - \pi)(p_L - p_H)),
\]
(33)
\[
\frac{\partial H_2}{\partial \pi} = -\delta (p_L - p_H)c'(p_L) - \delta (1 - c(p_L)),
\]
(34)
\[
\frac{\partial H_3}{\partial p_H} = -\delta d \frac{(R - 1)(1 - \delta_d p_H)}{1 - \delta_d p_H},
\]
(35)
\[
\frac{\partial H_3}{\partial p_L} = \delta_d (1 - \delta_d p_H),
\]
(36)
\[
\frac{\partial H_3}{\partial \pi} = 1.
\]
(37)

We have \( \frac{\partial H_1}{\partial p_H} > 0 \), \( H_1(p_H = 0) < 0 \). If \( c'(1) \) is large enough, we also have \( H_1(p_H = 1) > 0 \).

Thus (24) defines \( p_H \) as a continuous function of \( (p_L, \pi) \) that assumes values between 0 and 1.

Same is true for (25) and \( p_L \) and, by construction, for (28) and \( \pi \). The existence of equilibrium
follows from the Brouwer fixed-point theorem.

Now we prove that \( \pi > \frac{1}{2} \). Divide (24) by \( \delta c'(p_H) \), and subtract from the result (25) divided
by \( \delta c'(p_L) \). One gets
\[
\frac{(1 - \pi)(1 - c(p_H))}{c'(p_H)} = \frac{(1 - \pi)(1 - c(p_L))}{c'(p_L)}.
\]
(38)

It follows that \( \pi > \frac{1}{2} \) if and only if
\[
\frac{1 - c(p_H)}{c'(p_H)} < \frac{1 - c(p_L)}{c'(p_L)}.
\]
(39)

This is true, because \( p_L > p_H \) and
\[
\left( \frac{1 - c(p)}{c'(p)} \right)' = \frac{-c^2(p) - c''(p)(1 - c(p))}{c^2(p)} < 0.
\]
(40)

If \( \delta = 1 \), then the system of equations (24), (25), and (28) has a solution at \( p_H = p_L = \pi = 1 \).

Q.E.D.

27
Proof of Theorem 2.

Denote

\[ DH = \begin{pmatrix} \frac{\partial H_1}{\partial p_H} & \frac{\partial H_1}{\partial p_L} & \frac{\partial H_1}{\partial \pi} \\ \frac{\partial H_2}{\partial p_H} & \frac{\partial H_2}{\partial p_L} & \frac{\partial H_2}{\partial \pi} \\ \frac{\partial H_3}{\partial p_H} & \frac{\partial H_3}{\partial p_L} & \frac{\partial H_3}{\partial \pi} \end{pmatrix}. \] (41)

We have

\[ \frac{\partial H_1}{\partial \delta} = -(p_H + \pi(p_L - p_H))c'(p_H) - (1 - \pi)(1 - c(p_L)), \] (42)
\[ \frac{\partial H_1}{\partial \delta_d} = 0, \] (43)
\[ \frac{\partial H_1}{\partial \pi} = 0, \] (44)
\[ \frac{\partial H_2}{\partial \delta} = -(p_L - (1 - \pi)(p_L - p_H))c'(p_L) - \pi(1 - c(p_L)), \] (45)
\[ \frac{\partial H_2}{\partial \delta_d} = 0, \] (46)
\[ \frac{\partial H_2}{\partial \pi} = 0, \] (47)
\[ \frac{\partial H_3}{\partial \delta} = 0, \] (48)
\[ \frac{\partial H_3}{\partial \delta_d} = \frac{(p_L - p_H)}{(R - 1)(1 - \delta_d p_L)^2}, \] (49)
\[ \frac{\partial H_3}{\partial \pi} = -\delta_d(p_L - p_H)(R - 1)^2(1 - \delta_d p_L), \] (50)

Let \( \delta \ll 1 \). We know from (24), (25) that \( p_H \to 0 \) and \( p_L \to 0 \) as \( \delta \to 0 \). Ignoring all terms in (29)-(37) that have smallness of 2 or greater, we have

\[ |DH| = c''(p_H)c''(p_L) + \frac{\delta \delta_d(c''(p_H) + c''(p_L))}{R - 1} > 0 \] (51)

and

\[ DH^{-1} = \frac{1}{|DH|} \begin{pmatrix} c''(p_L)(R - 1) + \delta \delta_d & \delta \delta_d & -\delta c''(p_L)(R - 1) \\ \delta \delta_d & c''(p_H)(R - 1) + \delta \delta_d & \delta c''(p_H)(R - 1) \\ \delta d c''(p_L) & -\delta d c''(p_H) & c''(p_H)c''(p_L)(R - 1) \end{pmatrix}. \] (52)
In order to see how $p_H$, $p_L$, and $\pi$ change with $\delta$, $\delta_d$ and $R$, we apply the implicit function theorem:

\[
\frac{\partial p_H}{\partial \delta} = -\frac{1}{|DH|} \frac{\delta_d(p_L - p_H)}{(R-1)(1-\delta_d p_L)} \delta c''(p_L) < 0, \tag{53}
\]

\[
\frac{\partial p_L}{\partial \delta} = \frac{1}{|DH|} \frac{\delta_d(p_L - p_H)}{(R-1)(1-\delta_d p_L)} \delta c''(p_H) > 0, \tag{54}
\]

\[
\frac{\partial \pi}{\partial \delta} = \frac{1}{|DH|} \frac{\delta_d(p_L - p_H)}{(R-1)(1-\delta_d p_L)} c''(p_H) c''(p_L) > 0, \tag{55}
\]

\[
\frac{\partial p_H}{\partial \delta_d} = \frac{1}{|DH|} \frac{(p_L - p_H)}{(1-\delta_d p_L)^2} \delta c''(p_L)(R-1) > 0, \tag{56}
\]

\[
\frac{\partial p_L}{\partial \delta_d} = -\frac{1}{|DH|} \frac{(p_L - p_H)}{(1-\delta_d p_L)^2} \delta c''(p_H) < 0, \tag{57}
\]

\[
\frac{\partial \pi}{\partial \delta_d} = -\frac{1}{|DH|} \frac{(p_L - p_H)}{(1-\delta_d p_L)^2} c''(p_H) c''(p_L) < 0. \tag{58}
\]

If $\delta = \pi = p_H = p_L = 1$, then $DH$ has an inverse which is given by

\[
DH^{-1} = \begin{pmatrix}
\frac{1}{c''(1)} & \frac{c'(1)}{c''(1)^2} & 0 \\
0 & \frac{1}{c''(1)} & 0 \\
\frac{\delta_d}{(R-1)(1-\delta_d c''(1))} & \frac{\delta_d}{(R-1)(1-\delta_d)} & \frac{c'(1)-c''(1)}{c''(1)^2} & 1
\end{pmatrix}. \tag{60}
\]

It follows that

\[
\frac{\partial p_H}{\partial \delta} = \frac{c'(1)}{c''(1)} (1 + \frac{c'(1)}{c''(1)}) > 0 \tag{61}
\]

\[
\frac{\partial p_L}{\partial \delta} = \frac{c'(1)}{c''(1)} > 0 \tag{62}
\]

\[
\frac{\partial \pi}{\partial \delta} = \frac{\delta_d c'(1)}{(R-1)(1-\delta_d c''(1)^2)} > 0. \tag{63}
\]

Now let $\delta_d \ll 1$ or $R \gg 1$. We know that $\pi \to 1$ and $p_H \to 0$ as $\delta_d \to 0$ or $R \to \infty$. Ignoring all terms in (29)-(37) that have smallness of 1 or greater, we have

\[
|DH| \to c''(p_H)c''(p_L)(1 - \delta p_L)^2 \tag{64}
\]
and
\[
DH^{-1} = \frac{1}{|DH|} \begin{pmatrix}
    c''(p_L)(1 - \delta p_L) & 0 & -\delta c''(p_L)(1 - \delta p_L)(1 - c_H) \\
    0 & c''(p_H)(1 - \delta p_L) & \delta c''(p_H)(1 - \delta p_L)(c'(p_L)p_L + 1 - c(p_L)) \\
    0 & 0 & c''(p_H)c''(p_L)(1 - \delta p_L)^2
\end{pmatrix}.
\]

It follows that
\[
\frac{\partial p_H}{\partial R} = -\frac{1}{|DH|}(R - 1)^2 \frac{\delta p_L}{p_L} \delta c''(p_L)(1 - c_H) < 0,
\]
\[
\frac{\partial p_L}{\partial R} = \frac{1}{|DH|}(R - 1)^2 \frac{\delta p_L}{p_L} \delta c''(p_H)(c'(p_L)p_L + 1 - c(p_L)) > 0,
\]
\[
\frac{\partial \pi}{\partial R} = \frac{1}{|DH|}(R - 1)^2 \frac{\delta p_L}{p_L} c''(p_H)c''(p_L)(1 - \delta p_L) > 0,
\]
\[
\frac{\partial p_H}{\partial \delta_d} = \frac{1}{|DH|}(R - 1)(1 - \delta p_L) \delta c''(p_L)(1 - c_H) > 0,
\]
\[
\frac{\partial p_L}{\partial \delta_d} = -\frac{1}{|DH|}(R - 1)(1 - \delta p_L) \delta c''(p_H)(c'(p_L)p_L + 1 - c(p_L)) < 0,
\]
\[
\frac{\partial \pi}{\partial \delta_d} = -\frac{1}{|DH|}(R - 1) c''(p_H)c''(p_L) < 0,
\]
\[
\frac{\partial p_H}{\partial \delta} = \frac{1}{|DH|} p_L c'(p_H)c''(p_L)(1 - \delta p_L) > 0,
\]
\[
\frac{\partial p_L}{\partial \delta} = \frac{1}{|DH|} (p_L c'(p_L) + 1 - c(p_L)) c''(p_H)(1 - \delta p_L) > 0.
\]

The sign of \(\frac{\partial \pi}{\partial \delta}\) cannot be determined if one evaluates the matrix \(DH^{-1}\) with small terms. At \(\pi = 1\), (25) gives us (3).

Finally, let \(R \approx 1\). If so, from (28) we must have \(p_H - p_L \to 0\) but \(p_L > p_H\); (24) and (25) gives us \(\pi \to \frac{1}{2}\) as \(R \to 1\). We rewrite the condition (28), multiplying it by \(R - 1\). At \(R - 1\) we have
\[
DH^{-1} = \begin{pmatrix}
    \frac{1}{DH} & \frac{1}{DH} & -\frac{1 - \delta p_L}{28_d} \\
    \frac{1}{DH} & \frac{1}{DH} & \frac{1 - \delta p_L}{28_d} \\
   \frac{1}{28(1 - c(p_L))} & \frac{1}{28(1 - c(p_L))} & \frac{(1 - \delta p_L)D_H}{48_d(1 - c(p_L))}
\end{pmatrix}.
\]
where
\[
\tilde{DH} = 2c''(c_L)(1 - \delta p_L) - \delta' c'(p_L).
\] (76)

Implicit function theorem gives us
\[
\frac{\partial p_H}{\partial R} = -\frac{1 - \delta_d p_L}{2\delta_d} \frac{\delta_d(p_L - p_H)}{(R - 1)^2(1 - \delta_d p_L)} < 0,
\] (77)
\[
\frac{\partial p_L}{\partial R} = \frac{1 - \delta_d p_L}{2\delta_d} \frac{\delta_d(p_L - p_H)}{(R - 1)^2(1 - \delta_d p_L)} > 0,
\] (78)
\[
\frac{\partial p_H}{\partial \delta_d} = \frac{1 - \delta_d p_L}{2\delta_d} \frac{(p_L - p_H)}{(R - 1)(1 - \delta_d p_L)^2} > 0,
\] (79)
\[
\frac{\partial p_L}{\partial \delta_d} = -\frac{1 - \delta_d p_L}{2\delta_d} \frac{(p_L - p_H)}{(R - 1)(1 - \delta_d p_L)^2} < 0.
\] (80)

Taking \( p_H = p_L \) and \( \pi = \frac{1}{2} \) and substituting (25) into (42) and (45), we find that \( \frac{\partial H_1}{\partial \delta} = \frac{\partial H_2}{\partial \delta} < 0 \).

Hence the sign of both \( \frac{\partial p_H}{\partial \delta_d} \) and \( \frac{\partial p_L}{\partial \delta_d} \) is equivalent to the sign of \( \tilde{DH} \). The sign of \( \frac{\partial \pi}{\partial R} \) is equal to the sign of \( \tilde{DH} \), and the sign of \( \frac{\partial \pi}{\partial \delta_d} \) is opposite to the sign of \( \tilde{DH} \). For \( \pi = \frac{1}{2} \), (25) gives us (4).

Q.E.D.

Proof of Theorem 3.

Denote by \( U_H, U_{Hh} \), and \( U_{Hl} \) the continuation values of the competent subordinate if he is employed, if he is unemployed and another competent subordinate is employed, and if he is unemployed and an incompetent subordinate is unemployed. Similarly define \( U_L, U_{Lh}, \) and \( U_{Ll} \).

Let \( \pi \) be the probability that a dictator whose type is unknown chooses a capable subordinate.
We can then write out the continuation values:

\[
U_H = 1 - c(p_H) + \delta \left(p_H U_H + (1 - p_H)(1 - \pi)U_H + (1 - p_H)^2 \pi \left(U_H \frac{1}{N_H} + U_{Hh} \frac{N_H - 1}{N_H} \right)\right), \tag{81}
\]

\[
U_{Hh} = \delta \left(p_H U_{Hh} + (1 - p_H)(1 - \pi)U_{Hh} + (1 - p_H)^2 \pi \left(U_{Hh} \frac{1}{N_H} + U_{Hh} \frac{N_H - 1}{N_H} \right)\right), \tag{82}
\]

\[
U_{Hl} = \delta \left(p_H U_{Hl} + (1 - p_H)(1 - \pi)U_{Hl} + (1 - p_H)^2 \pi \left(U_{Hl} \frac{1}{N_H} + U_{Hl} \frac{N_H - 1}{N_H} \right)\right), \tag{83}
\]

\[
U_L = 1 - c(p_L) + \delta \left(p_L U_L + (1 - p_L)(1 - \pi)U_L + (1 - p_L)^2 \pi \left(U_L \frac{1}{N_L} + U_{Ll} \frac{N_L - 1}{N_L} \right)\right), \tag{84}
\]

\[
U_{Lh} = \delta \left(p_H U_{Lh} + (1 - p_H)(1 - \pi)U_{Lh} + (1 - p_H)^2 \pi \left(U_{Lh} \frac{1}{N_L} + U_{Lh} \frac{N_L - 1}{N_L} \right)\right), \tag{85}
\]

\[
U_{Ll} = \delta \left(p_L U_{Ll} + (1 - p_L)(1 - \pi)U_{Ll} + (1 - p_L)^2 \pi \left(U_{Ll} \frac{1}{N_L} + U_{Ll} \frac{N_L - 1}{N_L} \right)\right). \tag{86}
\]

The first-order condition for utility maximization gives us

\[
\frac{\partial U_H}{\partial p_H} = -c'(p_H) + \delta U_H - \delta(1 - \pi)U_{Hl} - \delta \pi \left(U_H \frac{1}{N_H} + U_{Hh} \frac{N_H - 1}{N_H} \right) = 0, \tag{88}
\]

\[
\frac{\partial U_L}{\partial p_L} = -c'(p_L) + \delta U_L - \delta \pi U_{Lh} - \delta(1 - \pi) \left(U_L \frac{1}{N_L} + U_{Ll} \frac{N_L - 1}{N_L} \right) = 0. \tag{89}
\]

As the second-order are always satisfied, equations (81)–(88) together with (28) describe the equilibrium.

For \(N_L = \infty\), the only solution to (84)-(86) involves \(U_{Lh} = U_{Ll} = 0\) and, hence, \(p_L = \tilde{p}\). Similarly we get \(p_H = \tilde{p}\) for \(N_H = \infty\). As \(p_H \to \tilde{p}\), we also must have \(p_L \to \tilde{p}\), since \(p_L > p_H\) and \(p_L \leq \tilde{p}\) (with equality possible only if \(U_{Lh} = U_{Ll} = 0\)). Thus we have \(r \to 1\) and \(\pi \to 1\) as \(N_H \to \infty\).

Q.E.D.

References


[65] 17th congress of the All-Union Communist Party (Bolsheviks). Minutes. Partizdat, 1934