Collaterization of Global Assets and Stochastic Leverage

Seminar at the
Higher School of Finance and Management of the ANE
May 15, 2014
Moscow
Methodological Remarks

The model should be simple: as simple as possible, but no simpler
Albert Einstein

There are two kinds of forecasters: those who don’t know, and those who don’t know they don’t know
John Kenneth Galbraith
Presentation Structure

Leverage and Financial Instability
(Voprosy Economiki, #9, 2012)

Logistic Model of Financial Leverage

Financial Assets Collaterization and Stochastic Leverage
(HSE Economic Journal, vol.18, #2, 2014)

The Papers Overview:
1. The Problem Formulation
2. The Logistic Model of Leverage
3. The Model Microfoundations
4. Wicksellian Analysis of Macrofinance
5. Stochastic Leverage Dynamics
6. Collaterization of Global Assets
7. Some Conclusions
1. The Problem Formulation

Crisis:


*Supra-systemic* approach – logistic models (2012-14)

Theoretical and Empirical Aspects of the Problem

Collaterization of Assets and the Wicksellian concept of the Natural Rate of Interest

Leverage and collaterized loans with margin calls
W. Shakespeare, The Merchant of Venice

The logistic model:
Th. Malthus hypothesis and P.-F. Verchulst equation

BIS and the Fed on the leverage monitoring and control

The IMF, Global Financial Stability Reports

The Problem: Does a stable measure of macrofinancial assets collaterization exist in the long and short run? Whether is it possible to make such estimations for the Global Financial System?
Financial and Real Markets

Diagram showing the flow of resources through Real Markets, Financial Markets, and the connection to aggregate income and noise.
Logistic Model in Finance: General Approach

2. The logistic model of leverage

The leverage differential, $dl(t)$, and its level, $l(t)$ are connected

$$dl = f[l(t)] dt,$$

where both functions depend parametrically upon time.

RHS can be decomposed to represent the coupling (in terms of the complex systems theory) between the leverage and its rate of change

$$f[l(t)] = l(t) \ast g[l(t)].$$
The Verchulst Equation in Finance

A Taylor series expansion at \( l(t) = l^* \), where \( g(l^*) = 0 \):
\[
g(l) \equiv g(l^*) + g'(l^*)(l - l^*) = -g'(l^*)l^* + g'(l^*)l = a - bl.
\]

leads to the **logistic model of leverage dynamics**:

\[
dl_t = l_t(a - bl_t)dt,
\]

where \( a = -g'(l^*)l^* \) and \( b = -g'(l^*) \).

Yet this approach is too general to provide a convincing description of the financial process.
MacroFinancial Balances and Flows

The balance of the three *state variables* of an aggregate financial system:

\[ A(t) = x(t) + e(t). \]

The balance of *financial flows*:

*Aggregate Saving* = *Aggregate Debt* + *Aggregate Investment*

\[ dA(t) = dx(t) + de(t). \]
Yields: $ROE = \rho; \text{ and } de(t) = \rho e(t)dt$.

$ROA = \mu; \text{ and } dA(t) = \mu A(t)dt$. Refinancing rate, $r > 0$ \( dx(t) = r x(t)dt \).
Global Financial System Empirical Data for 2004-12

Changes in global assets, debts and capital in $ tn (GFSR, 2003-13)

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>$A_t$</td>
<td>16.4</td>
<td>7.1</td>
<td>38.6</td>
<td>39.3</td>
<td>-15.3</td>
<td>17.8</td>
<td>17.9</td>
<td>5.8</td>
<td>12.7</td>
</tr>
<tr>
<td>$e_t$</td>
<td>6.0</td>
<td>0</td>
<td>13.6</td>
<td>14.3</td>
<td>-31.6</td>
<td>13.6</td>
<td>7.9</td>
<td>-8.0</td>
<td>5.4</td>
</tr>
<tr>
<td>$\chi_t$</td>
<td>10.4</td>
<td>7.1</td>
<td>25.0</td>
<td>25.0</td>
<td>16.3</td>
<td>4.2</td>
<td>10.0</td>
<td>13.8</td>
<td>7.3</td>
</tr>
</tbody>
</table>
Given the *Global Capital of $65.1$ tn in 2007* the following system

\[
\begin{pmatrix}
A \\
x
\end{pmatrix} = \begin{pmatrix}
-0.0666 & -0.0990 \\
1 & -1
\end{pmatrix}^{-1} \begin{pmatrix}
-31.6 \\
65.1
\end{pmatrix}
\]

has a solution of $229.7$ tn (Global Assets) and $164.6$ tn (Global Debt) which coincides with empirical volumes for 2007 year.
Financial Flows and Leverage

**Static relations** among the rates of financial flows and the leverage:

\[ l(t) = \frac{A(t)}{e(t)}, \]

are connected for \( 1 \leq l < \infty \), and define the equation of the balanced financial market:

\[ \rho = r + (\mu - r)l, \]

or \[ \rho = \mu + (\mu - r)[l - 1] \]

for positive and negative spreads \( (\mu - r) \).
Hence there are two conjugate regimes for the “normal” and the “irrational” balanced market.
The ROE surface for the feedback dynamics of leverage
Isoquants of the ROE surface
A Mergers & Acquisitions Deal

3. The Model MicroFoundations

Given two companies, L and U, with the same assets \( A_U = A_L = A \) and generating the same return \( \Delta A \)

<table>
<thead>
<tr>
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<th>t</th>
<th>T</th>
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</thead>
<tbody>
<tr>
<td>Buy company L</td>
<td>(-e_L = e)</td>
<td>(+ \rho e_L = \rho e)</td>
</tr>
<tr>
<td>Buy company U</td>
<td>(-e_U = -A)</td>
<td>(+ \rho_U e_U = \mu A)</td>
</tr>
<tr>
<td>borrow</td>
<td>(+x)</td>
<td>(-r x)</td>
</tr>
<tr>
<td></td>
<td>(-e = -A + x)</td>
<td>(\rho e = \mu A - r x)</td>
</tr>
<tr>
<td></td>
<td>(e = A - x)</td>
<td>(\rho = r + (\mu - r)a)</td>
</tr>
</tbody>
</table>

If credit equals the debt of the first company, then the MM Proposition takes place.
A Particular Case of a Leverage Equation

The DuPont model:

$$\rho = \frac{\text{net income}}{\text{capital}} = \frac{\text{net income}}{\text{sales}} \times \frac{\text{sales}}{\text{assets}} \times \frac{\text{assets}}{\text{capital}}$$

or

$$\rho = \frac{\Delta A}{A} \times \frac{A}{e} = \mu l,$$

is a particular case of a leverage equation, for $r = 0$. 
Leverage Dynamics with a Feedback

The proposed methodology differs from the analysis of collaterized loans by J. Geanakoplos and the leverage by Shin.

It is assumed that leverage dynamics follow differential equation

$$dl(t) = (\mu - \rho)l(t)dt.$$ 

Being added with a feedback loop:

$$\mu - \rho = (\mu - r)[1 - l],$$

it forms the logistic leverage model:

$$dl(t) = (\mu - r)[1 - \frac{l(t)}{l^*}]l(t)dt; \ l(0) = l_0.$$
ROE and Leverage for Five Largest BHCs

Figure 2.5.1. Leverage Levels of U.S. Dealer-Banks

Source: Company reports.
¹For the period shown.
Leverage Logistic Model

The alternative representation of a leverage model:

\[ dl(t) = a \left[ 1 - \frac{1}{K} l(t) \right] l(t) dt \equiv [a - bl(t)] l(t) dt \]

where \( l^* = K = \frac{a}{b} = \frac{\rho - r}{\mu - r} \) is the stationary leverage;

\( a = \mu - r \) is the WACC/refinancing (deposit) rate;

\( aK = \rho - r \) is the spread ROE/refinancing (deposit) rate;

\( b = \frac{(\mu - r)^2}{\rho - r} \) is the drag parameter.
Empirical Cost and Return on Equity

Figure 2.5.3. Return on Equity versus Cost of Equity for All U.S. Dealer-Banks (In percent)

Cost of equity: Red line
Return on equity: Green line

Sources: Bloomberg L.P.; and SNL Financial.
4. Investors’ behavior near stationary Wicksellian point $W$
Investors’ behavior near unstable stationary point.

The Minsky and the Fisher markets: $a < 0$.
Local Behavior of Leverage

Wicksellian Rules of Investors’ Behavior

<table>
<thead>
<tr>
<th></th>
<th>( \alpha = \mu - r )</th>
<th>( \rho - r )</th>
<th>( b = a / K )</th>
<th>( l^* = K )</th>
<th>( l = 3 )</th>
<th>( l = 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point W</td>
<td>0.01</td>
<td>0.04</td>
<td>0.0025</td>
<td>4</td>
<td>0.013 &gt; 0.01</td>
<td>0.008 &lt; 0.01</td>
</tr>
<tr>
<td>Point B</td>
<td>-0.01</td>
<td>-0.04</td>
<td>-0.0025</td>
<td>4</td>
<td>-0.0013 &lt; -0.01</td>
<td>-0.008 &gt; -0.01</td>
</tr>
</tbody>
</table>

\[
\text{positive yield} > \text{positive cost} \implies \text{borrow} \implies \text{leverage} \uparrow \\
\text{positive yield} < \text{positive cost} \implies \text{sell - off} \implies \text{leverage} \downarrow \\
\text{negative yield} > \text{negative cost} \implies \text{loss} < \text{economy} \implies \text{buy} \\
\text{negative yield} < \text{negative cost} \implies \text{loss} > \text{economy} \implies \text{sell}
\]
Empirical Verification of the Logistic Model

Credit Crunch 2007–08 Parameters

<table>
<thead>
<tr>
<th>Year</th>
<th>$\rho$</th>
<th>$\mu$</th>
<th>$r$</th>
<th>$a = \mu - r$</th>
<th>$c = \rho - r$</th>
<th>$b = \frac{(\mu - r)^2}{\rho - r}$</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>0.2805</td>
<td>0.2064</td>
<td>0.1791</td>
<td>0.0273</td>
<td>0.1024</td>
<td>0.00728</td>
<td>3.75</td>
</tr>
<tr>
<td>2008</td>
<td>−0.4854</td>
<td>−0.0666</td>
<td>0.0990</td>
<td>−0.1656</td>
<td>−0.5844</td>
<td>−0.04691</td>
<td>3.53</td>
</tr>
</tbody>
</table>
Logistic Model: Empirical Verification

Attractor and Repeller of the Global Finance in 2007–08
Financial Market Phase Diagram
The phase diagram of the leverage dynamics showed that
the system bifurcates at the origin \((\mu - r) = 0\).
Solution to the Logistic Equation

Trajectories (in time) of the logistic equation solution are given as

\[ l(t) = l^* \left\{ 1 + \left( \frac{l^*}{l_0} - 1 \right) \exp\left[ -(\mu - r)t \right] \right\}^{-1}. \]

Solution to the logistic equation is the weighted harmonic average

\[ l(t) = \frac{l^* l_0}{l_0 (1 - \exp[-at]) + l^* \exp[-at]} \]

where \( a = \mu - r. \)

of the initial leverage, \( l_0, \) and its stationary state, \( l^*. \)
Financial Leverage Trajectories
# Financial Leverage Asymptotics

<table>
<thead>
<tr>
<th>States of the Market</th>
<th>( I^* &gt; I_0 )</th>
<th>( I^* &lt; I_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu - r &gt; 0 )</td>
<td>credit stabilization, ( I_0 \to I^* )</td>
<td>credit stabilization, ( I_0 \to I^* )</td>
</tr>
<tr>
<td>( \mu - r &lt; 0 )</td>
<td>deleveraging, ( I_0 \to 0 )</td>
<td>credit expansion, ( I_0 \to \infty )</td>
</tr>
</tbody>
</table>
Bifurcation diagram of credit market
The capital ratio dynamics

The weighted arithmetic average of the stationary, $w^*$, and the initial, $w_0$, capital ratios:

$$w(t) = w^*(1 - \exp[-(\mu - r)t]) + w_0 \exp[-(\mu - r)t].$$
Capital Ratio Dynamics

\[ w^* (1 - \exp[-(\mu - r)t]) + w_0 \exp[-(\mu - r)t] = 0 \]

The expected time of the credit market collapse:

\[ t_c = -\frac{1}{\alpha} \log \frac{w^*}{w^* - w_0}. \]
Empirical data for the global leverage model

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>(\mu_t)</td>
<td>0.1278</td>
<td>0.0491</td>
<td>0.2543</td>
<td>0.2064</td>
<td>−0.0666</td>
<td>0.0830</td>
<td>0.0771</td>
<td>0.0232</td>
<td>0.0496</td>
<td></td>
</tr>
<tr>
<td>(\rho_t)</td>
<td>0.1923</td>
<td>0</td>
<td>0.3656</td>
<td>0.2815</td>
<td>−0.4854</td>
<td>0.4090</td>
<td>0.1674</td>
<td>−0.1452</td>
<td>0.1148</td>
<td></td>
</tr>
<tr>
<td>(r_t)</td>
<td>0.1071</td>
<td>0.0660</td>
<td>0.2182</td>
<td>0.1791</td>
<td>0.0990</td>
<td>0.0232</td>
<td>0.0529</td>
<td>0.0713</td>
<td>0.0350</td>
<td></td>
</tr>
<tr>
<td>((\mu - r)_t)</td>
<td>0.0207</td>
<td>−0.0169</td>
<td>0.0361</td>
<td>0.0273</td>
<td>−0.1656</td>
<td>0.0598</td>
<td>0.0242</td>
<td>−0.0481</td>
<td>0.0146</td>
<td></td>
</tr>
<tr>
<td>((\rho - r)_t)</td>
<td>0.0852</td>
<td>−0.0660</td>
<td>0.1474</td>
<td>0.1024</td>
<td>−0.5844</td>
<td>0.3858</td>
<td>0.1145</td>
<td>−0.2165</td>
<td>0.0747</td>
<td></td>
</tr>
<tr>
<td>(K_{t-1})</td>
<td>4.12</td>
<td>3.9</td>
<td>4.08</td>
<td>3.75</td>
<td>3.53</td>
<td>6.45</td>
<td>4.73</td>
<td>4.50</td>
<td>5.46</td>
<td></td>
</tr>
<tr>
<td>(l_t^*)</td>
<td>3.97</td>
<td>3.89</td>
<td>4.08</td>
<td>3.74</td>
<td>3.53</td>
<td>6.4</td>
<td>4.92</td>
<td>4.54</td>
<td>5.43</td>
<td>5.12</td>
</tr>
</tbody>
</table>

\[
l(t) = 4.58\left\{1 + \left(\frac{4.58}{5.12} - 1\right)\exp[-\alpha t]\right\}^{-1},
\]
Possible evolution of global leverage
The surface of leverage trajectories for 2012-26
Leverage under Uncertainty

5. Stochastic Logistic Model of Leverage

Parameters of the leverage usually contain multiplicative noise:

\[ dl_t = l_t [(a - bl_t) dt + \sigma dz_t], \]

where \( \sigma \) is the volatility of leverage; \( z_t = \int_0^t dz_u \) is the standard Brownian motion; \( dz_t = \varepsilon_t (dt)^{0.5}, \varepsilon_t \sim \mathcal{N}(0,\tau^2) \).

The standard Brownian motion is a nonstationary process since \( (dz_t)^2 = dt \).
The Soros Financial Reflexivity and Expectations

“There is a two-way reflexive connection between perception and reality which can give rise to initially self-reinforcing but eventually self-defeating boom-bust processes, or bubbles. Every bubble consists of a trend and a misconception that interact in a reflexive manner.”

Stochastic Logistic Equation Solution

The “strong” solution of the stochastic logistic equation (Sciadas, 2010):

\[
l(t) = \frac{l_0 K \exp[(a - 0.5\sigma^2)t + \sigma z_t]}{K + a l_0 \int_0^t \exp[(a - 0.5\sigma^2)u + \sigma z_u] du}.
\]

For zero volatility, \( \sigma = 0 \), solutions of the deterministic and the stochastic models are coincided.

The **expected rate of growth** of the leverage is also the same:

\[
\frac{1}{dt} \left\langle \frac{dL_t}{L_t} \right\rangle = a \left(1 - \frac{1}{K} L_t\right).
\]
Asymptotic Behavior of Stochastic Leverage

The stationary forward Kolmogorov-Fokker-Plank equation for

\[
\frac{\partial}{\partial l} [l(a-bl)p(l)] - \frac{1}{2} \frac{\partial^2}{\partial l^2} [\sigma^2 l^2 p(l)] = 0
\]

where \( p(l) \) is the pdf of the random leverage process \( L(t) \).
The Trivial Solution to the Stationary Equation

Solutions of the Kolmogorov equation (Pascuali, 2001)

A trivial solution:

\[ p(l) = \delta(l) \]

which is a \( \delta - \text{Dirac} \) distribution. It is associated with the stationary state, \( I_1^* = 0 \).
A non-trivial solution is a gamma-distribution:

\[ p(l) = \frac{\beta^\alpha}{\Gamma(\alpha)} l^{\alpha-1} e^{-\beta l} \]

It is defined for positive parameters of the shape and rate(scale), respectively:

\[ \alpha = \frac{2a}{\sigma^2} - 1; \quad \beta = \frac{2b}{\sigma^2} \]

Hence it exists for the values of:

\[ 0 < \sigma^2 < 2a \]
## Parameters of the random leverage

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$, spread</td>
<td></td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>$\sigma$, volatility</td>
<td>0.14</td>
<td>0.2</td>
<td>0.22</td>
</tr>
<tr>
<td>$\sigma^2$, variance</td>
<td>0.02</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>$\alpha$, shape</td>
<td>3</td>
<td>1</td>
<td>0.6</td>
</tr>
<tr>
<td>$\beta$, growth rate</td>
<td>1</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>$1/\beta$, scale</td>
<td>1</td>
<td>2</td>
<td>2.5</td>
</tr>
<tr>
<td>$\langle \lambda \rangle$, The Lyapunov exponent</td>
<td>$-0.02$</td>
<td>0</td>
<td>0.01</td>
</tr>
<tr>
<td>$\langle L \rangle$, expected value</td>
<td>3</td>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td>$Mode[L]$</td>
<td>2</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>

Curve 1: $\sigma^2 - a = 0.02 - 0.04 < 0$ is a peaked gamma distribution of leverage;
Curve 2: $\sigma^2 - a = 0.04 - 0.04 = 0$ is an exponential distribution of leverage;
Curve 3: $\sigma^2 - a = 0.05 - 0.04 > 0$ is a J-shaped gamma distribution of leverage.
Gamma Distributed Leverage
3D Plot of a Gamma Distribution for 
$l \in [0, 7]; \alpha \in [0, 6]$
Contour Plot of a Gamma Distribution
Expectation and Mode of Leverage

The expected value of a gamma-distributed random leverage is

$$\langle L \rangle = \int_0^\infty l p(l) dl = \frac{\alpha}{\beta} = K - \frac{\sigma^2}{2b},$$

while its mode for $\alpha > 1$ is equal to

$$\text{Mode}[L] = \frac{\alpha - 1}{\beta} = K - \frac{\sigma^2}{b}.$$
The market can stay irrational longer than you remain solvent. The Lyapunov Exponent is a measure of investors’ confidence in the financial market solvency.

The Lyapunov Exponent is the expected value of

\[
\langle \lambda \rangle = \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^\infty (a - 2bl) l^{\alpha-1} \exp[-\beta l] dl.
\]

It defines stability conditions of stochastic leverage

\[
\langle \lambda \rangle = \langle a - 2bL \rangle = a - 2b\langle L \rangle = a - 2b\left(\frac{a - \sigma^2}{b}\right) = \sigma^2 - a.
\]

\[
\langle \lambda \rangle = (\sigma^2 - a) \begin{cases} < & \text{stability or confidence;} \\ = & 0; \text{ the system neutrality;} \\ > & \text{instability or the loss of confidence.} \end{cases}
\]
Stochastic and Deterministic Models could be very different

It is important that stability of a deterministic system does not imply stability of its stochastic analogue. Hence the actual random leverage dynamics could differ significantly from its deterministic forecast.

<table>
<thead>
<tr>
<th>Deterministic model is valid</th>
<th>$\sigma^2 \ll K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stochastic model is stable</td>
<td>$\sigma^2 &lt; a$</td>
</tr>
<tr>
<td>Stochastic model is unstable</td>
<td>$a &lt; \sigma^2 &lt; 2a$</td>
</tr>
<tr>
<td>Noise induced chaos</td>
<td>$\sigma^2 &gt; 2a$</td>
</tr>
</tbody>
</table>
The Global Financial Assets Collaterization

6. Collateral Ratio for the Global Financial System

Collateral Ratio Estimation is a two-stage process:

1. Stationary (long term) parameters of gamma distribution are to be found;
2. Collateral ratio is to be estimated.

Global Financial System Leverage Dynamics in 2003-12

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</thead>
<tbody>
<tr>
<td>$l_t$</td>
<td>3.97</td>
<td>3.89</td>
<td>4.08</td>
<td>3.74</td>
<td>3.53</td>
<td>6.4</td>
<td>4.92</td>
<td>4.54</td>
<td>5.43</td>
<td>5.12</td>
</tr>
<tr>
<td>$I/l$</td>
<td>1.0</td>
<td>0.98</td>
<td>1.049</td>
<td>0.917</td>
<td>0.944</td>
<td>1.813</td>
<td>0.769</td>
<td>0.923</td>
<td>1.196</td>
<td>0.943</td>
</tr>
<tr>
<td>$l_y$</td>
<td>3.42</td>
<td>3.54</td>
<td>3.42</td>
<td>3.95</td>
<td>4.21</td>
<td>3.52</td>
<td>4.02</td>
<td>3.98</td>
<td>3.66</td>
<td>3.72</td>
</tr>
<tr>
<td>$I/l_y$</td>
<td>1.0</td>
<td>1.035</td>
<td>0.966</td>
<td>1.155</td>
<td>1.065</td>
<td>0.836</td>
<td>1.142</td>
<td>0.990</td>
<td>0.920</td>
<td>1.016</td>
</tr>
</tbody>
</table>
Parameters of the Gamma Distributed Global Leverage

*Stationary Gamma distribution* for the Global Financial System, 2003-12

<table>
<thead>
<tr>
<th></th>
<th>$a_H$</th>
<th>$b_H$</th>
<th>$\langle I \rangle$</th>
<th>$\langle I \rangle_H$</th>
<th>$K = a_H / b_H$</th>
<th>$\langle L \rangle = \alpha / \beta$</th>
<th>Mode[$L$] = $(\alpha - 1) / \beta$</th>
<th>$\sigma_1^2$</th>
<th>$\sigma_2^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.034</td>
<td>0.0063</td>
<td>5.0</td>
<td>4.98</td>
<td>5.35</td>
<td>4.67</td>
<td>3.94</td>
<td>0.0092</td>
<td>0.0313</td>
</tr>
</tbody>
</table>

The collateral ratio for the global financial system

$$I_Y = I_t \ast q_t,$$

where $q_t = e_t / Y_t$ is the Tobin’s q parameter.
Variance of $\sigma^2 = 0.0092$ scenario is given by equation
\[ dI(t) = 0.034[1 - 0.0063 I(t)] I(t)dt + 0.0959 I(t)dz. \]
and a stationary pdf function
\[ p(l; 6.39, 1.37) = 0.032 l^{5.39} \exp[-1.37 l]. \]
The Global Collateral Ratio components

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>$l_y$</td>
<td>3.42</td>
<td>3.54</td>
<td>3.42</td>
<td>3.95</td>
<td>4.21</td>
<td>3.52</td>
<td>4.02</td>
<td>3.98</td>
<td>3.66</td>
<td>3.72</td>
</tr>
<tr>
<td>$l_t$</td>
<td>3.97</td>
<td>3.89</td>
<td>4.08</td>
<td>3.74</td>
<td>3.53</td>
<td>6.4</td>
<td>4.92</td>
<td>4.54</td>
<td>5.43</td>
<td>5.12</td>
</tr>
<tr>
<td>$q_t$</td>
<td>0.87</td>
<td>0.91</td>
<td>0.84</td>
<td>1.05</td>
<td>1.19</td>
<td>0.55</td>
<td>0.82</td>
<td>0.88</td>
<td>0.67</td>
<td>0.73</td>
</tr>
<tr>
<td>$l_y = l_t * q_t$</td>
<td>3.45</td>
<td>3.54</td>
<td>3.42</td>
<td>3.93</td>
<td>4.2</td>
<td>3.52</td>
<td>4.03</td>
<td>3.99</td>
<td>3.64</td>
<td>3.74</td>
</tr>
</tbody>
</table>

For the leverage expectation $\langle L \rangle = 4.67$ and Tobin’s parameter $q_{2012} = 0.73$ the short run collateral ratio, $l_y$, is

$$l_y = \frac{\alpha}{\beta} q_{2012} = 4.67 * 0.73 = 3.41.$$  

Given the World GDP in $72.2$ tn, the amount of collaterized assets for the year 2012 should be equal to $246.2$ tn:

$$A = l_y * Y = 3.41 * 72.2 = 246.2.$$  

Thus about $16.6$ tn of global financial assets (or 94 per cent) were toxic assets.
7. Some conclusions

a) Inertia of macrofinance, its asymptotic features. Logistic model in the short and long run. Logistic maps and strange attractors. Stochastic logistic model and stationary distributions;

b) An ability of the logistic model to forecast should be viewed in the context of indeterminacy of the economic proportions {the Reinhart-Rogoff confusion regarding the 90 percent threshold of debt};

c) The role of uncertainty in finance seems to be rather ambiguous. At least, it is not unilaterally negative. Since

\[ \text{Mode} [L] \leq \langle L \rangle \leq K \]

the central bank is able to use the uncertainty to some extent to minimize the amplitude of a business cycle.
Thank you for your kind attention!
Динамика мирового ВВП и финансовых активов за 2003-2011 гг.

<table>
<thead>
<tr>
<th></th>
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<td>128,3</td>
<td>144,7</td>
<td>151,8</td>
<td>190,4</td>
<td>229,7</td>
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<td>232,2</td>
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<td>1,05</td>
<td>1,25</td>
<td>1,21</td>
<td><strong>0,93</strong></td>
<td>1,08</td>
<td>1,08</td>
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<td>1,00</td>
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<td>1,28</td>
<td><strong>0,51</strong></td>
<td>1,41</td>
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<td>92,1</td>
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<td>1,11</td>
<td>1,02</td>
<td>1,17</td>
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<td><strong>1,05</strong></td>
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<td>93,0</td>
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<td>1,12</td>
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<td>1,14</td>
<td><strong>0,95</strong></td>
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<td>Мировой валовой внутренний продукт, трлн долл.</td>
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<td>1,09</td>
<td>1,09</td>
<td>1,13</td>
<td>1,12</td>
<td><strong>0,95</strong></td>
<td>1,09</td>
<td>1,11</td>
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<tr>
<td>Отношение глобальных финансовых активов к ВВП</td>
<td>3,42</td>
<td>3,54</td>
<td>3,42</td>
<td>3,95</td>
<td>4,21</td>
<td>3,52</td>
<td>4,02</td>
<td>3,98</td>
<td>3,66</td>
</tr>
<tr>
<td>Индекс изменения отношения финансовые активы/ВВП</td>
<td>1,0</td>
<td>1,04</td>
<td>0,96</td>
<td>1,15</td>
<td>1,07</td>
<td>0,83</td>
<td>1,19</td>
<td>0,99</td>
<td>0,92</td>
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