Andrei Akhremenko, Alexander Petrov

EFFICIENCY, POLICY SELECTION, AND GROWTH IN DEMOCRACY AND AUTOCRACY: A FORMAL DYNAMICAL MODEL

BASIC RESEARCH PROGRAM

WORKING PAPERS

SERIES: POLITICAL SCIENCE
WP BRP 16/PS/2014

This Working Paper is an output of a research project implemented within NRU HSE’s Annual Thematic Plan for Basic and Applied Research. Any opinions or claims contained in this Working Paper do not necessarily reflect the views of HSE.
EFFICIENCY, POLICY SELECTION, AND GROWTH IN DEMOCRACY AND AUTOCRACY: A FORMAL DYNAMICAL MODEL

The main focus of this paper is the impact of efficiency losses, related to public capital stock, on the prospects of economic growth in democratic and autocratic political environments. We introduce a distinction between two types of efficiency loss: along with the loss of public capital during its accumulation, we take into account the process of capital stock depreciation. We demonstrate that the decrease in efficiency of any type makes the probability of long-run growth higher for autocracies; however, in the presence of high efficiency, democracies tend to perform better. The results were obtained by formal analysis and computational experiments, realized on the basis of an original dynamical model.

JEL Classification: C02, P16, Z18

Keywords: dynamical formal model, policy space, democracy, autocracy, economic growth, efficiency, public capital
I. Introduction

The interrelation between political regimes and economic growth has been one of the major focuses of political economy studies over the last several decades. Pioneering works by Schumpeter (1942), Lipset (1959), Olson (1982), Przeworski and Limongi (1993) gave birth and momentum to the enormous and still expanding body of literature. A large portion of it examines the effects of democracy on growth; a lesser proportion, originating from modernization theory, studies the reverse impact.

In this paper we are trying to approach this issue from a different angle. Rather than a recipe for growth, democracy is considered a way to abandon policies that have led to decline by replacing the ruling party. This does not mean that the abandoned policy would have detrimental in the long run. It also does not mean that the next ruler’s policy would be more successful. In this perspective, democracy is considered just a mechanism to find better policy by replacing the ruling party. Accordingly, the main question of our paper is whether this mechanism is good for growth.

This is, of course, a simplified approach, but it allows developing a new mathematical model of economic performance in democratic and autocratic environments. This model contains dynamic equations and implies more advanced methodology than plain or sophisticated statistical methods commonly employed in empirical research. Methodological flaws are the first that come to mind when looking at controversies across empirical results.

Some scholars, such as Acemoglu et al. (2014), Bates, Fayad and Hoeffler (2012), Bhalla (1997), Gerring et al. (2005), Papaioannou and Siourounis (2008), Halperin, Siegle and Weinstein (2005), Leblang (1997), and Rodrik and Wacziarg (2005), argue that democracy has a positive effect on growth. Other authors, e.g. Barro (1996), Burkhart and Lewis-Beck (1994), Feng (2003), Kurzman et al. (2002), Przeworski et al. (2002), Giavazzi and Tabellini (2005), Wood (2007), on the contrary, observe an insignificant or even negative impact of democracy upon development. Sometimes the picture becomes more complicated: the relation between democracy and economic performance is reported to be curve-shaped (non-linear). Thus, according to one of Barro’s models, “growth is increasing in democracy at low levels of democracy, but the relation turns negative once a moderate amount of political freedom is attained” (Barro 1997).

The range of disagreement among scholars was quantitatively demonstrated by Doucouliagos and Ulubasoglu (2008), who conducted meta-analytical research based on 483 estimates from 84 published democracy-growth studies. They state that “15% of the estimates are negative and statistically significant, 21% of the estimates are negative and statistically insignificant, 37% of the estimates are positive and statistically insignificant, and 27% of the
estimates are positive and statistically significant” (Doucouliagos and Ulubasoglu 2008: 62). Their own analysis of all the available published evidence found no direct impact of democracy on economic growth.\(^4\)

The idea of a reverse causation, from economic development to democracy (“Lipset hypothesis”), has not been confirmed fully. Some empirical evidence in support of it (Barro 1999) has been challenged by some authors (e.g. Acemoglu et al. 2009) on methodological grounds.

One of the conventional explanations of such ambiguity refers to the very nature of the regime-growth nexus. The relation between democracy (or autocracy) and development is often stated to be indirect, in the recent literature especially. From this point of view, regime types affect growth through a set of channels, and in each case the direction of impact may be different. This makes the net effect murky (and, perhaps, necessitates the use of mathematical modeling to make it clear).

One basic way to achieve economic development is to accumulate capital inputs. In modern economic research, two types of capital are in focus – physical\(^5\) and human\(^6\). Current conventional wisdom in political economy is that democracy is better than autocracy in stimulating human capital-induced growth (see, Baum and Lake 2003; Doucouliagos and Ulubasoglu 2008). High priority, attributed to education and health by the majority of people, creates incentives for democratic governments to invest more in public education and healthcare systems.

The situation with physical capital is more complicated, but the major arguments are in favor of autocracy. The absence of strong unions and real political accountability reduces the pressures on dictatorial regimes to support immediate public consumption. This provides opportunities to channel recourses to capital accumulation (Rodrick 1999; Tavares and Wacziarg 2001; Knutsen 2011). However, democracies also have some advantages. They tend to provide more suitable conditions for private investors (especially foreign ones) via better property rights protection (Boix and Payne 2003; Clague et al. 2003)\(^7\) and restrictions on state predation (Robinson 2001).

Another way to achieve growth is to induce technological innovation and diffusion. Some theoretical and empirical arguments evidence that democracies are more capable of doing this (Rivera-Batiz 2002; Halperin et al. 2005; Acemoglu 2008). The main consideration here is that hierarchical and closed structures are generally less adoptive to new ideas and are apt to prevent

---

\(^4\) Although, some important indirect and regional-specific effects were observed.

\(^5\) The former is sometimes decomposed into public and private capital – we will describe this issue in detail below.

\(^6\) Some authors study more specific types of capital, like social (Guiso et al. 2004, Woolcock 1998) or even political (Gerring et al. 2005).

\(^7\) But see Przeworski and Limongi (1993) for another view.
the free circulation of knowledge. Democracies with their horizontally distributed decision-making, on the other hand, are viewed as more flexible and more likely to take advantages of new technologies.

Some scholars link growth opportunities with “state autonomy” – the capacity of the government to implement policies independent of special interests. Democracies are generally considered to be more exposed to large pressure groups in comparison to autocracies (Olson 1982). These lobby groups can (and very often do) act as strong veto-players in the democratic political process, able to block changes to the status quo, including those that potentially lead to improvements in productivity(Tsebelis 2002; but see also, Gehlbach and Malesky 2010).

At the same time, the autonomy of a dictator may be strongly overestimated. Autocratic policy-making is vulnerable to particularistic pressures from small, elite groups that form the leader’s “selectorate” (Bueno de Mesquita et al. 2003). But even if we accept the “autocracy insulation” hypothesis, its obvious downside would be a possible rupture of feedback loops between state and society, which makes the whole process of policy-making problematic. It should be noticed here that in the literature, political decision makers are inexplicitly presupposed to be aware of what policies are “bad” (leading to recession or collapse) and what policies are “good” (leading to growth). From this point of view, a dictator consciously chooses a “bad” policy in order to secure her power and to maximize her personal wealth, or, on the contrary, decides in favor of growth and development. In reality, it is difficult or even impossible to make such a judgment before the actual process of policy implementation is launched.

All the factors (or channels) mentioned above – namely, human and physical capital accumulation, property rights protection and state predation, technological change, state autonomy – are important in understanding the regime-growth problem. To conclude this very brief review, we ask two questions that will become departing points in our own story. First, is the list of important channels, linking regime type and growth, complete? Second, do we have a proper methodology to analyze those complex phenomena?

In this article we show that at least one additional factor, participating in shaping a regime-growth link, should be taken into consideration. We are speaking about efficiency or, more precisely, efficiency losses. For example, there is a hypothesis that a democratic environment encourages the government to invest in building hospitals – one of the concrete forms of increasing human capital. But how do these investments transform into the real public assets, i.e. will the people receive the best possible hospitals with given prices and technologies for the money spent from the budget? This is the “efficiency-centered” way to put the question.

Our study concentrates on the efficiency of public capital (or public infrastructure), particularly on the processes of its accumulation and deterioration. Such a focus is natural for
political economy since public capital, being a public good, falls into the government’s area of responsibility. We distinguish two types of efficiency loss: 1) one connected with the incomplete transformation of budget investment into productive public assets; and 2) one connected with the quality of those assets and, thus, their depreciation rates. The main goal of this work is to examine the effects of both types of efficiency loss on economic performance in democratic and autocratic environments. The authors are not aware of any other attempts to research this specific question in the literature.

We demonstrate that efficiency loss highly (and differently) influences the capabilities of democracy and autocracy to provide growth. Unsurprisingly, a decrease in any type of loss raises the chances to achieve economic development for any type of political regime. The point is that democracy and autocracy have different “sensitivities” to efficiency alterations. When efficiency is low, autocracies have higher chances to outperform democracies in terms of the development rates achieved. An increase in efficiency provides democratic regimes with better growth capacities. We hope that our findings shed some light on the existing inconsistencies in the theoretical understanding and empirical examination of the regime-growth problem.

Moreover, omitted factors are not a single source of ambiguity; there are other reasons related to research design and analytical strategies. A conventional approach assumes a direct passage from verbally formulated theory to statistical estimations via regressions of various kinds. This approach misses one important link of the chain, namely a formal model. And in this area of research such a model should have specific features determined by the subject of inquiry. Such phenomena as “capital accumulation”, “technology diffusion”, “depreciation” and “growth” are processes which evolve in time, and per se they should be studied in a dynamical framework with temporal feedback loops.8

Furthermore, such a complex entity as a political regime can hardly be reduced to one component of a regression equation; another commonplace in contemporary studies. We argue that formal representation of regime type should reflect at least some substantial differences between democracy and autocracy.

Following these guidelines and aiming at the research problem formulated above, we present a dynamic formal model that includes a) a simple economy affected by two different types of inefficiency (economic block), and b) two regimes of choosing policy, namely autocracy and democracy (political block). For analytical purposes the dynamic system takes the form of a

---

8 That is one of the reasons why the mainstream game theoretic modelling, even in its dynamic version (e.g. Besley et al. 2010, Battaglini et al. 2012), is not the best instrument here. It tends to oversimplify the dynamics of a system and does not provide it with proper feedback loops.
continuous-time model (ordinary differential equations). Computational experiments are conducted on the basis of a discrete-time model (difference equations).

The major dynamic variables (stocks) are private capital and public capital. The dynamics are governed by two control parameters: tax rate and proportion of budget channeled to public investment. These parameters are dimensions of the “policy plane”. The latter is divided into two areas: the “long-run growth area”, containing a set of policies (i.e. combinations of control parameter values) that generate exponential growth in the long run; and the “long-run decline area”, policies lead to decline in the long run. The basic difference between democracy and autocracy in this framework is modeled via the presence/absence of the feedback between private capital dynamics and policy change over time.

The rest of the paper is organized as follows. In Section II we outline briefly the existing approaches to public capital studies, and argue to the importance of distinguishing two types of efficiency loss. In Section III a formal model is constructed. Section IV describes the mathematical analysis of the model dynamics, and Section V presents the results of the computational experiments. Section VI concludes.

II. Public capital, types of efficiency and growth

Public capital (or “public infrastructure” in the earlier literature) can be understood in a more or less broad way. In the narrowest sense, it includes material assets created and maintained by the government: railways, roads, sewer systems, electricity distribution networks, etc. A broader definition includes all public goods that can be expressed, at least potentially, in monetary form provided by state, such as property rights protection, public security, healthcare services, etc. For the time being this distinction is not crucial and we shall not take a firm stand on either side. Practically, making the first approach to public capital models, it is easier to consider it as a physical entity. Much more essential to our model is that we consider public capital to be non-rivalrous and non-excludable – in other words, a “pure” public good.9 The latter means that it can be produced only at the expense of state budget funds with the efforts of a bureaucratic system.

The tradition of analyzing public capital as a separate input factor of production originates from 1990s macroeconomic literature (Aschauer 1989, 1990; Hulten 1996). From a formal point of view, this approach transformed traditional Cobb-Douglas production function with aggregated capital and labor as input factors into the following form:

\[ Y_t = AK_t^{\alpha} G_t^{\beta} X_t^{\gamma}, \]

9 In another – more complicated view public capital is a congestible - rival when heavily used – public good, organized in capital-intensive networks, which efficiency depends on its size and configuration (Hulten 1996)
where $K$ stands for private capital, $G$ for public capital, $X$ for other input factors (labor and human capital mainly), $\alpha$, $\beta$, $\gamma$ denote corresponding elasticities, and $A$ is multifactor productivity (or total factor productivity, TFP).

After such a decomposition of capital had been made, it led directly to the question about the impact of public capital on total output ($Y$) and its change over time, i.e. economic growth. Over the following two decades multiple empirical works examining this issue were published, and the results appear to be mixed. On the one hand, Crowder and Himarios (1997), Pereira & Roca Sagales (2003), Kamps (2004), World Bank (2007), Commission on Growth and Development (2008), and Arslanalp et al. (2010) all witnessed a significant and positive effect of public capital accumulation on growth and/or private capital productivity and employment. On the other hand, Devarajan et al. (1996), Ghali (1998), Ferrara & Marcellino (2000), and Albala-Bertrand & Mamatzakis (2004) reported ambiguous, insignificant or even a negative influence of investment in public infrastructure on economic development. Moreover, substantial cross-national heterogeneity in public capital productivity was detected. And, surprisingly, the countries with lower levels of productivity tended to make the largest investments in public assets in terms of budget shares (Keefer and Knack 2007). On the way to resolving these puzzles, the notion appeared that not only is the accumulated value of public capital important, but also how effectively it is allocated and utilized.

The first steps in that direction were made by Hulten (1996) and Pritchett (2000). In Hulten’s analysis a special “effectiveness parameter” $\theta$ enters the production function as the coefficient of public capital:

$$Y_t = AK_t^\alpha G_t^\beta X_t^\gamma$$

Hulten made use of this model to empirically demonstrate that inefficient utilization of infrastructure leads to a vast reduction in benefits from public investments. His empirical analysis (real GDP growth 1970–1990) shows that a 1% increase in the efficiency parameter has more effect on economic growth than a 7% increase in the rate of public investment.

Pritchett (2000) went further in the same direction and split TFP into two separate productivity parameters for public ($A_g$) and private ($A$) capital:

$$Y_t = AK_t^\alpha A_g G_t^\beta X_t^\gamma$$

The total factor productivity $A$ could be interpreted in different ways. Basically, it is a residual component – change in outputs is not explained by a change in inputs (Hulten 2001), and hence they could be described as the “measures of our ignorance”. Low public capital

---

10 Hereinafter we use the uniform denotation instead of the original formulas of the cited works. It is done to make the text more comprehensive.
productivity may be a result of a recourse misallocation, like highways leading nowhere in some African countries; but it also could be due to poor maintenance, inadequate technologies or many other reasons.

Pritchett’s second idea, which has become the guideline for many modern studies in this area, touches on the very dynamics of public capital – its accumulation and depreciation. The traditional approach to it is expressed in the so-called perpetual inventory equation:

\[ G_{t+1} = G_t - \delta G_t + I_t \]

where the stock of public capital \( G \) depreciates over time at rate \( \delta \) and is increased by budget investment inflow \( I \). The formula above supposes that all infrastructure spending translates directly and fully into productive capital assets. Such an understanding has one strong implication upon empirical estimations: research takes an accumulated investment – overall budget spent for public projects – as a proxy for productive capital stock (minus depreciation effects, of course). The main contribution of Pritchett’s seminal paper is a clear-cut statement that this can never be true in a general case. Cumulated and Depreciated Investment Effort (\( I \), CUDIE) is not equal to capital stock. He makes a distinction between the “accounting” cost of capital (actual expenditures, CUDIE) and the “economic” cost – the minimum achievable cost of capital formation with given technologies and prices. The ratio of economic cost to accounting cost is the efficacy of investment parameter \( \gamma \). It modifies the perpetual inventory equation in the following way:

\[ G_{t+1} = G_t - \delta G_t + \gamma I_t \]

where \( \gamma \) represents a fraction of budget investment \( I \) which creates useful capital. It varies between 0 (total waste of invested resources) to 1 (full efficiency). The unit efficiency could neither be achieved in the private nor in public sector, but the latter is much more vulnerable from this point of view because of various agency problems and the monopolistic property of the goods provided. The state, due to its monopoly on institutionalized coercion, is able to provide investment, irrespective of citizens’ views on the value of the good to be created. The state is also capable of restricting citizens’ capacities to monitor public projects implementation. The monopolistic nature of public good provision and the absence of an actual market make the evaluation of expenditure efficiency an extremely difficult task.

Further, efficiency loss related to a transformation of budget money into useful public assets is referred to as \( \gamma \)-loss. One of its obvious determinants is the level of corruption. More precisely, we speak about a type of corruption based on raising actual costs of public projects in comparison to economic ones (and thus decreasing \( \gamma \)). It can take the form of “traditional” corruption (venality), when bureaucrats receive kickbacks from investment contract distribution
and public purchases. In this case a part of public investment \((1-\gamma)I\) becomes a bureaucrat’s personal monetary benefit. A more complicated form of corruption that affects \(\gamma\)-loss is political patronage, when investment costs are raised to provide extra payments to supporters.

Despite its seeming simplicity, Pritchett’s idea of public investment efficiency has been realized empirically only recently. In the indicatively titled article, “Efficiency-Adjusted Public Capital and Growth”, Gupta and colleagues (2014) introduced their novel dataset of private and public capital stocks for 52 developing countries. Public capital was estimated with adjustment for investment efficiency on the basis of Pritchett’s methodology. Country-specific efficiency parameters were measured by Public Investment Management Index (PIMI\(^{11}\)) and constructed by Dabla-Norris et al. (2011). Not surprisingly, PIMI correlates significantly with the measures of corruption, Transparency International CPI in particular. The results of Gupta’s group analysis demonstrated that the stock of public capital in developing countries is only about half of the amount suggested by non-adjusted estimations. Thus, up to fifty per cent of public investment did not pass where it was supposed to. The usefully accumulated part of it, at the same time, was confirmed to be an important (and grossly underestimated in previous studies) factor of economic growth.

Along with investment-related efficiency loss \(\gamma\), there is one more important parameter in the perpetual inventory equation, namely the depreciation rate \(\delta\). Gamma reflects the \textit{quantitative} aspect of public investment: what share of the original amount of money transforms into productive assets? Delta deals more with the \textit{quality} of public capital. The lesser the quality of materials and labor used in the construction of public infrastructure, the higher the speed at which it deteriorates. As in the previous case, the poor quality of public goods may have (and does have in many countries) corruptive origins. Another behavioral source of \(\delta\)-loss, as we will call it, is poor maintenance (Pritchett 2000).

Consider the following stylized example to better capture the difference between \(\gamma\) and \(\delta\) types of efficiency. Suppose $100 has been appropriated for the construction of a road in last year budget. The economic (“fair”) cost of one mile of the road is $10 and the “natural” depreciation rate of high quality materials is 0.01. With full efficiency 10 miles \((100/10)\) of a high quality road would be built. In the presence of \(\gamma\)-loss (say, \(\gamma = 0.5\) and \(\delta = 0.01\)) we would get 5 miles \((100 \times 0.5/10)\). With \(\delta\)-loss (\(\gamma = 1\) and \(\delta = 0.03\)) we would get 10 miles, as in the “efficient” case, but it would deteriorate three times more rapidly than the previous shorter road.

\(^{11}\) PIMI is composed of 17 indicators grouped into four stages of the public investment management cycle: (a) Project Appraisal; (b) Project Selection; (c) Project Implementation; and (d) Project Evaluation.
Very few papers, as far as we know, have addressed the efficiency/depreciation nexus directly. A rare example of a formal theoretical work is from Ellis & Fender (2006), who focus on the positive impact of depreciation on corruption. A number of empirical studies (e.g. Arestoff and Hurlin 2006; Arslanalp et al. 2010) concentrate on the problem of depreciation rate estimations that appear to be one of the most challenging in public capital studies. Behavioral matters like the level of corruption and the quality of operation and upkeep are not the only factors affecting this parameter. Among the others, one can find different life spans for different types of assets (up to 100 years for traditional materials like cement, versus 3–5 years for IT-based assets). It is not an easy analytical task to separate these effects from each other, let alone the problems with the availability of corresponding statistical data.

Although $\gamma$-loss and $\delta$-loss, as we call them, are a phenomena worth studying in themselves, in reality one faces some combination of both types rather than their discrete or “pure” forms. In this framework, it could be viewed as a country-specific vector \( \begin{bmatrix} \gamma \\ \delta \end{bmatrix} \). What type of efficiency is more important for development? How do they affect growth in democratic and autocratic environments? We are not familiar with studies focusing upon these questions, and this paper aims to fill in some of the gaps.

III. Model

We start by constructing a simple economic environment that is supposed to integrate fruitful ideas found in the literature (Section II). It is based on Cobb-Douglas production technology:

\[
Y(t) = AK^\alpha(t)G^{1-\alpha}(t)
\]

where \( Y(t) \) is aggregate output, \( K(t) \) is private capital, \( G(t) \) is public capital, and \( A \) is a time-invariant total factor productivity parameter. The sum of elasticities with respect to all inputs is equal to unity, so that the function in general exhibits constant returns to scales (CRS) and a diminishing return to each input. For simplicity only two production factors – public and private capital – enter the baseline specification. Nevertheless, it should be noticed that adding extra inputs to the model does not change any basic results of the formal analysis (Section IV) since the CRS condition holds. A more unfolded specification of inputs\(^{12}\), however, is critical in an empirical study – this is now being planned by the authors.

The output is taxed at a fixed rate \( \tau \), and tax revenue fills the state budget \( T(t) \):

\[
T(t) = \tau Y(t) = \tau AK^\alpha(t)G^{1-\alpha}(t)
\]

\(^{12}\) Otherwise we have to accept an unrealistic assumption that labor and human capital are included in the TFP index \( A \).
The budget could be spent two-fold. Coefficient \( \mu \) defines the public capital investment’s share:

\[
I(t) = \mu T(t)
\]

The rest of the budget \((1 - \mu)\) goes to private producers in direct transfers. Thus, private capital is composed of two components: part of the output left after taxation, and transfers from the budget:

\[
K(t+1) = (1 - \tau)Y(t) + (1 - \mu)T(t)
\]

Speaking in terms of political decision-making, the government in this model has to decide about two policy parameters: tax rate \( \tau \), and public investment budget share \( \mu \). These parameters can be considered as the dimensions of the policy space. A political party or political leader, or any decision making unit in power in this model, is fully characterized by her location in that policy space. To put it another way, government policy in the model is a vector of parameters \( \left[ \begin{array}{c} \tau \\ \mu \end{array} \right] \).

The dynamics of public capital \( G(t) \) is determined by its depreciation rate \( \delta \), on the one hand, and efficiency-adjusted public investment inflow, on the other:

\[
G(t + 1) = G(t) - \delta G(t) + \gamma I(t)
\]

As was already described in Section II, \( \gamma \) denotes the fraction of public investment \( I(t) \) that transforms into actual public capital. The other type of efficiency, \( \delta \)-loss, is associated with the change in public capital depreciation rate (via the usage of low quality materials, etc). The larger the value of \( \delta \), the smaller the rate of efficiency. In our simplified framework, all the depreciation is considered to be due to efficiency; the same is true with investment losses. Both efficiency parameters are kept time-invariant in order to achieve more generality in formal analysis. Thus, together with the policy vector \( \left[ \begin{array}{c} \tau \\ \mu \end{array} \right] \) we have got the loss vector \( \left[ \begin{array}{c} \gamma \\ \delta \end{array} \right] \).

In the political block of the model, we formalize the electoral mechanism of replacing the ruling party (i.e. the current policy \( \left[ \begin{array}{c} \tau \\ \mu \end{array} \right] \)). Since there are no separate individuals in the model, the voting choice is made by society as a whole. The electoral behavior of the society is retrospective and, of course, economically motivated. The decision whether or not to support the current policy is made on the grounds of changes in private capital stock. The rule is simple: if the difference between present and previous private capital stock is negative, the decision is also negative, and vice versa:
\[ v(t) = \begin{cases} 0, & K(t) - K(t-1) < 0 \\ 1, & K(t) - K(t-1) \geq 0 \end{cases} \]

In democratic regimes, a vote for the party in power \([v(t) = 1]\) means simply the prolongation of the current governmental policy for one more time period. A vote against it leads to the shift in the policy space from the present point to another one. The latter is predetermined, thus the competition can be described as dualistic or “bipartisan”. In computational experiments, we just examine all the pairs of policy vectors. In autocratic regimes the results of elections have no impact on economic policy.

Democracy is modeled by switching the dynamical system, which is too complicated to be investigated analytically. The only opportunity to investigate the effects of efficiency on economic growth in different political environments is via computational experiments (Section V).

**IV. Analysis**

The continuous-time model takes the form

\[
Y(t) = AK^\alpha G^{1-\alpha}(t), \quad 0 < \alpha < 1 
\]

\[
T(t) = \tau Y(t) = \tau AK^\alpha G^{1-\alpha}(t) 
\]

\[
I(t) = \mu T(t) 
\]

\[
\frac{dG}{dt} = -\delta G + \gamma I(t), \quad 0 < \gamma \leq 1
\]

\[
\frac{dK}{dt} = -(1 - \tau \mu) AK^\alpha G^{1-\alpha}(t)
\]

The dynamical system (1)-(5) can be reduced to the following coupled system of nonlinear ordinary differential equations for functions \(G(t), K(t)\):

\[
\frac{dG}{dt} = -\delta G + \gamma \tau \mu AK^\alpha G^{1-\alpha} 
\]

\[
\frac{dK}{dt} = -(1 - \tau \mu) AK^\alpha G^{1-\alpha}
\]

The equilibrium solution is given by

\[
\begin{pmatrix} G(t) \\ K(t) \end{pmatrix} = \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix} e^{\alpha t} 
\]

where \(\psi_0, \psi_1\) are constants to be found. An equilibrium solution refers to a proportional change in the value of private and public capital. If \(\alpha > 0\), then they increase proportionally; hence the society is effective. If \(\alpha < 0\), then \(G(t)\) and \(K(t)\) decrease proportionally, and the
society is ineffective. Our immediate purpose is to determine whether they increase for a particular set of parameters.

After some simplification, substituting (10) into (8), (9) gives

\[(a + \lambda)\psi_0 = \gamma\tau\mu A\psi_1^\alpha \psi_0^{1-\alpha}\]  
\[(a + 1)\psi_1 = (1 - \tau\mu)A\psi_1^\alpha \psi_0^{1-\alpha}\]  

Finally, we get the following equation for system effectiveness \(a\):

\[\left[\frac{\gamma\tau\mu A}{a + \delta}\right]^{1-\alpha} = \left[\frac{(a + 1)}{(1 - \tau\mu)A}\right]^\alpha\]  

(11)

It can be easily proved that \(a > 0\) if and only if

\[(\tau\mu)^{1-\alpha} (1 - \tau\mu)^\alpha > \frac{1}{A}\left(\frac{\delta}{\gamma}\right)^{1-\alpha}\]  

(12)

Given \(A, \gamma, \delta, \alpha\) inequality, (12) defines an area in policy space \(0 \leq \tau \leq 1, 0 \leq \mu \leq 1\).

The policy \((\tau, \mu)\) is said to be a long-run growth policy if it satisfies (12). Otherwise it is called long-run decline policy. If inequality (12) is unsolvable in policy space, then there is no effective policy.

Let us now consider the following problem. Suppose that values of \(A, \gamma, \delta, \alpha\) are such that there exists at least one solution to inequality (12), i.e. at least one long-run growth policy exists. We may presume that there are some other such policies, which cover an area in the policy space. What does this area look like? How can we describe the range of long-run growth policies? And how can we describe the impact of losses on this range?

Let us remark that the left-hand side of the inequality (12) depends on the product \(\tau\mu\), not on \(\tau\) and \(\mu\) separately. Thus consider the function

\[f(\tau\mu) = (\tau\mu)^{1-\alpha} (1 - \tau\mu)^\alpha, \quad 0 \leq \tau\mu \leq 1\]  

(13)

Since

\[f'(\tau\mu) = (\tau\mu)^{-\alpha} (1 - \tau\mu)^{\alpha-1} [1 - \alpha - \tau\mu]\]

then the function \(f(\tau\mu)\) has maximum value at \(\tau\mu = 1 - \alpha\). Hence if

\[f(1 - \alpha) = (1 - \alpha)^{1-\alpha} (\alpha)^\alpha > \frac{1}{A}\left(\frac{\delta}{\gamma}\right)^{1-\alpha}\]  

(14)

then long-run growth policies exist, and the gamut of such policies is given by

\[C_1 < \tau\mu < C_2\]  

(15)
(here \( C_1 < 1 - \alpha < C_2 \)). This area is bounded by the two hyperbolae \( \tau \mu = C_1 \) and \( \tau \mu = C_2 \) (Fig. 1). In the sequel, this area is called the croissant.

![Figure 1. The area of effective policies (the croissant)](image)

The difference \( W = C_2 - C_1 \) gives a convenient measure for how large the gamut of long-run growth policies is. Let us call it the width of the croissant (in fact, it is the width at the very edge of the croissant).

The explicit formula for \( W \) can be easily derived for the special case \( \alpha = 0.5 \). Putting this into inequality (12), we obtain

\[
\sqrt{\tau \mu (1 - \tau \mu)} > \frac{1}{A} \sqrt{\frac{\delta}{\gamma}}
\]  

(16)

Solving inequality (16) yields

\[
\frac{1}{2} - \frac{1}{2} \sqrt{\frac{1}{4} - \frac{\delta}{\gamma A^2}} < \tau \mu < \frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{4} - \frac{\delta}{\gamma A^2}}
\]  

(17)

Thus, the width of the croissant is

\[
W = 2 \sqrt{\frac{1}{4} - \frac{\delta}{\gamma A^2}} = \sqrt{1 - \frac{4\delta}{\gamma A^2}}
\]  

(18)

(there are no long-run growth policies if \( 1 - 4\delta / \gamma A^2 < 0 \)).
Two examples of croissants are shown in Fig. 2. Here $\delta/\gamma = 0,1$; $A = 0,67$ for the left image, and $A = 1$ for the right one. Greater total factor productivity allows more possibilities for long-run growth policy. The width of the croissant tends to 1 as $A \to \infty$.

On the other hand, inefficiency inhibits economic growth. In terms of our approach this means that the greater the losses are, the smaller the width of the croissant is. If efficiency losses are too great, the croissant vanishes. It follows from (18) that $W > 0$ if and only if

$$1 - \frac{4\delta}{\gamma A^2} > 0$$

(19)

From this we get

$$\frac{\delta}{\gamma} < \frac{A^2}{4}$$

(20)

This inequality defines the admissible level of inefficiency. If $\delta$-loss is too great (i.e. depreciation rate $\delta >> 1$), or if $\gamma$-loss is too great (i.e. $\gamma << 1$), then inequality (20) is violated, and there are no long-run growth policies. In other words, a small quotient $\delta/\gamma$ means high efficiency.

For example, if $A = 0,8$, then should be $\delta/\gamma < 0,16$, otherwise $a < 0$ for any policy. If $A = 1$ or $A = 1,2$, then should be $\delta/\gamma < 0,25$ or $\delta/\gamma < 0,36$.

In Fig. 3 the triangle between the lines $\delta = 0, \gamma = 1$ and the upward-sloping straight line (for example, $\delta = 0,16\gamma$ in the left image) represents the area of admissible efficiency losses.
Figure 3. Inefficiency is admissible if the point \((\gamma, \delta)\) is in the triangle if the point falls within the triangle.

The important question is: which kind of efficiency affects more strongly the size of the gamut of long-run growth policies? In terms of our model, this size is referred to the width of the croissant. Thus, let us consider the width of the croissant to be a function of efficiency-related parameters \(\gamma\) and \(\delta\), defined on the area of admissible efficiency losses. In other words, consider the function

\[
W(\gamma, \delta) = \sqrt{1 - \frac{4\delta}{\gamma A^2}}
\]

(21)

The measurement of how responsive the function \(W(\gamma, \delta)\) is to a change in an argument is elasticity. By definition, the elasticity of \(W\) with respect to \(\gamma\) is

\[
E_\gamma = \frac{\gamma dW/W}{d\gamma/\gamma} = \frac{\gamma}{W} \frac{dW}{d\gamma}
\]

Using (21), we obtain

\[
E_\gamma = \frac{\gamma}{W} \frac{\partial W}{\partial \gamma} = \frac{\gamma}{W} \frac{1}{\sqrt{1 - \frac{4\delta}{\gamma A^2}}} \frac{4\delta}{2 \left(1 - \frac{4\delta}{\gamma A^2}\right)^{\frac{3}{2}}} = -\frac{2\delta}{\gamma A^2 - 4\delta}
\]

Similarly, the elasticity of \(W\) with respect to \(\delta\) is

\[
E_\delta = \frac{\delta}{W} \frac{\partial W}{\partial \delta} = \frac{\delta}{W} \frac{1}{\sqrt{1 - \frac{4\delta}{\gamma A^2}}} \frac{-4\gamma^2}{2 \left(1 - \frac{4\delta}{\gamma A^2}\right)^{\frac{3}{2}}} = -\frac{2\delta}{\gamma A^2 - 4\delta}
\]

Hence, we have \(E_\gamma = -E_\delta\). The one percent change in \(\gamma\)-loss leads to the same effect as the one percent change in \(\delta\)-loss. This is not surprising, since it is obvious from (21) that, for example, doubling \(\gamma\) yields the same effect on \(W\), as halving \(\delta\). But doubling \(\gamma\) is sometimes impossible because the point \((\gamma, \delta)\) should fall within the triangle shown in Fig. 3. In this sense,
the size of the gamut of long-run growth policies is affected equally by $\gamma$-loss and $\delta$-loss within given restrictions.

But in the real world, an increase in $\delta$ is more dangerous than a decrease in $\gamma$. Any value of $\gamma$ (except $\gamma = 0$, of course) leaves us some faint hope to switch to a growth trajectory. Dealing with $\delta$ we face some “red tape”, some definite sharp threshold, crossing which leads directly to the collapse of the long-run growth area in the policy space. In this sense $\delta$-loss is more “restrictive” in comparison to the $\gamma$-loss.

The results of this section can be summarized as follows. If the TFP is high enough, and the efficiency losses are small enough, there is a set of long-run growth policies, represented by a croissant in the policy space. The lower efficiency is, the thinner the croissant is. If the level of efficiency is beyond definite threshold, then there are no effective policies.

V. Computational experiment

The goal of the computational experiment is to examine the effects of various levels of $\gamma$- and $\delta$-loss on the system’s economic performance under autocracy and democracy. Democratic decision making, as opposite to the autocratic, preserves the possibility of policy changes over time. We formalize this distinction as follows. Let the policy vector $\left( \frac{\tau_1}{\mu_1} \right)$ represent the incumbent Party I, and the policy vector $\left( \frac{\tau_2}{\mu_2} \right)$ represent the challenger Party II. The incumbent loses power ($\nu = 0$ and policy change occurs) if there is a decrease in private capital stock (see Section III for details). But if $\left( \frac{\tau_1}{\mu_1} \right) = \left( \frac{\tau_2}{\mu_2} \right)$, the loss of power is bogus and we find ourselves in an autocratic environment; otherwise we face democracy.

The main independent parameters of interest are, on the one hand, variables $\lambda$ and $\delta$ (efficiency loss plane), and economic variables $\tau$ and $\mu$ (policy plane), on the other. Each of the parameters are normalized to unity, which makes the grid-sweeping technique (Laver & Sergenti 2012) a natural choice of the experimental design. The policy of each party sweeps the two-dimensional grid on ($\tau, \mu$)-plane ($0 < \tau < 1, 0 < \mu < 1$) as the point ($\gamma, \delta$) sweeps the two-dimensional grid on ($\gamma, \delta$)-plane ($0 < \gamma < 1, 0 < \delta < 1$). There are 10 divisions (step=0.1) in each dimension. Therefore, we have 100 points in ($\gamma, \delta$)-plane and 10,000 pairs of parties’ policies.

The main outcome variable indicates the economic dynamics of the system. We reduce all trajectories of economic development to two basic (qualitative) scenarios:
• long-run growth (1) – both private and public capital dynamics evolve into a stationary regime described by positive exponential function (i.e. $a > 0$ – see (8));
• long-run decline (0) – both public and private capital converges asymptotically to zero (i.e. $a < 0$).

These two scenarios of development exhaust all the possibilities (as $a = 0$ is unstable and cannot be obtained in a numerical experiment).

We initialized the model with one unit of private capital and one unit of public capital, private capital elasticity $\alpha$ equalized to 0.5, and TPF is set at the value 1.1. The model is simulated using the MatLab algorithm.

First, we check the general fit of the computational model, performing the total set of runs without the distinction between democracy and autocracy (Fig. 4).

These results are in full accordance with the analysis in Section IV (Fig. 3). Generally, the level of $\delta$-loss sets tighter limits on the growth perspectives. When delta exceeds the critical value (0.31 for this experiment), a successful scenario becomes impossible.

The next and the core stage of the experiment is the decomposition of the overall computational output into democratic and autocratic “components”. In Fig. 5 we demonstrate that the probability of achieving stationary growth is different for democracy and autocracy, and values of delta and gamma are responsible for this difference. When delta is very small (i.e. $\delta$-loss is high, Fig. 5A), democracy outperforms autocracy irrespective of gamma values (except for when gamma is extremely low, see the very left side of the graph). When $\delta = 0.1$ autocracy
works better than democracy for \( \gamma = (0.3, 0.09) \), but a very high rate of \( \gamma - \text{loss} \) levels their performance (very right side of Fig. 5B). With larger values of delta (Fig. 5C, D) autocracy is always more successful than democracy (for non-zero probabilities).

![Figure 5. Probabilities of achieving growth under democracy and autocracy with different depreciation rates](image)

The effect of \( \gamma - \text{loss} \) is even more pictorial with higher values of TFP. In Fig. 6 \((A = 1.2, \delta = 0.1)\) we observe better chances of autocracy to provide growth when \( \gamma - \text{loss} \) is low (right side of the graph) and the opposite result for high values of gamma (left side of the graph).
In general, the increase in the efficiency of the system entails a comparative disadvantage for autocracy – in comparison to democracy – in providing growth. In other words, when the overall level of efficiency is high, democracy outperforms autocracy. A decrease in either type of efficiency leads to a reduction in the chances of the system to achieve growth (squeezing of the croissant in the policy space), and democracy is more sensitive to this effect than autocracy. This is the most important and non-trivial result of the study. It holds up to various robustness checks, including changes in total factor productivity and input factor elasticities.

VI. Conclusions

This paper makes three contributions to the existing literature that studies the linkage between regime type and growth. The first one is a novel dynamic model that brings together a process of production and a political mechanism of choosing policy. The model has been investigated analytically and by means of computational experiment to evaluate comparative growth prospects of democratic and autocratic regimes. Our analytical strategy is equipped with some useful and comprehensible tools and representations – policy space and efficiency space in the first place.

Second, we introduced a distinction between two types of efficiency loss. Along with the loss of public capital during its accumulation, which has already become a recognized element of a formal framework in this research area, we take into account the process of capital stock’s deterioration. This deterioration may considerably exceed its “natural” level due to corruption.

Finally, we demonstrated that the efficiency of public capital is an important factor affecting the difference in growth capacities of democracy and autocracy. The increase in
efficiency makes the probability of long-run growth higher for democracies. Vice versa, in the presence of low efficiency (related above all to corruption practices) autocracies tend to perform better.

The results obtained by means of formal analysis and computational experiments encourage us to make a few predictions that could be tested empirically. The main hypothesis is that democracy brings advantages (i.e. higher probability to achieve growth) when effective economies are competing, but it is disadvantageous in competition between ineffective economies.

Next, taking into consideration the difference in $\delta$- and $\gamma$- effects upon a system’s growth capacities, we theorize that $\delta$-loss will prevail when some institutional restrictions on budget expenses are combined with the high intensity of corruption networks and the predominance of informal rules and practices. The presuppositions behind this thesis are that a) either type of loss is caused by corruption primarily and b) $\gamma$-loss is less costly and risky for a bureaucrat than $\delta$- corruption. In the absence of any actual monitoring of budget flows the choice would be made in favor of the direct appropriation of the resources instead of a more complicated “quality-diminishing” strategy. We believe that the combination mentioned above is more typical for “defective” democracies or, more generally, hybrid regimes.

In conclusion, we sketch some directions for the future improvement of our methodology. First of all, in the present version of the model, autocratic policy selection is obviously oversimplified. In fact, it is defined “negatively” – as the absence of feedback from the economy. In reality, autocracies – in contrast to their model behavior – demonstrate considerable flexibility. The main problem here is rather theoretical than methodological, as political scientists still lack a solid theory of autocratic policy making, despite important contributions in the general understanding of authoritarian rule (e.g., Bueno de Mesquita 2003; Gandhi and Przeworski 2007; Besley and Kudamatsu 2007; Svolic 2012).

Democratic policy selection can also be made more complicated and rich, and the most promising option here is to include some learning mechanisms into voter’s behavior model. Another possibility is to tie the voter’s choice to the change in public capital along with existing dependence on the change in private stock.

One of potentially fruitful directions of upgrading the model is also to endogenize some parameters of the economic block, linking them to political environment. The most important problem here is to equip the model with a description of influence of democratic or autocratic environment to the value of efficiency loss.
References


Andrei Akhremenko
National Research University Higher School of Economics (Moscow, Russia). Laboratory for Qualitative and Quantitative Analysis of Political Regimes, Director.
Faculty of social sciences, Department of political science, professor.
E-mail: aakhremenko@hse.ru
Tel. +7 (916) 657-64-93

Any opinions or claims contained in this Working Paper do not necessarily reflect the views of HSE.

© Akhremenko, Petrov 2014