## Rationality of Voting and Voting Systems: Lecture III The Relevance of Social Choice Theory

Hannu Nurmi

Department of Political Science
University of Turku

# Three Lectures at National Research University 'Higher School of Economics' 

## Arrow's theorem

## Theorem

Arrow'(1963): No social welfare function satisfies the following conditions:
(1) unrestricted domain (U)
(2) independence of irrelevant alternatives (IIA)
(3) Pareto ( $P$ )
(4) non-dictatorship (D)

Remark: social welfare functions assigns to each $n$-tuple of connected and transitive individual preference relations a (collective) connected and transitive preference relation.

## Definitions

## Definition

The set V of individuals is almost decisive for x against y , if $x \succ y$ whenever $x \succ_{i} y, \forall i \in V$ and $y \succ_{j} x, \forall j \notin V$.

## Definition

$V$ is decisive for $x$ against $y$, if $x \succ y$ whenever $x \succ_{i} y, \forall i \in V$.

## Definition

$D(x, y)$ means that J is almost decisive for x against $\mathrm{y} . \bar{D}(x, y)$ means that J is decisive for x against y .

Remark: $\bar{D}(x, y) \rightarrow D(x, y)$.

## Lemma

## Lemma

If there is an individual $J$ that is almost decisive for any pair of alternatives, then a social welfare function that satisfies $U, I I A$ and $P$ implies that $J$ is a dictator.

Proof: Let J be almost decisive for some x against some y , i.e. $\exists x, y \in X: D(x, y)$. Let z be an alternative different from x and y and let i denote all other individuals.
Assume that $x \succ_{j} y$ and $y \succ_{j} z$ that $y \succ_{i} x$ and $y \succ_{i} z$.
Now, $\left[D(x, y) \& x \succ_{j} y \& y \succ_{i} x\right] \rightarrow x \succ y$.
Moreover, $\left[y \succ_{j} z \& y \succ_{i} z\right] \rightarrow y \succ z$ (Pareto).
But $[x \succ y \& y \succ z] \rightarrow x \succ z$ (by transitivity).
Hence we have derived $x \succ z$ assuming nothing about other persons' but $J$ preferences regarding $x$ and Z. I.e.

$$
\begin{equation*}
D(x, y) \rightarrow \bar{D}(x, z) . \tag{1}
\end{equation*}
$$

## Lemma, cont'd

Assume now that $z \succ_{J} x \& x \succ_{J} y$ and that $z \succ_{i} x \& y \succ_{i} x$. Then, Pareto implies that $z \succ x$.
And since $D(x, y) \& x \succ_{J} y \& y \succ_{i} x$, then $x \succ y$. Hence, by transitivity $z \succ y$. Now we have obtained this result assuming nothing about other persons' but J's preferences about y and z. I.e.

$$
\begin{equation*}
D(x, y) \rightarrow \bar{D}(y, z) \tag{2}
\end{equation*}
$$

Interchanging $y$ and $z$ in (2), we see that

$$
\begin{equation*}
D(x, z) \rightarrow \bar{D}(y, z) \tag{3}
\end{equation*}
$$

## Lemma, cont'd

Make now the following substitutions in Eq. (1): $x$ for $z, z$ for $y, y$ for $x$. Now (1) becomes

$$
\begin{equation*}
D(y, z) \rightarrow \bar{D}(y, x) \tag{4}
\end{equation*}
$$

Now,

$$
\begin{array}{ll}
D(x, y) & \rightarrow \bar{D}(x, z) \text { (due to (1)) } \\
& \rightarrow D(x, z) \text { (due to Remark) } \\
& \rightarrow \bar{D}(y, z) \text { (due to (3)) } \\
& \rightarrow \bar{D}(y, z) \text { (due to Remark) } \\
\text { I.e. } & \rightarrow \bar{D}(y, x) \text { (by (4)) }
\end{array}
$$

$$
\begin{equation*}
D(x, y) \rightarrow \bar{D}(y, x) \tag{5}
\end{equation*}
$$

## Lemma, cont'd

By interchanging $x$ and $y$ in (1), (2) and (5), we get:

$$
\begin{equation*}
D(y, x) \rightarrow[\bar{D}(y, z) \& \bar{D}(z, x) \& \bar{D}(x, y)] \tag{6}
\end{equation*}
$$

Now, $D(x, y) \rightarrow \bar{D}(y, x)$ (due to (5))
$\rightarrow D(y, x)$ (due to Remark)
Thus,

$$
\begin{equation*}
(6) \rightarrow D(x, y) \rightarrow[\bar{D}(y, z) \& \bar{D}(z, x) \& \bar{D}(x, y)] \tag{7}
\end{equation*}
$$

Combining (1), (2), (5) and (7) we see that $D(x, y)$ implies that $J$ is decisive for all pairs that can be formed of $x, y$ and $z$ when conditions $\mathrm{U}, \mathrm{P}$ and IIA are satisfied. Thus, J is a dictator for all triplets where x and y are present.

## Lemma, cont'd

To expand the alternative set, take any 2 alternatives u and v . If $u=x$ and $v=y$ then (by what is said above) $\bar{D}(u, v)$ holds. If $u=x$ and $v \neq y$, we choose $x, y$ and $v$. Since $D(x, y)$ holds, we have $\bar{D}(u, v)$ and $\bar{D}(v, u)$.
If both x and y differ from u and v , we take first $(x, y, u)$. Then $\bar{D}(x, u) \rightarrow D(x, u)$. Next, take $(x, u, v)$. Since we have $D(x, u)$, we also have $\bar{D}(u, v)$ and $\bar{D}(v, u)$. Therefore, $D(x, y)$ for some $x, y$ implies that $\bar{D}(u, v)$ for any $u$ and v. I.e. $J$ is a dictator. Q.E.D.

## The proof of the theorem

It will be shown that if conditions IIA, P and U are satisfied, there always exists a person who is almost decisive for some pair of alternatives.
For all pairs of alternatives, there is at least one decisive set, viz. N (Pareto).
Consider now all sets of individuals that are almost decisive for some pair. Choose the smallest of them and denote it by V. Let the alternative pair for which it is almost decisive be ( $\mathrm{x}, \mathrm{y}$ ).
If $|V|=1$ we are done.
If $|V|>1$, we divide $V$ in two parts:

- $V_{1}$ consisting of one person,
- $V_{2}$ consisting of the others in V , and
- $N \backslash V=V_{3}$.


## The proof, cont'd

Assume that we have the following profile:
(1) $\forall i \in V_{1}: x \succ_{i} y \& y \succ_{i} z$,
(2) $\forall j \in V_{2}: z \succ_{j} x \& x \succ_{j} y$, and
(3) $\forall k \in V_{3}: y \succ_{k} z \& z \succ_{k} x$.

Since V is almost decisive for x against y and since $\forall i \in V: x \succ_{i} y$ and $\forall i \in V_{3}: y \succ_{i} x$ we have $x \succ y$.
In the comparison of y and z , only $\forall i \in V_{2}: z \succ_{i} y$, while $\forall i \in V_{1}$ and $\forall i \in V_{3}$ we have: $y \succ_{z}$. So, if $z \succ y$, then $V_{2}$ has to be almost decisive for $z$ against $y$ and hence for $x$ against $y$ (by Lemma). But V was chose as the smallest almost decisive set. Moreover, $V_{2} \subset V$. Therefore, $\neg(z \succ y)$.
By completeness we thus must have $y \succeq z$. But ( $x \succ y \& y \succeq z$ ) $\rightarrow x \succ z$. However, only $V_{1}$ (one person) prefers x to z . Others have $z \succ_{i} x$. I.e. $V_{1}$ is almost decisive. Therefore, by Lemma $V_{1}$ a dictator. Q.E.D.

## Gibbard-Satterthwaite theorem

## Definition

A social choice function is manipulable (by individuals) iff there is a situation and an individual so that the latter can bring about a preferable outcome by preference misrepresentation than by truthful revelation of his/her preference ranking, ceteris paribus.

## Definition

A social choice function is non-trivial (non- degenerate) iff for each alternative x , there is a preference profile so that x is chosen.

## Theorem

(Gibbard-Satterthwaite 1973-75). Every universal and non-trivial resolute social choice function is either manipulable or dictatorial.

Strategy of proof:
(1) It is shown that any universal, non-trivial and non-manipulable SCF must satisfy the Pareto condition if the number of voters is two.
(2) One goes through all 36 different preference profiles (of two voters) and determines the possible winners excluding outcomes that fail of Pareto condition. It turns out that the non-excluded outocmes are either manipulable at some profiles or one of the voters is a dictator.

## Gärdenfors' theorem

## Theorem

Gärdenfors. If a social choice function is anonymous and neutral and satisfies the Condorcet winning criterion, then it is manipulable.

Strategy of proof:

- One begins with a specific 3-voter, 3-alternative profile, where the same alternative is ranked first by two voters. One postulates that this alternative is chosen in this profile.
- Another specific 3-voter, 3-alternative profile is then focused upon and all logically possible choice from this profile are analyzed.


## Gärdenfors, cont’d

- For each choice form the latter profile, one shows that if this were the actual choice, then the SCF would be manipulable by some voter at some other profile. Since the Condorcet winner is chosen in the first profile, the conclusion is that all Condorcet extensions are manipulable.


## Examples of non-manipulable SCF's:

- If every voter's preference ranking is strict (no ties), then SCF that chooses the Condorcet winner when one exists and all alternatives, otherwise, is non-manipulable.
- Under the same assumption concerning voter preferences any SCF that chooses the Condorcet winner when one exists and the set of Pareto-undominated outcomes, otherwise, is also non-manipulable.


## Young and Moulin

## Theorem

Young: all consistent methods are incompatible with the Condorcet winning criterion.

## Theorem

Moulin: all procedures that satisfy the Condorcet winning criterion are vulnerable to no-show paradox.

## How the incompatibilities are dealt with?

Most politicians of today and in the past are unaware of voting paradoxes and other incompatibilities. Yet, there are institutional arrangements that can be see as ways out of certain types of voting paradoxes. An example of those arrangements is the rule adhered to in all parliamentary bodies that resort to the amendment type binary voting: of $k$ alternatives only $k-1$ pairwise votes are taken and the winner of the last vote is declared to overall winner. This amounts to assuming something that we know is not in general true, viz. that the collective preference relation is transitive.

## Dealing with incompatibilities, cont'd

Another general stratagem is not to disclose individual preference rankings. Reasons for this can be practical, e.g. in plurality voting the computation of the winner does not require full information about preference rankings. In the absence of this information it is impossible to determine whether some desiderata have been satisfied or not. Some voting rules can be viewed as direct remedies of shortcomings of other rules. E.g. the plurality runoff may be seen as an improvement of the plurality rule and Nanson's rule as an improvement of Borda's. That these alleged remedies may come with a price of being accompanied with additional weaknesses, is often overlooked.

## The role of culture

- impartial culture: each ranking is drawn from uniform probability distribution over all rankings
- impartial anonymous culture: all profiles (i.e. distributions of voters over preference rankings) equally likely
- unipolar cultures
- bipolar cultures


## Lessons from probability and simulation studies

- cultures make a difference (Condorcet cycles, Condorcet efficiencies, discrepancies of choices)
- none of the cultures mimics "reality"
- IC is useful in studying the proximity of intuitions underlying various procedures


## What makes some incompatibilities particularly dramatic?

The fact that they involve intuitively plausible, "natural" or "obvious" desiderata. The more plausible etc. the more dramatic is the incompatibility.

## Theorem

Moulin, Pérez: all Condorcet extensions are vulnerable to the no-show paradox.

## Example

| $26 \%$ | $47 \%$ | $2 \%$ | $25 \%$ |
| :---: | :---: | :---: | :---: |
| A | B | B | C |
| B | C | C | A |
| C | A | A | B |

## Some "difficult" counterexamples: Black

Black' procedure is vulnerable to the no-show paradox, indeed, to the strong version thereof.

## Example

| 1 voter | 1 voter | 1 voter | 1 voter | 1 voter |
| :---: | :---: | :---: | :---: | :---: |
| D | E | C | D | E |
| E | A | D | E | B |
| A | C | E | B | A |
| B | B | A | C | D |
| C | D | B | A | C |

Here D is the Condorcet winner and, hence, is elected by Black. Suppose now that the right-most voter abstains. Then the Condorcet winner disappears and E emerges as the Borda winner. It is thus elected by Black. E is the first-ranked alternative of the abstainer.

## Another difficult one: Nanson

| 5 voters | 5 voters | 6 voters | 1 voter | 2 voters |
| :---: | :---: | :---: | :---: | :---: |
| A | B | C | C | C |
| B | C | A | B | B |
| D | D | D | A | D |
| C | A | B | D | A |

Here Nanson's method results in B.
If one of the right-most two voters abstain, C - their favorite - wins.
Again the strong version of no-show paradox appears.
The twin paradox occurs whenever a voter is better off if one or several individuals, with identical preferences to those of the voter, abstain. Here we have an instance of the twin paradox as well: if there is only one CBDA voter, $C$ wins. If he is joined by another, $B$ wins.

## Dodgson's method and the twins' paradox

| 42 voters | 26 voters | 1 voters | 11 voters |
| :---: | :---: | :---: | :---: |
| B | A | E | E |
| A | E | D | A |
| C | C | B | B |
| D | B | A | D |
| E | D | C | C |

In this profile $B$ is the (strong) Condorcet winner. Adding 20 copies of the one voter with ranking EDBAC leads to $A$ being closest to Condorcet winner. This is worse than B from the point of view of the clones. Hence we have an instance of the twins' paradox.

## Learning from proofs

Some proofs are (almost) constructive, i.e. tell us how to generate paradoxes. Pérez uses the following auxiliary result. Let $p(x, y)=$ the no. of voters preferring $x$ to $y$.

## Theorem

For any Condorcet extension which is invulnerable to no-show paradox, for any situation $(X, p)$ and for any pair $x, z$ of alternatives, if $p(x, z)<\min _{y \in X} p(z, y)$, then $x \notin f(X, p)$.

In words, the antecedence says that the minimum support for $z$ is larger than the no. of votes $x$ receives in comparison with $z$. The consequence says that then $x$ is not elected (provided that the $f$ is Condorcet and invulnerable).

## Learning ..., cont'd

The theorem is then used to construct an example.

| 5 | 4 | 3 | 3 |
| :---: | :---: | :---: | :---: |
| $t$ | $y$ | $x$ | $x$ |
| $y$ | $z$ | $t$ | $t$ |
| $z$ | $x$ | $z$ | $y$ |
| $x$ | $t$ | $y$ | $z$ |

Applying the Theorem to pairs $(z, y),(\mathbf{y}, \mathbf{t}),(t, x)$ it turns out that only $x$ is chosen.
Add now 4 voters with ranking zxyt and apply Theorem to pairs $(t, x),(\mathbf{x}, \mathbf{z}),(z, y)$ to find that $y$ is chosen.

## Why tournaments?

- rankings just aren't always plausible
- individual decision making with multiple criteria
- best variant choice problems
- much background work is already available


## All ranking profiles can be mapped into tournaments

## Example

$\left.$| 4 | 3 | 2 |
| :--- | :--- | :--- |
| A | B | C |
| B | C | B |
| C | A | A |$|\Longrightarrow|$|  | A | B | C |
| :---: | :---: | :---: | :---: |
| A | - | 4 | 4 |
| B | 5 | - | 7 |
| C | 5 | 2 | - |$|\Longrightarrow|$|  | A | B | C |
| :---: | :---: | :---: | :---: |
| A | - | 0 | 0 |
| B | 1 | - | 1 |
| C | 1 | 0 | - | \right\rvert\,

## Remark

What we have above is a simple majority tournament.

## Remark

Ties call for special arrangements, e.g. $\frac{1}{2}$ points to each element.

## . . . and all tournaments into profiles

## Theorem

McGarvey 1953. Given an arbitrary preference pattern [relation], over a set of $n$ elements, a group of individuals exists with strong individual preference orderings [complete, asymmetric and transitive] such that the group preference pattern as determined by the method of simple majority decision is the given preference pattern.

## Illustration

Suppose that we want to find a preference profile that would translate into the following tournament:

|  | A | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | - | 1 | 0 | 1 | 0 |
| B | 0 | - | 1 | 1 | 1 |
| C | 1 | 0 | - | 0 | 1 |
| D | 0 | 0 | 1 | - | 1 |
| E | 1 | 0 | 0 | 0 | - |

## Illustration, cont'd

Consider all elements $(i, j), i, j=A, \ldots, E$ that equal 1 , e.g. $(D, E)=1$. Introduce now two voters with the following preferences

| voter 1 | voter 2 |
| :---: | :---: |
| D | C |
| E | B |
| A | A |
| B | D |
| C | E |

Thereby $D$ will beat $E$ with 2 votes to none, while all other pairwise comparisons result in $1-1$ tie. Thus all other collective preferences remain unaffected.

## Illustration, cont'd

McGarvey also allows for indifference. Suppose the desired collective preference relation includes a pair of indifferent elements, say $F$ and $G$. Then one introduces two individuals with inverted preferences rankings, e.g. the following
voter 1 voter 2
A
G
B
F
C
D
E C
F B
G $\quad A$

## Illustration, cont'd

## Remark

The technique is based on the idea that each pair of introduced voters determines the collective preference concerning one and only one pair $x, y$ of alternatives and has no effect on other pairs. This is accomplished by making the preferences of these two voters "cancel out" each other in all pairwise comparisons that include alternatives that are not $x$ or $y$. In the case of collective indifference, the voter pair cancels each other's preference in all pairs of alternatives, that is, also between $x$ and $y$.

## Another illustration

Suppose we want a collective preference relation: $A \succ B \sim C \succ D$. The following profile does the trick:

| A | D | B | D | C | B |
| :--- | :--- | :--- | :--- | :--- | :--- |
| B | C | C | A | D | A |
| C | A | A | C | A | C |
| D | B | D | B | B | D |

According to the theorem we could use $2 \times\binom{ k}{2}=k(k-1)$ pairs to generate any desired (collective) preference pattern, transitive or nontransitive.

## Another, cont'd

## Remark

We have used only 6 ( not 12) individuals. Why? We need to consider only half (upper-diagonal) of the number of all pairs since the other half is determined by asymmetry of strict preference or symmetry of indifference.

## How many voters are needed?

Thus, with $k$ alternatives, there are $k(k-1) / 2$ pairwise comparisons. Consequently, $k(k-1)$ is the maximum number of voters one needs to generate a preference profile that translates into any given tournament. Is this also the minimum? No, says McGarvey:
... the actual minimum number of individuals necessary to express all possible patterns over $n$ elements has not been ascertained, but we conjecture that it is approximately $n$.

## How many, cont'd

## Theorem

Stearns 1959. The number of voters need not be larger than $k+2$.

## Theorem

Knoblauch 2013. Head-to-head (absolute) majority membership voting with voters having complete and transitive preferences can implement an arbitrary binary relation over the set of alternatives. The number of voters can be chosen to be smaller than $4 \times k-2$.

## How many, cont'd

## Remark

Head-to-head membership voting defines a binary relation V over alternatives as follows:

$$
x V y \Leftrightarrow|\{v \in N \mid x \succ y\}|>|N| / 2 .
$$

## Remark

In Knoblauch's theorem, the collective preference relation does not have to be complete. Hence, it is a generalization of McGarvey and Stearns.

## Zermelo on chess tournaments

Zermelo's (1926) approach to tournaments is based on observations of chess playing contests which often take the form of a tournament. Each player plays against every other player several times. The outcome of each game is either a victory of one player or a tie. We assume that the games are independent binomial trials so that the probability of player $i$ beating player $j$ is $p_{i j}$. Zermelo then introduces the concept Spielstärke, playing strength, denoted by $V_{i}$, that determines the winning probability as follows:

$$
p_{i j}=\frac{V_{i}}{V_{i}+V_{j}}
$$

## Zermelo, cont'd

The order of the $V_{i}$ values is the ranking of the players in terms of playing strength. Apparently player $i$ is ranked no lower than player $j$ if and only if $p_{i j} \geq 1 / 2$, i.e. players with greater strength defeat contestants with smaller strength more often than not. Now, given the matrix $M=\left\{m_{i j}\right\}$ of contest schedules and matrix $R$ of results, i.e. a $k \times k$ matrix of 0's and 1's denoting losses and victories of the alternatives represented by the rows, Zermelo defines maximum likelihood estimates, denoted by $v_{i}$, for the playing strengths of players. Consider any $k$ vector of strengths $v$. One can associate with it the probability that the observed matrix $M$ of game schedules and matrix $R$ of outcomes is the result of the tournament when the strengths are distributed according to $v$.

## Zermelo, cont'd

The probability is the following:

$$
p(v)=\prod_{i, j}\binom{m_{i j}}{r_{i j}}\left(\frac{v_{i}}{v_{i}+v_{j}}\right)^{r_{i j}} \times\left(\frac{v_{j}}{v_{i}+v_{j}}\right)^{r_{i j}}
$$

and this is what is to be maximized. (Here $m_{i j}$ is the number of times player $i$ has competed with player $j$ and $r_{i j}$ is the number of victories of $i$ over $j$ in those contests.)

## Zermelo, cont'd

Conditions under which a unique maximizing vector of strengths can be found are discussed by Zermelo and found to be rather general. A particularly noteworthy property of the Zermelo rankings is that they always coincide with the rankings in terms of scores defined above. So, were one interested in rankings only, the easy way to find them is simply to compute the scores. However, the $v_{i}$ values give us more information about the players than just their order of strength; it also reveals how much stronger player $i$ is when compared with player $j$.

## Early results of graph theory

## Theorem

Landau 1953. A sequence of nonnegative integers $s_{1} \leq s_{2} \leq \ldots \leq s_{k}$ is a score (outdegree) sequence iff their sum satisfies

$$
\sum_{i=1}^{k} s_{i}=\frac{1}{2} k(k-1)
$$

and the following inequalities hold for any $m<k$ :

$$
\sum_{i=1}^{m} s_{i} \geq \frac{1}{2} m(m-1)
$$

## Theorem

Harary and Moser 1966. In any tournament, the distance from a point with maximum score to any other point is 1 or 2 .

## Theorem

Harary and Moser 1966. The following are equivalent in any tournament $T$ of $k$ points: (1) $T$ is transitive, (2) $T$ is acyclic, (3) $T$ has a unique complete path, (4) the score sequence is $(0,1, \ldots, k-1)$, (5) $T$ has $k(k-1)(k-2) / 6$ transitive triples.

## Theorem

Harary and Moser 1966. The number b of transitive triples in a tournament $T$ with score sequence $\left(s_{1}, s_{2}, \ldots, s_{k}\right)$ is

$$
b=\sum_{i=1}^{k} \frac{1}{2} s_{i}\left(s_{i}-1\right)
$$

## Theorem

Harary and Moser 1966. The maximum number $c_{\max }(k)$ of cyclic triples is: $\left(k^{3}-k\right) / 24$ if $k$ is odd, and $\left(k^{3}-4 k\right) / 24$ if $k$ is even.

This result has been used in defining the coefficient of consistency (of pairwise comparisons) by Kendall and Babington Smith (1940):

$$
\xi=1-\frac{c}{c_{\max }(k)}
$$

where $c$ is the number of cyclic triples in the tournament.

## Axiomatic approach

## Definition

Ranking method is a function that ascribes a ranking (complete, reflexive and transitive relation) to any tournament.

## Definition

Point system is the ranking method defined by $i \succeq j$ if $S_{i} \geq S_{j}$, where $S_{i}$ is the number of individuals that $i$ beats in the tournament.

## Axioms

(1) anonymity: the ranking method does not discriminate for or against individuals
(2) positive responsiveness: suppose that $i \succeq j$ in $T$. Then $T^{\prime}$ is formed so that $k$ beats $i$ in $T$, but $i$ beats $k$ in $T^{\prime}$, ceteris paribus. Then $i \succ j$ in $T^{\prime}$.
(0) independence: the relative ranking of two individuals is independent of those matches in which neither of them is involved.

## Rubinstein's theorem

## Theorem

Rubinstein 1980. The point system is the only ranking method that satisfies axioms 1. - 3. above.

## Tournament solutions

## Definition

An alternative $x$ covers another alternative $y$ iff it beats $y$ and everything that $y$ beats. The uncovered set UC consists of alternatives not covered by any alternative.

## Definition

A Copeland winner is an alternative that beats the largest number of alternatives (has a maximum outdegree).

## Definition

A Banks chain from $x$ consists of alternatives $x_{1}, \ldots, x_{m}$ so that each alternative beats all its predecessors. The Banks set consists of end points of all Banks chains.

## Slater's rule

- given a set of $k$ alternatives, generate all $k$ ! rankings
- convert these into tournaments
- measure the distance between these and the individual tournaments
- pick the closest one(s): the underlying ranking is the solution


## Miller's and Moulin's findings

- Copeland winners and the Banks set are always subsets of UC
- the Banks set represents all outcomes of sophisticated voting in binary voting games
- when the number of alternatives is no larger than 12 some Copeland winners are in the Banks set


## Discrepancies

- the Slater winner may be in an position in Dodgson ranking (Klamler)
- the Slater and Copeland rankings can be far from each other (Charon and Hudry)
- prudent order (Arrow and Raynaud) may be exact opposite of the Slater ranking (Lamboray)
- the unique Slater winner may be in any position of a prudent order (Lamboray)
- the Banks and Slater sets can be disjoint when the number of alternatives is at least 14 (Östergård and Vaskelainen)


## Is tournament aggregation the way to go?

- calls for new criteria of performance
- loses order information (which may be unreliable anyway)
- a well-researched field with a lot of computational complexity results
- analogous to pairwise comparison voting (and should thus not be viewed as voter-degrading)
- more demanding than approval voting (requires preference statement for every pair of alternatives)


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