A Model of Decision Making on Stock Exchange

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Introduction

«The central idea of this book concerns our blindness with respect to randomness, particularly the large deviations: Why do we, scientists or nonscientists, hotshots or regular Joes, tend to see the pennies instead of the dollars? Why do we keep focusing on the minutiae, not the possible significant large events, in spite of the obvious evidence of their huge influence?»

Nassim Nicolas Taleb

«The Black Swan. The Impact of The Highly Improbable»

Introduction

- [Kahneman 2011] there is no extra knowledge that analysts bring to receive a premium or remuneration as they often fail to achieve even a 50% success rate.
- [Proskurin and Penikas 2013] only 56.8% of expert recommendations on selling or buying stocks of Russian companies were profitable.
- [Odean, 1999] the stocks individual investors buy subsequently underperform those they sell.
- [Barber and Odean 2008] 66,465 households during 1991 to 1996 were analyzed, the average household earns an annual return of 16.4 percent, while the market returns 17.9 percent, and those who trade most earn an annual return less (11.4 percent).

Problem

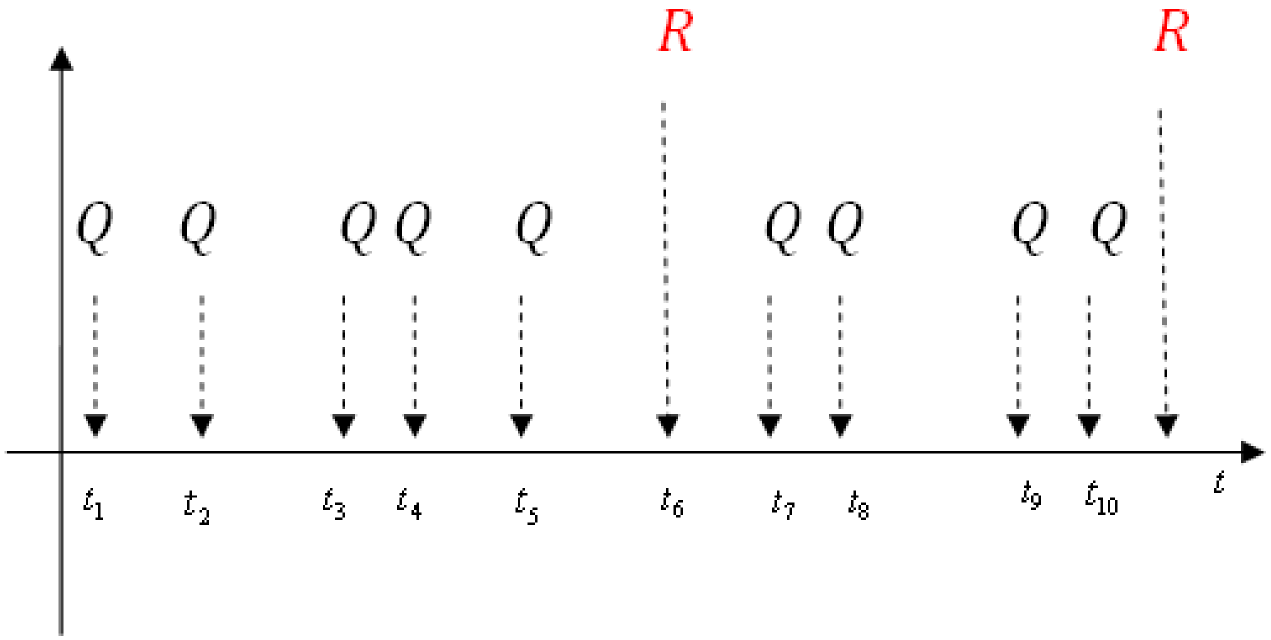
We will model economic fluctuations representing them as a flows of events of two types:

- *Q*-event reflects the “normal mode” of an economy;
- *R*-event is responsible for a crisis.

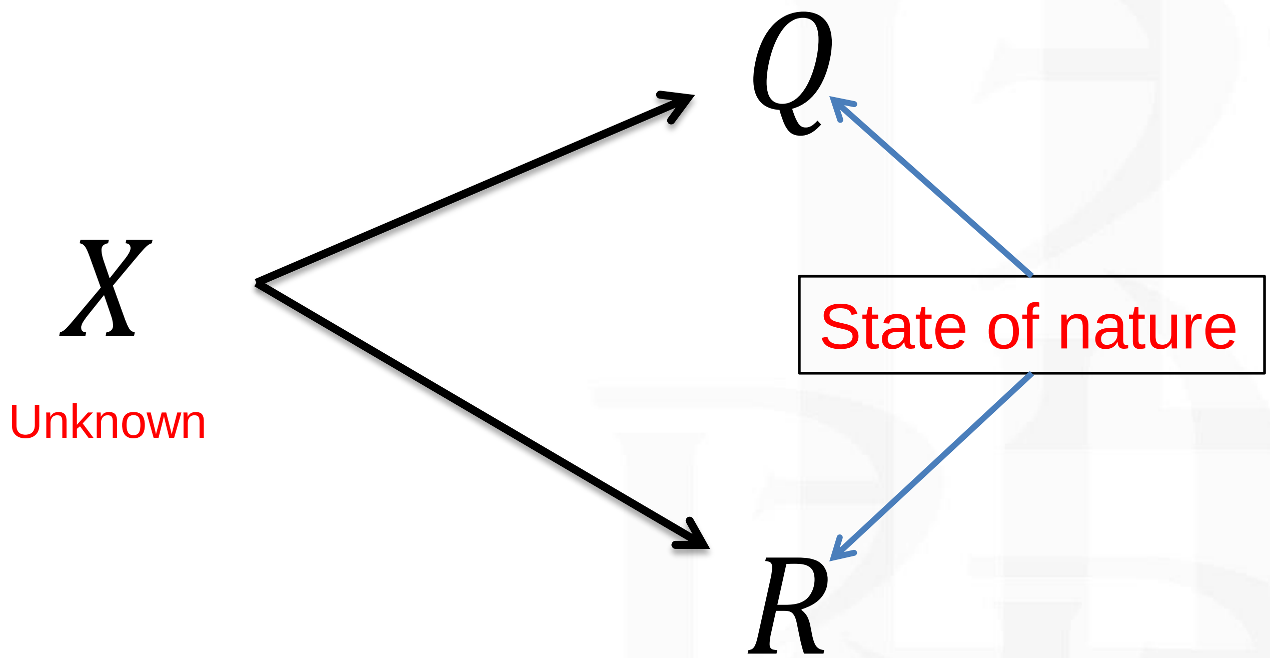
The number of events in each time interval has a Poisson distribution with constant intensity.

- λ is the intensity of the flow of regular events *Q.*
- μ is the intensity of the flow of crisis events *R*.
- $\lambda >> \mu$ holds (that is, *Q*-type events are far more frequent than the *R*-type events).

Problem



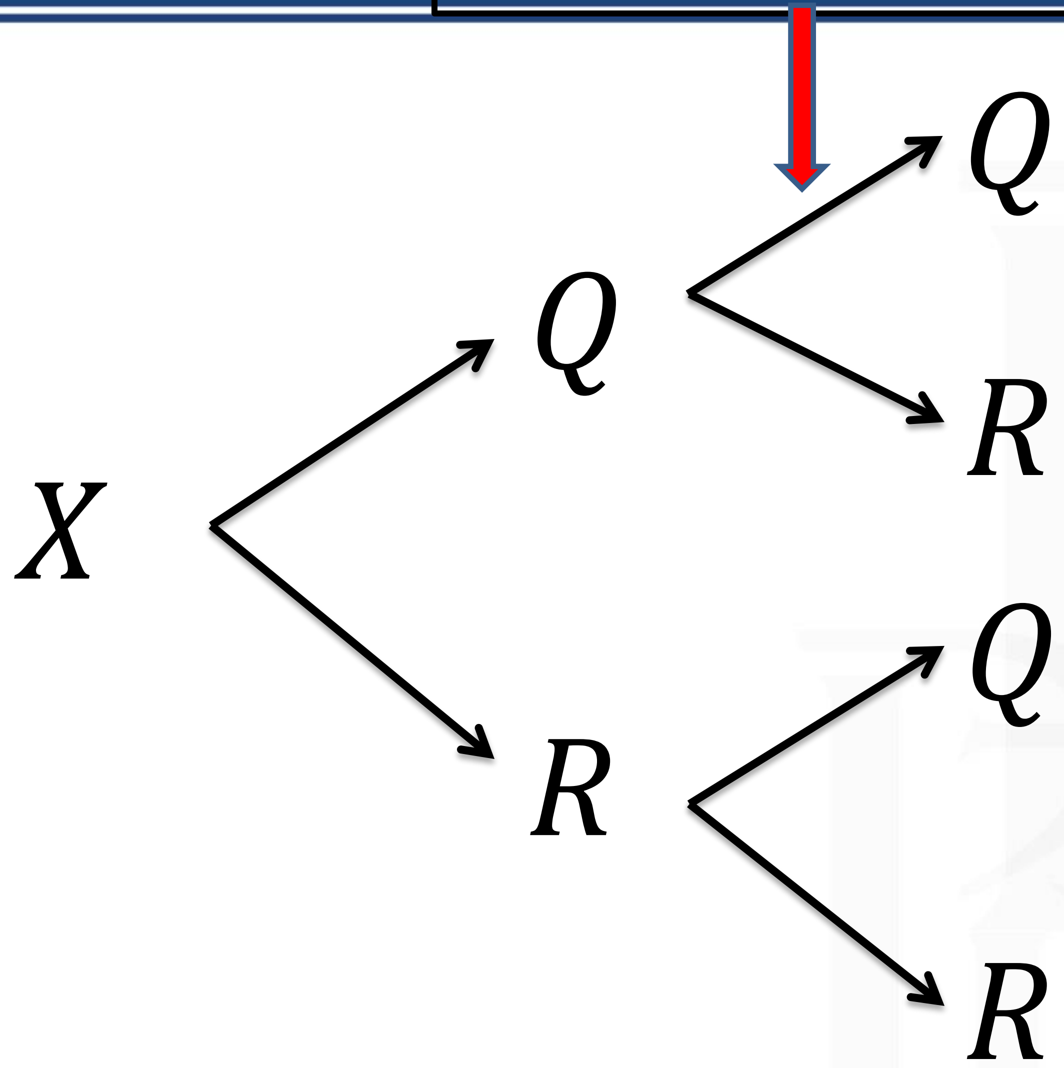
Problem



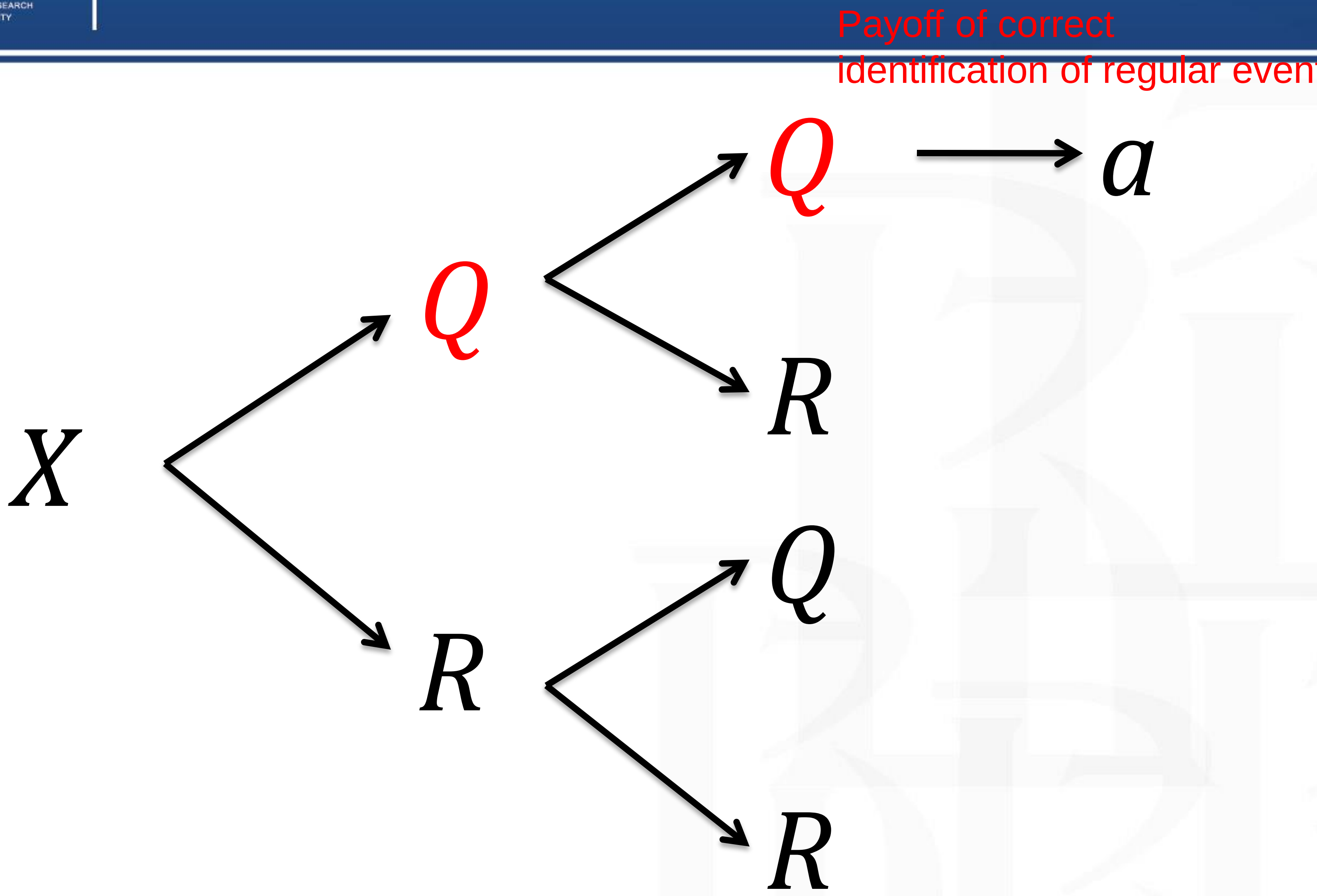
The problem of correct identification (recognition)

Problem

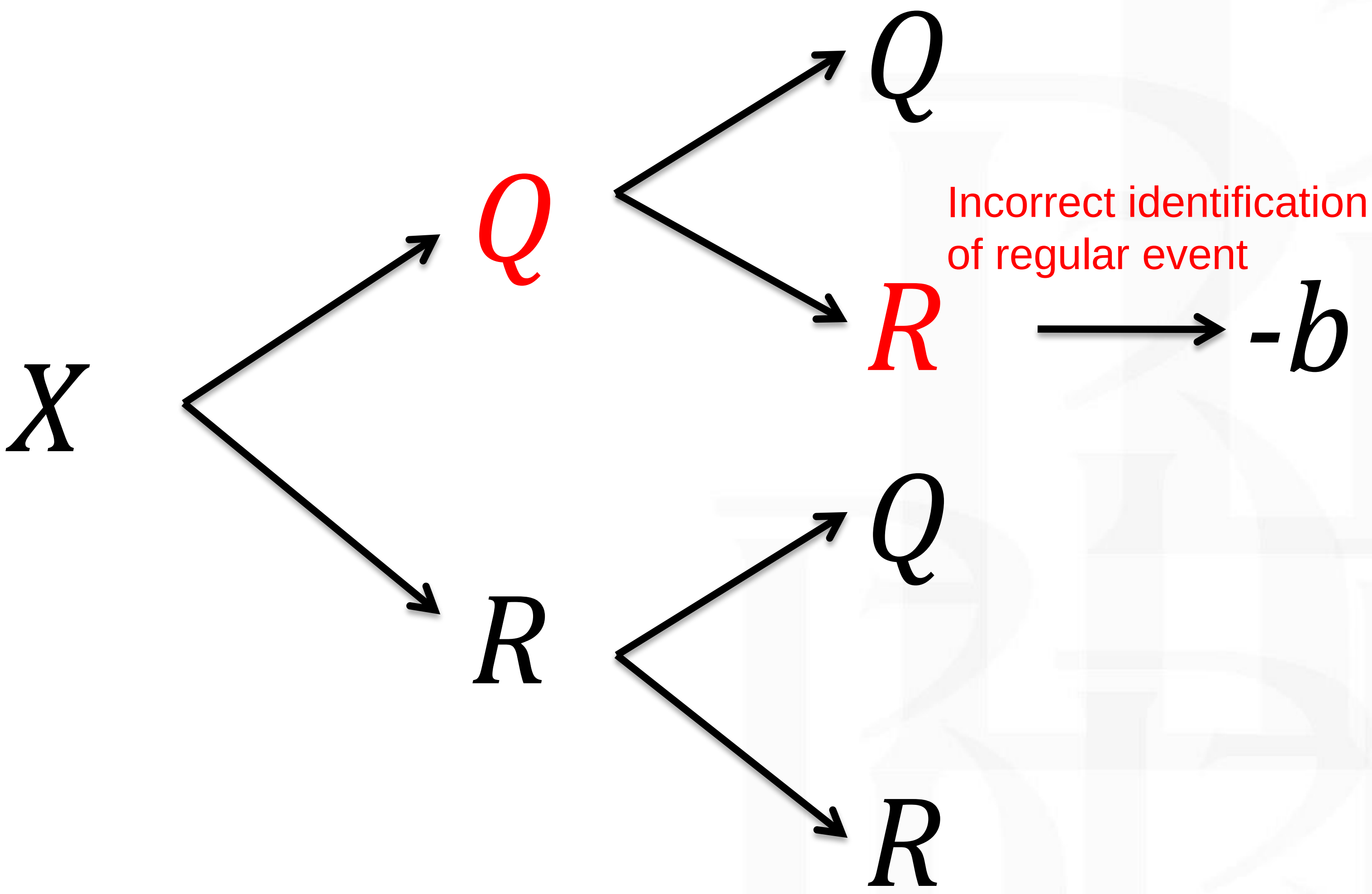
Player´s perceived identification of the state of nature



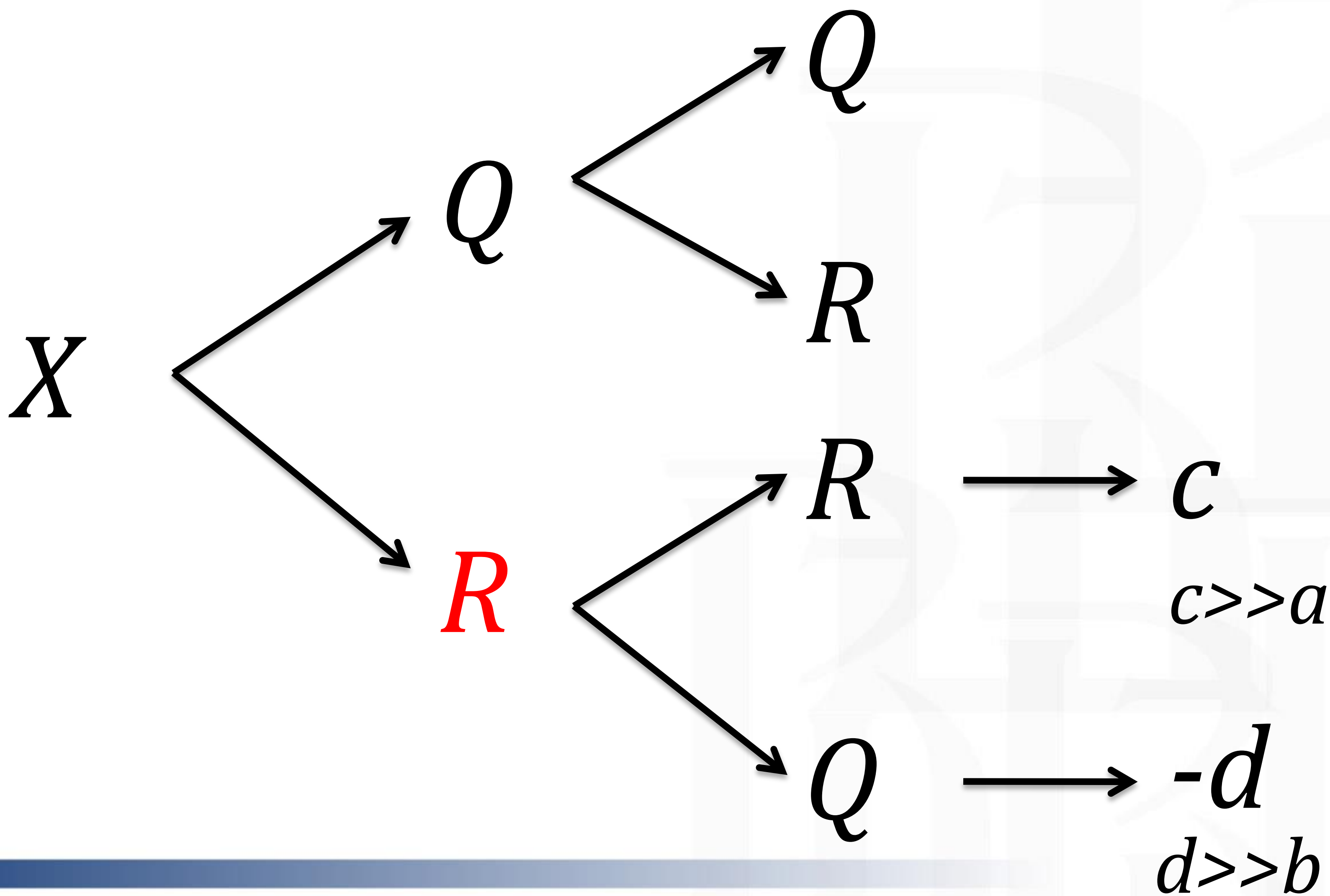
Problem


Payoff of correct identification of regular event
X
Q
R
Q
R
Q
R
a

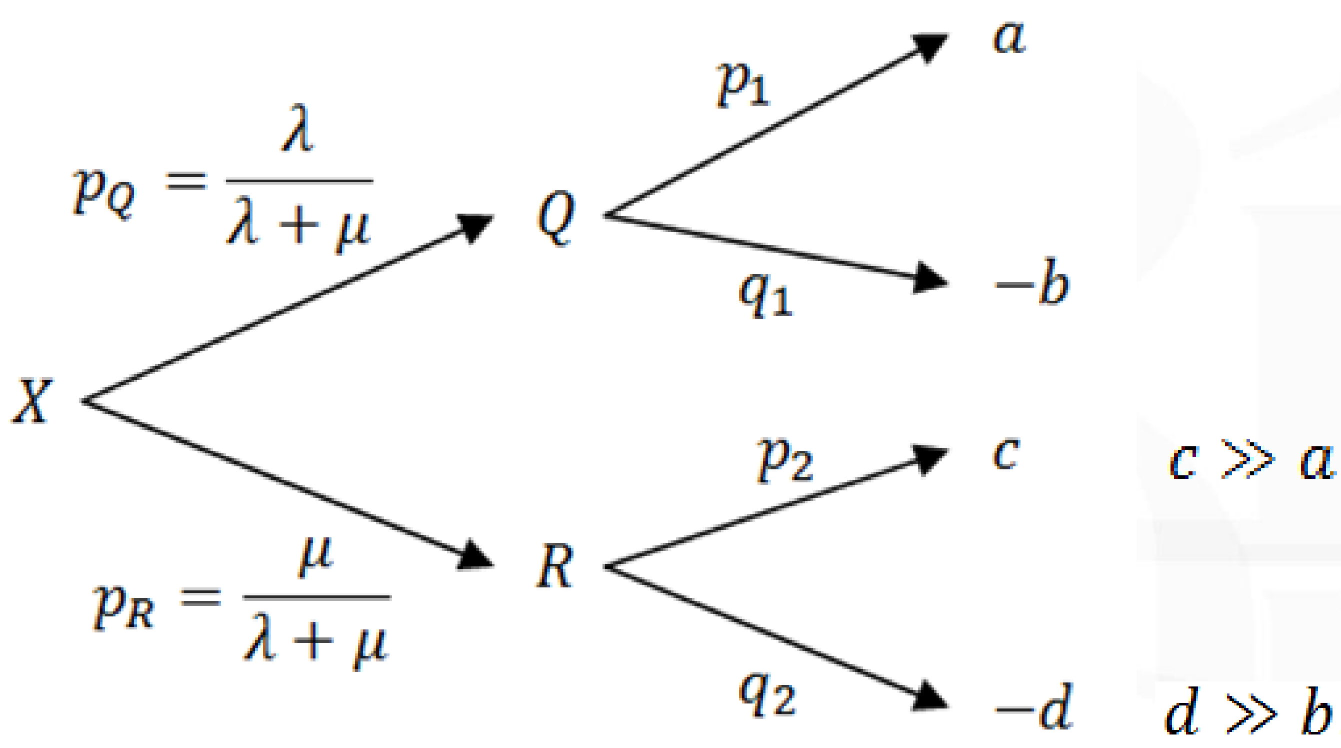
Problem



Problem



Problem



How large will be the sum of payoffs received up to time t?

Solution

Random value *Z* of the total sum of the received payoffs during the time *t* is a compound Poisson type variable.

We give the expression for the expectation of a random variable payoff:

$$E(Z) = (\lambda t)E(W) + (\mu t)E(Y) =$$

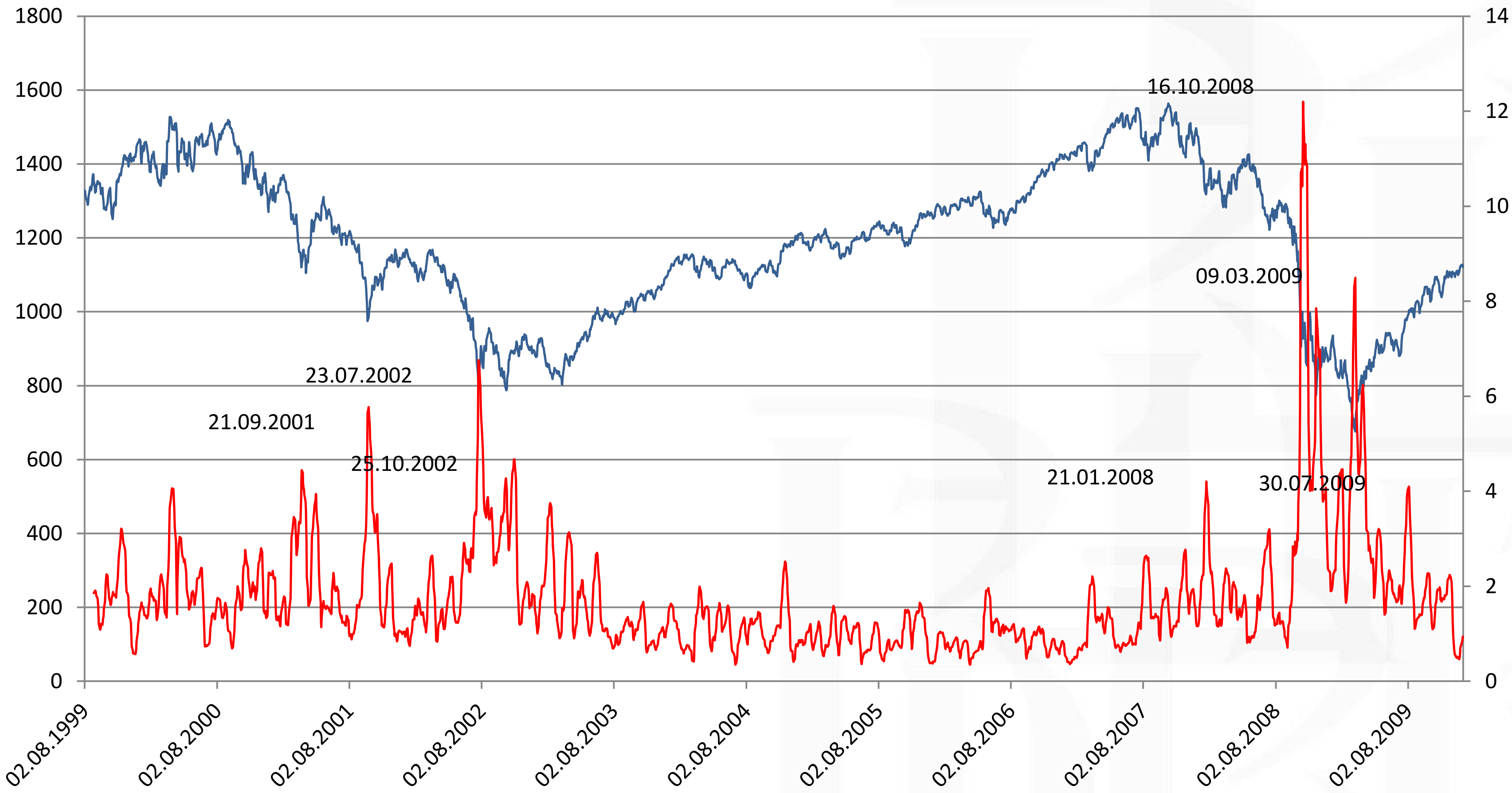
$$= \lambda t\big((1 - q_1)a - q_1 b\big) + \mu t\big((1 - q_2)c - q_2 d\big)$$

Application to real data

We consider a stock exchange and events *Q* and *R* which describe a 'business as usual' and a 'crisis', respectively.

The unknown event *X* can be interpreted as a signal received, e.g. by an economic analyst or by a broker, about the changes of the economy that helps him to decide whether the economy is in 'a normal mode' or in a crisis.

Parameters for S&P 500



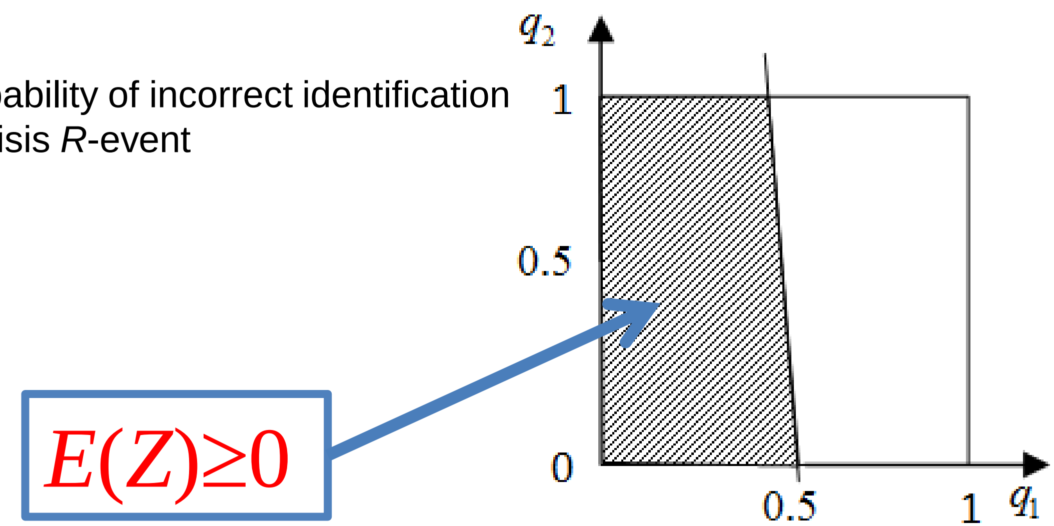
Parameters for S&P 500

Estimates for indices with the threshold 6%						
Index	λ	μ	*a*, %	-*b*, %	*c*, %	-*d*, %
S&P 500	246	4	0,6	-0,6	2,8	-2,9
Dow Jones	246	4	0,6	-0,6	1,9	-2,4
CAC 40	243	7	0,8	-0,8	3,0	-2,5
DAX	239	11	0,8	-0,9	2,1	-2,5
Nikkei 225	245	5	0,8	-0,9	2,6	-3,2
Hang Seng	241	9	0,9	-0,9	2,6	-3,0

Parameters for S&P 500

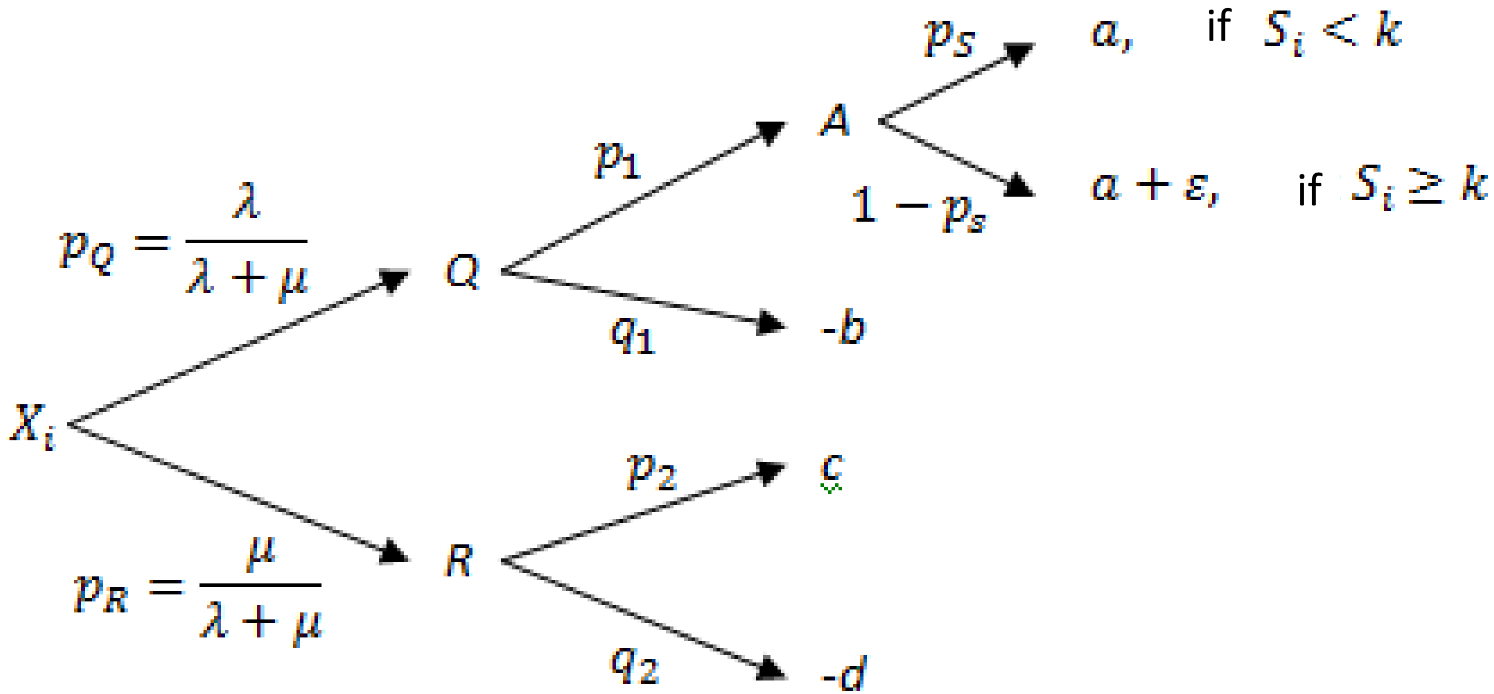
In fact, it is enough to identify regular *Q*-events in half of the cases to ensure a positive outcome of the game.

Probability of incorrect identification of crisis *R*-event



Probability of incorrect recognition of regular *Q*-event

Model with stimulation



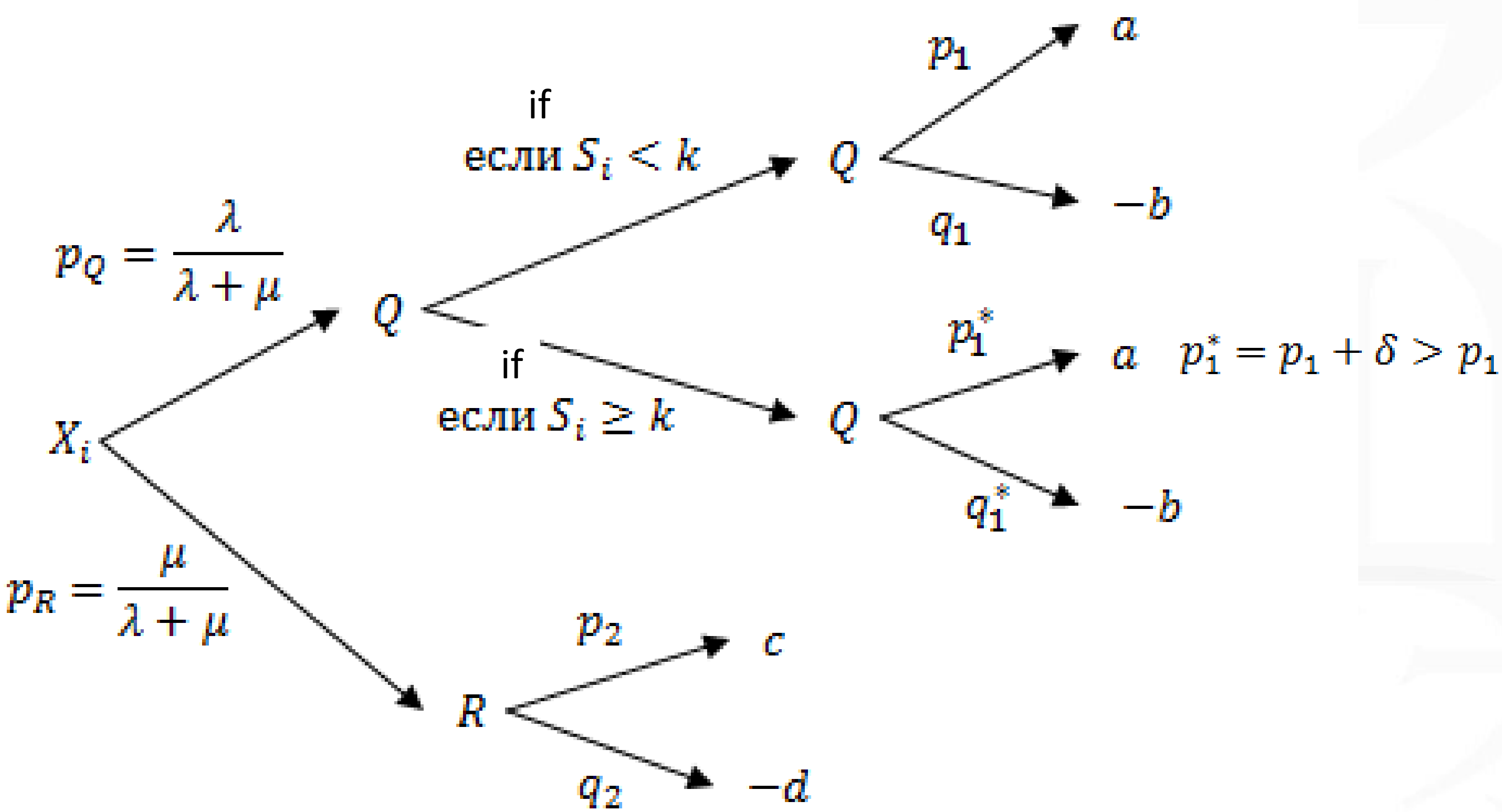
$$S_i = \begin{cases} S_{i-1} + 1, \text{if came } A, \\ 0, \text{if came } \bar{A} \text{ (didn't come } A\text{)}, \end{cases} \qquad S_0 = 0.$$

Model with stimulation

$$E(Z) = E(X)\lambda_{N_Z} +$$

$$+\varepsilon p_a^{k+1}\left[\lambda_{N_Z}\left(1-\frac{\Gamma(k-1,\lambda_{N_Z})}{\Gamma(k-1)}\right)+(1-k)\left(1-\frac{\Gamma(k,\lambda_{N_Z})}{\Gamma(k)}\right)\right].$$

Model with learning



$$S_i = \begin{cases} S_{i-1} + 1, \text{if occured } a, \\ 0, \text{if occured } \bar{a} \text{ (didn't occur } a), \end{cases} \qquad S_0 = 0.$$

Estimations for advanced models

	$k = 3$ Critical q_1	$k = 5$ Critical q_1	$k = 10$ Critical q_1
The model with stimulation $\varepsilon = 0.05$	0.464	0.462	0.461
The model with training $\delta = 0.1$	0.477	0.466	0.461
The model with stimulation $\varepsilon = 0.05$	0.464	0.462	0.461
The model with training $\delta = 0.2$	0.497	0.473	0.461
The model with stimulation $\varepsilon = 0.05$	0.464	0.462	0.461
The model with training $\delta = 0.3$	0.522	0.485	0.462
The model with stimulation $\varepsilon = 0.05$	0.464	0.462	0.461
The model with training $\delta = 0.4$	0.554	0.504	0.465

Effectiveness of different trading strategies for price-takers: analysis via computational simulation

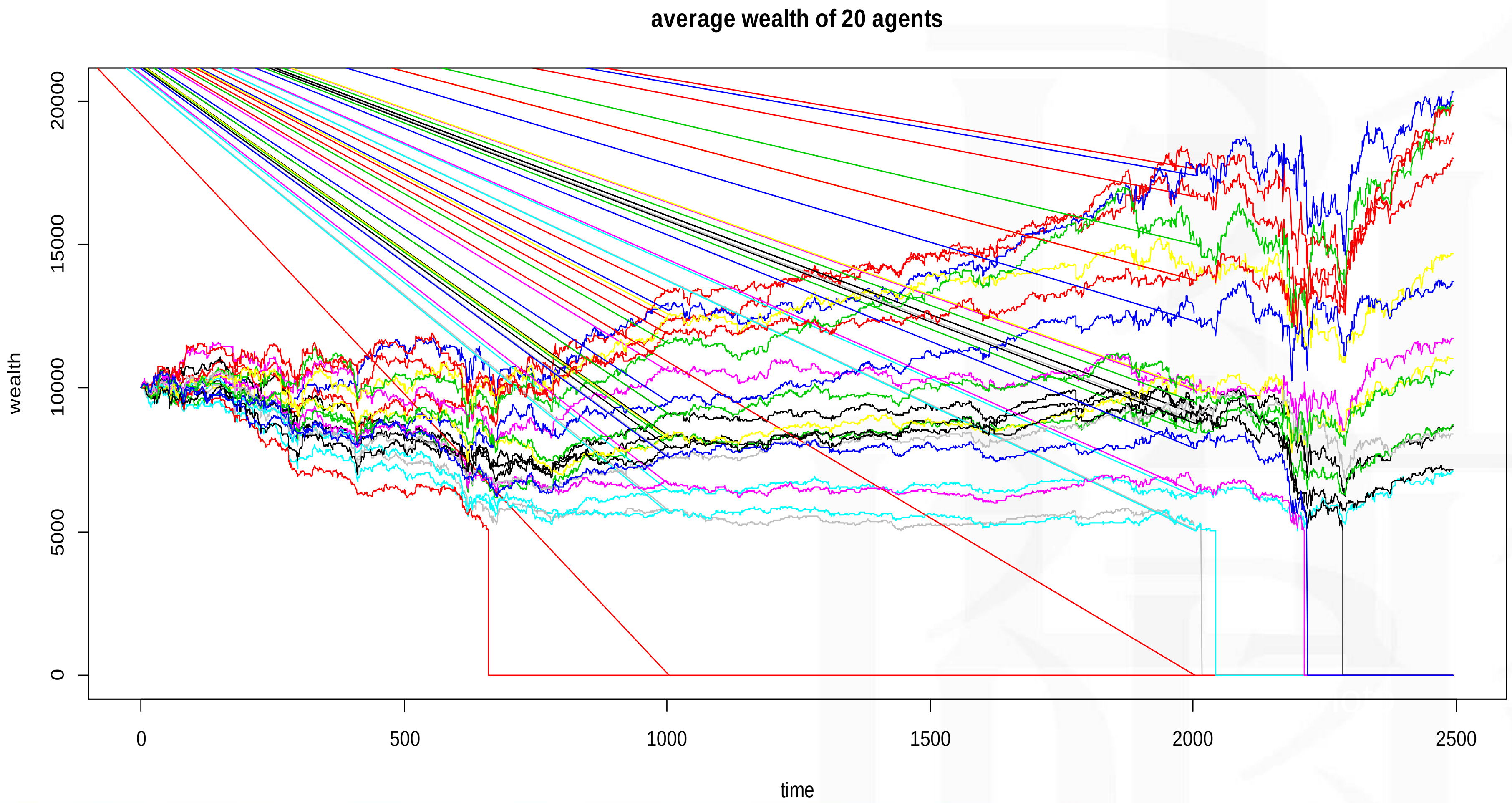
Description of models

- Market description and price formation
  - Market orders
  - Initial wealth allocation
  - Capital reference level
  - Margin trading with leverage rate 1:2, 1:5, 1:10

Description of models

- Traders and their strategies
  - All of them are price-takers
  - Basic characteristic: p is a probability of correct prediction of the price movement on the next day
  - Follower strategy
  - ‘Black swan seeker’ strategy

Results for the basic model



Results for the basic model

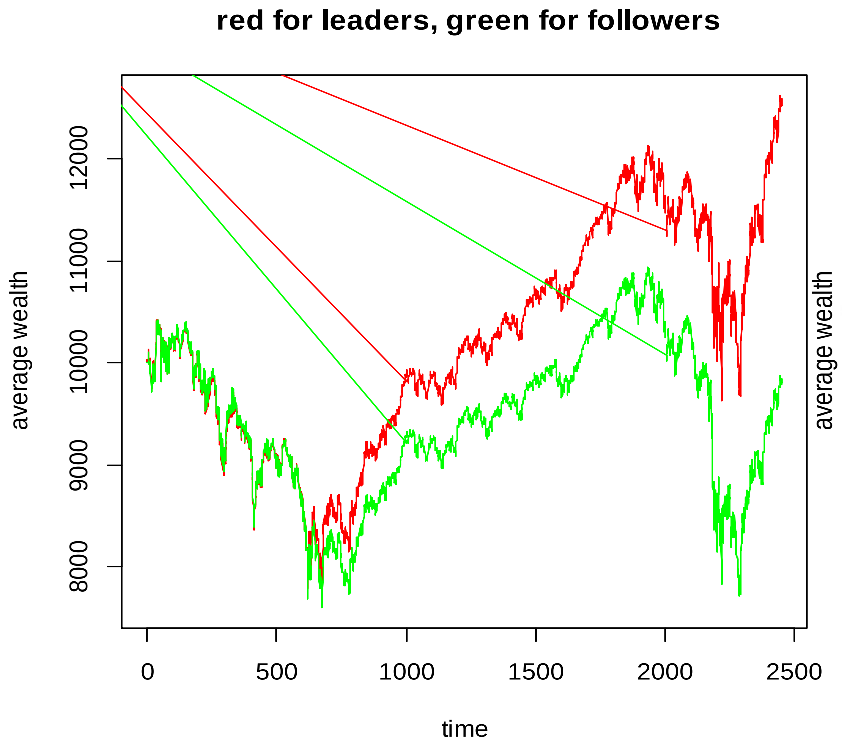
		The fraction of bankrupts,%			
	p	Leverage=0	Leverage=2	Leverage=5	Leverage=10
1	0.50	6.2	52.2	88.5	98.6
2	0.51	2.7	37.5	77.6	96.2
3	0.52	0.9	20.7	60.6	90.8
4	0.53	0.2	10.2	42.4	81.4
5	0.54	0.0	5.0	30.3	70.5
6	0.55	0.0	2.4	21.4	59.8
7	0.56	0.0	1.0	13.7	49.3
8	0.57	0.0	0.7	10.9	40.8
9	0.58	0.0	0.5	7.3	32.4
10	0.59	0.0	0.0	5.1	26.5
11	0.60	0.0	0.0	2.7	21.8
12	0.61	0.0	0.0	1.8	17.9
13	0.62	0.0	0.0	1.5	15.5
14	0.63	0.0	0.0	1.4	13.3
15	0.64	0.0	0.0	0.5	9.6
16	0.65	0.0	0.0	0.5	7.0

Results for the basic model

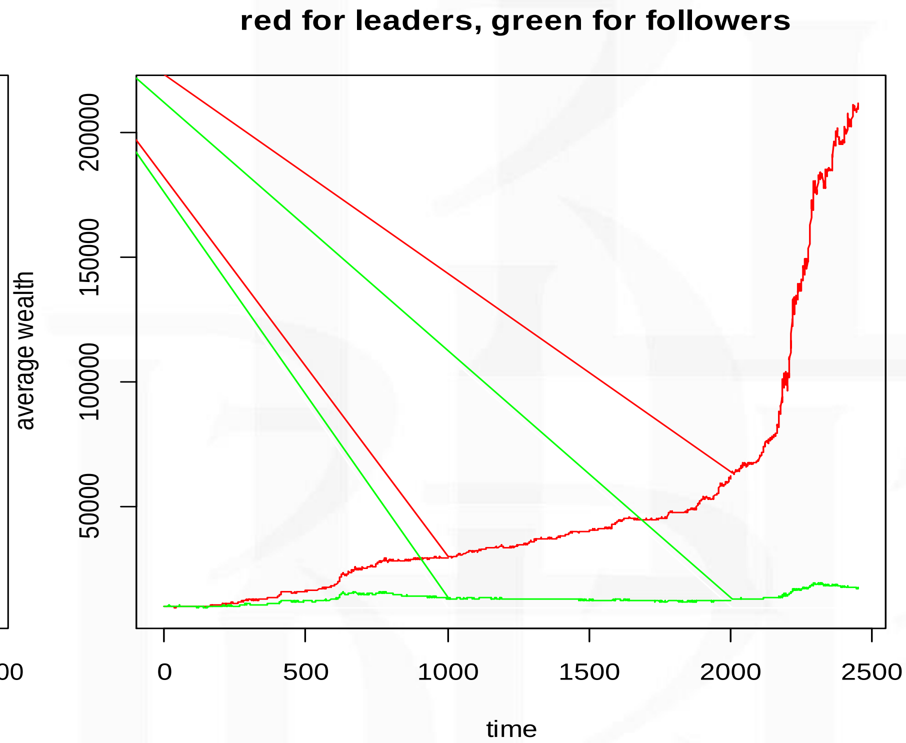
		Leverage=0					
		S&P	CAC	DAX	FTSE	Nikkei	HS
1	Average wealth of not-bankrupts	9 584	9 464	11 098	9 667	8 929	12 719
2	The fraction of agents with final wealth greater than initial wealth,%	35.35	28.05	45.35	37.36	18.83	71.12
3	The fraction of bankrupts,%	6.17	19.58	23.17	3.95	24.09	5.16
		Leverage=2					
1	Average wealth of not-bankrupts	14 131	19201	17 348	14 110	15 637	19 606
2	The fraction of agents with final wealth greater than initial wealth,%	29.80	20.44	25.73	29.25	18.87	34.99
3	The fraction of bankrupts,%	52.21	72.06	65.20	54.40	71.95	55.87
		Leverage=5					
1	Average wealth of not-bankrupts	43 427	64 072	66 643	44 156	56 847	61 566
2	The fraction of agents with final wealth greater than initial wealth,%	10.31	5.47	7.67	10.35	5.51	9.06
3	The fraction of bankrupts,%	88.54	93.95	91.16	88.12	93.66	89.86
		Leverage=10					
1	Average wealth of not-bankrupts	2e+05	3e+05	2e+05	3e+05	5e+05	3e+05
2	The fraction of agents with final wealth greater than initial wealth,%	1.32	0.21	0.67	1.26	0.47	0.89
3	The fraction of bankrupts,%	98.62	99.79	99.27	98.63	99.51	99.09

Results for the ‘follower’ strategy

Leverage = 0
$p \sim R[0.4; 0.6]$



Leverage = 5
$p \sim R[0.4; 0.6]$

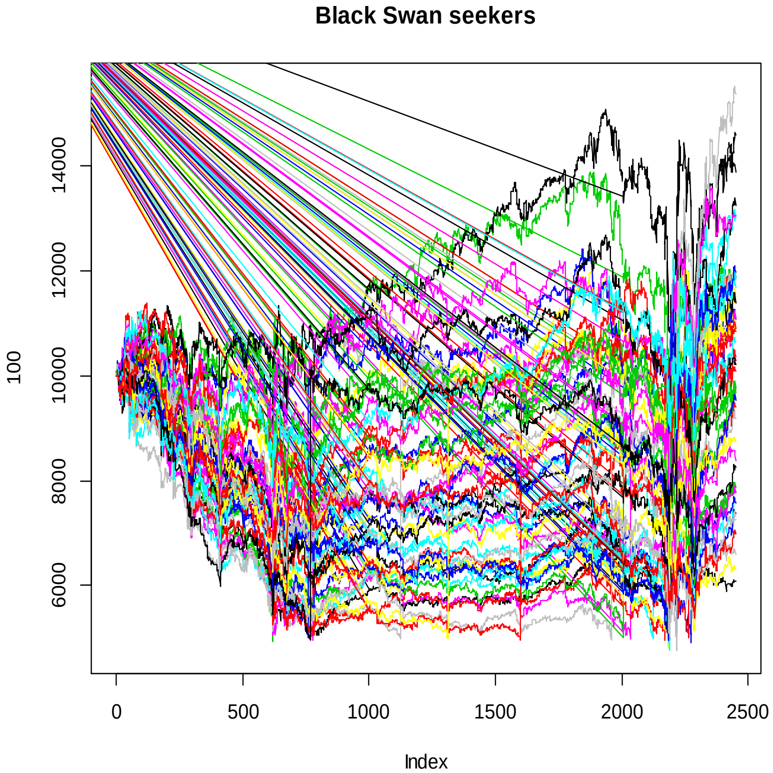


Value of leader's p	*leverage=0* Average wealth of follower	Better wealth of follower	Bankrupts among followers	*leverage=2* Average wealth of follower	Better wealth of follower	Bankrupts among followers	*leverage=5* Average wealth of follower	Better wealth of follower	Bankrupts among followers	*leverage=10* Average wealth of follower	Better wealth of follower	Bankrupts among followers
0.44	10 336	25.3	1.4	16 845	40.9	42.7	49 738	12.7	84.5	59 670	1.2	98.8
0.45	10 131	23.1	1.5	17 378	32.7	48.2	52 434	18.9	79.1	163 849	1.6	98.64
0.46	9 926	22.2	1.9	16 780	44.5	45.5	50 334	17.3	80.9	230 542	1.4	98 .6
0.47	9 970	21.2	2.1	15 188	38.2	44.5	85 579	8.3	89.1	18 023	1.3	98.7
0.48	9 946	19.8	2.3	15 383	28.2	59.1	37 389	14.8	83.6	59 015	0.7	99.3
0.49	9 603	19.3	2.5	13 030	27.3	60.9	65 388	19.1	79.1	29 822	1.9	98.1
0.50	9 590	17.5	3.2	13 006	31.8	50.8	39 833	10.6	87.3	28 308	2.8	97.2
0.51	9 559	17.0	3.1	12 995	28.2	48.2	52 156	16.1	82.7	18 291	1.5	98.5
0.52	9 408	15.8	3.6	13 141	25.5	54.5	39 057	8.5	87.3	-	0	100
0.53	9 443	13.9	4.4	12 949	21.8	58.2	87 119	4.5	95.5	14 227	0.1	99.9
0.54	9 050	12.9	4.7	12 185	16.4	67.3	36 900	10.9	88.2	-	0	100
0.55	8 839	11.7	5.1	12 551	20.9	64.5	28 826	7.3	90.9	639 577	0.9	99.1
0.56	9 012	11.7	6.0	12 314	14.5	66.4	18 045	2.4	96.4	-	0	100
0.57	9 069	10.0	6.8	11 164	13.6	73.6	16 397	6.4	90.9	55 243	1.4	98.6
0.58	8 592	9.2	7.5	10 527	10.0	70.0	95 561	5.9	93.6	18 908	2.1	97.9
0.59	8 414	8.1	8.1	10 961	8.2	81.8	28 376	1.8	97.3	-	0	100
0.60	8 590	7.4	9.0	11 061	12.7	76.4	16 725	2.7	95.5	75 147	2.2	97.8

Results for the Black Swan seekers' strategy

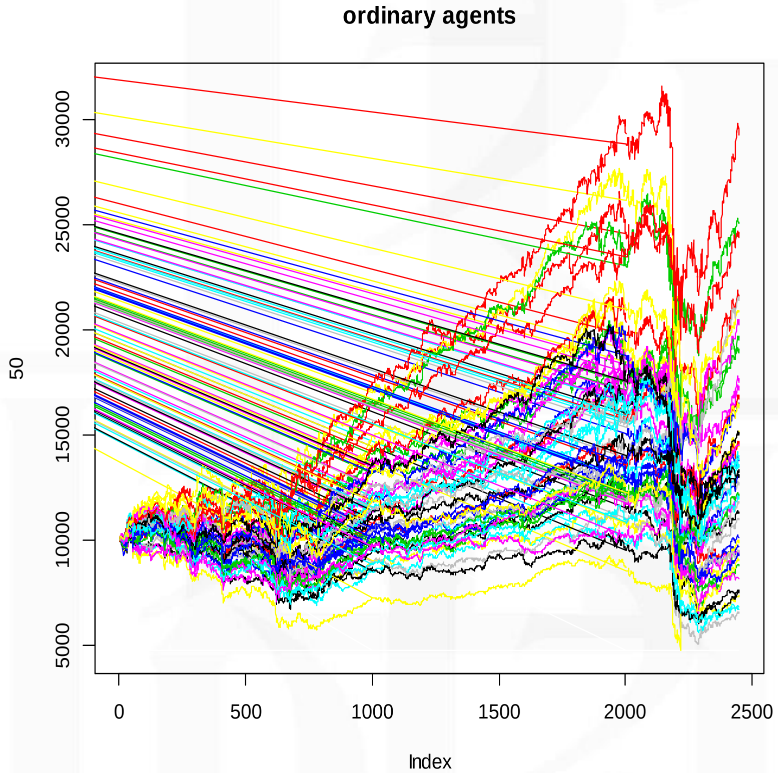
Black swan seekers

$p^{Q}_{sign} \sim R[0.4;0.5]$ $p^{R}_{sign} \sim R[0.8;0.9]$



Ordinary agents

$p^{Q}_{sign} \sim R[0.5;0.6]$ $p^{R}_{sign} \sim R[0.1;0.2]$



Conclusion

We showed in a very simple model that with a small reward for the correct (with probability slightly higher than ½) identification of the routine events (and if crisis events are identified with very low probability) the average player's gain will be positive.

In other words, players do not need to play more sophisticated games, trying to identify crises events in advance.

Publications

- Aleskerov F. T., Egorova L. G. Is it so bad that we cannot recognize black swans? // Working papers by NRU Higher School of Economics, Series WP7 «Mathematical Methods of Decision Analysis in Economics, Business and Politics» 2010, WP7/2010/03.
- Aleskerov F. T., Egorova L. G. Black Swans and Stock Exchange // Higher School of Economics Economic Journal, 2010, 14(4), p. 492-506. (in Russian)
- Egorova L. G. Recognition of Stock Exchange processes as a Poisson Process of Events of Two Types: Models with Stimulation and Learning // Working papers by NRU Higher School of Economics, Series WP7 «Mathematical Methods of Decision Analysis in Economics, Business and Politics» 2011, WP7/2011/02.
- Aleskerov F. T., Egorova L. G. Is it so bad that we cannot recognize black swans? // Economics Letters, 2012, 117(3), p. 563-565.
- Egorova L. G.   The Effectiveness Of Different Trading Strategies of Small Traders // Control Sciences, 2014, 5, p. 34-41. (in Russian)
- Egorova L. G. The Effectiveness Of Different Trading Strategies For Price-Takers // Working papers by NRU Higher School of Economics, Series FE «Financial Economics», 2014, WP BRP 29/FE/2014

Thank you!

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