Government of Russian Federation

Federal State Autonomous Educational Institution of High Professional Education

«National Research University Higher School of Economics»

National Research University
High School of Economics
Faculty of Computer Science

Syllabus for the course
«Probability Theory and Mathematical Statistics»

010402.68 «Applied Mathematics and Informatics»
«Data Sciences», Master Program

Authors:
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Approved by:

Recommended by:
1. Teachers

**Author, assistant professor:** Geoffrey G. Decrouez, National Research University Higher School of Economics, Department of Computer Science.

2. **Scope of Use**

The present program establishes minimum demands of students’ knowledge and skills, and determines content of the course.

The present syllabus is aimed at department teaching the course, their teaching assistants, and students of the Master of Science program 010402.68 «Applied Mathematics and Informatics».

This syllabus meets the standards required by:
- Educational standards of National Research University Higher School of Economics;
- Educational program «Data Sciences» of Federal Master’s Degree Program 010402.68, 2014;
- University curriculum of the Master’s program in «Data Science» (010402.68) for 2014.

**Summary**

This subject develops the building blocks of probability theory that are necessary to understand statistical inference. The axioms of probability theory are reviewed, discrete and continuous random variables are introduced, and their properties are developed in the univariate and bivariate setting. In particular, we discuss the most common probability distributions that arise in statistical applications. We then introduce methods for computing the distribution of sums of random variables, talk about convergence of random variables (in probability and in distribution) before stating two major theorems of probability theory: the law of large numbers, and the central limit theorem. The last part of this course introduces basic concepts in statistics, including properties of estimators (consistency, efficiency), maximum likelihood estimators and their asymptotic normal properties, and confidence intervals.

3. **Learning Objectives**

The learning objective of the adaptation course «Probability Theory and Mathematical Statistics» is to provide students with essential tools in probability theory to understand the theory of statistics and their applications, such as

- Common univariate probability distributions and their meaning;
- Multivariate distributions;
- Asymptotic normal approximations;
- Maximum likelihood inference;
- Interval estimation;
4. Learning outcomes

After completing the study of the discipline «Probability Theory and Mathematical Statistics» the student should:

- Know the most widely used probability distributions and recognize them in applications.
- Know the main tools to describe a random variable, such as the probability density function, the cumulative distribution function, and the moment generating function.
- Recognize the importance of the central limit theorem and understand when it is appropriate to use normal approximations for the distribution of a statistic.
- Be able to derive maximum likelihood estimators.
- Be able to construct exact and approximate confidence intervals.
- Possess techniques of proving theorems and thinking out counter-examples.
- Learn to develop complex mathematical reasoning.

5. Place of the discipline in the Master’s program structure

The course «Probability Theory and Mathematical Statistics» is an adaptation course taught in the first year of the Master’s program «Data Science». It is recommended for all students of the Master’s program who do not have fundamental knowledge in probability theory at their previous bachelor/specialist program.

Prerequisites

The course is based on basic knowledge in probability theory, linear algebra and analysis. No special knowledge is required.

The following knowledge and competence are needed to study the discipline:

- A good command of the English language, both orally and written.
- A basic knowledge in probability theory.

Main competences developed after completing the study this discipline can be used to learn the following disciplines:

- Probability theory
- Elements of mathematical statistics.

After completing the study of the discipline «Probability Theory and Mathematical Statistics» the student should have the following competences:

<table>
<thead>
<tr>
<th>Competence</th>
<th>Code</th>
<th>Code (UC)</th>
<th>Descriptors (indicators of achievement of the result)</th>
<th>Educative forms and methods aimed at generation and development of the competence</th>
</tr>
</thead>
<tbody>
<tr>
<td>The ability to reflect developed methods of activity.</td>
<td>C-1</td>
<td>SC-M1</td>
<td>The student is able to reflect developed mathematical methods in probability and statistics.</td>
<td>Lectures and tutorials.</td>
</tr>
<tr>
<td>Competence</td>
<td>Code</td>
<td>Code (UC)</td>
<td>Descriptors (indicators of achievement of the result)</td>
<td>Educative forms and methods aimed at generation and development of the competence</td>
</tr>
<tr>
<td>---------------------------------------------------------------------------</td>
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<td>------------------------------------------------------------------------------------------------------------------------</td>
<td>----------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>The ability to propose a model to invent and test methods and tools of professional activity</td>
<td>C-2</td>
<td>SC-M2</td>
<td>The student is able to model randomness in real-life examples using common probability distributions, and perform statistical inference to estimate the model parameters.</td>
<td>Examples covered during the lectures and tutorials. Assignments.</td>
</tr>
<tr>
<td>Capability of development of new research methods, change of scientific and industrial profile of self-activities</td>
<td>C-3</td>
<td>SC-M3</td>
<td>Students obtain necessary knowledge in probability and statistics, sufficient to develop new methods in other disciplines.</td>
<td>Assignments, additional material/reading provided.</td>
</tr>
</tbody>
</table>

6. Schedule

Two pairs consist of 2 academic hour for lecture followed by 2 academic hour for tutorial after lecture.

<table>
<thead>
<tr>
<th>№</th>
<th>Topic</th>
<th>Total hours</th>
<th>Contact hours</th>
<th>Self-study</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Total</td>
<td>Lectures</td>
<td>Seminars</td>
</tr>
<tr>
<td>1.</td>
<td>Foundations of Probability Theory</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2.</td>
<td>Discrete random variables.</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3.</td>
<td>Continuous random variables.</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4.</td>
<td>Multivariate random variables.</td>
<td>8</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5.</td>
<td>Convergence.</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>6.</td>
<td>Properties of estimators and maximum likelihood estimation.</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>7.</td>
<td>Interval estimation and hypothesis testing.</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Total:</td>
<td><strong>108</strong></td>
<td><strong>16</strong></td>
<td><strong>16</strong></td>
</tr>
</tbody>
</table>
Requirements and Grading

<table>
<thead>
<tr>
<th>Type of grading</th>
<th>Type of work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
<td>Optional test to withdraw from the subject: students with sufficient prior knowledge in probability theory need not attend this course.</td>
</tr>
<tr>
<td>Homework</td>
<td>Solving 3 homework tasks and examples.</td>
</tr>
<tr>
<td>Exam</td>
<td>Written exam. Preparation time – 180 min.</td>
</tr>
<tr>
<td>Final</td>
<td></td>
</tr>
</tbody>
</table>

9. Assessment

The assessment consists of three homeworks, handed out to the students throughout the semester. The homework problems are based on each lecture topics and have increasing complexity.

Final assessment is the final exam. Students have to demonstrate knowledge of probability and statistics theory.

The grade formula:

The exam will consist of 10 problems, giving ten marks each, worth 80% of the final mark.

Final course mark is obtained from the following formula: Final=0.2*(Homeworks)+0.8*(Exam).

The grades are rounded in favour of examiner/lecturer with respect to regularity of class and home works. All grades, having a fractional part greater than 0.5, are rounded up.

Table of Grade Accordance

<table>
<thead>
<tr>
<th>Ten-point Grading Scale</th>
<th>Five-point Grading Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - very bad</td>
<td>Unsatisfactory - 2</td>
</tr>
<tr>
<td>2 – bad</td>
<td></td>
</tr>
<tr>
<td>3 – no pass</td>
<td></td>
</tr>
<tr>
<td>4 – pass</td>
<td>Satisfactory – 3</td>
</tr>
<tr>
<td>5 – highly pass</td>
<td></td>
</tr>
<tr>
<td>6 – good</td>
<td>Good – 4</td>
</tr>
<tr>
<td>7 – very good</td>
<td></td>
</tr>
<tr>
<td>8 – almost excellent</td>
<td>Excellent – 5</td>
</tr>
<tr>
<td>9 – excellent</td>
<td></td>
</tr>
<tr>
<td>10 – perfect</td>
<td></td>
</tr>
</tbody>
</table>

FAIL

PASS
10. Course Description
The following list describes main mathematical definitions which will be considered in the course in correspondence with lecture order.

Topic 1. Foundations of probability theory.
Sample space, sigma algebra. Axioms of probability theory. Conditional probability, independent events. Baye’s theorem.

Topic 2. Discrete random variables
Definition of a random variable, Probability mass functions and probability distribution functions, Bernoulli trials and related distributions, Poisson distribution, Uniform distribution, Mean and variance.

Topic 3. Continuous random variables
Definition of a continuous random variable, Probability density function. Summarising a continuous RV, uniform, exponential and gamma distributions, normal distribution. Skewness and tail thickness. Distribution of functions of a continuous random variable.

Topic 4. Multivariate random variables

Topic 5. Convergence

Topic 6. Elements of statistical inference

11. Term Educational Technology
The following educational technologies are used in the study process:
- discussion and analysis of the results during the tutorials;
- solutions of exercises are posted on the subject website for the student to practice;
- regular assignments to test the progress of the student;
- consultation time on Monday mornings.

12. Recommendations for course lecturer
Course lecturer is advised to use interactive learning methods, which allow participation of the majority of students, such as slide presentations, combined with writing materials on board, and usage of interdisciplinary papers to present connections between probability theory and statistics. The course is intended to be adaptive, but it is normal to differentiate tasks in a group if necessary, and direct fast learners to solve more complicated tasks.
13. Recommendations for students
The course is interactive. Lectures are combined with classes. Students are invited to ask questions and actively participate in group discussions. There will be special office hours for students, which would like to get more precise understanding of each topic. The lecturer is ready to answer your questions online by official e-mails that you can find in the “contacts” section. Additional references found in section 15.1 are suggested to help students in their understanding of the material. This course is taught in English, and students can ask teaching assistants to help them with the language.

14. Final exam questions
The final exam will consist in ten questions equally weighted. No material is allowed for the exam. Each question will focus on a particular topic presented during the lectures. The first question of the exam will ask the students to prove a result or a theorem proved during the class, and will be one of the following:

1. Derivation of the Poisson distribution from the binomial distribution.
2. Derivation of the mean and variance of the binomial, geometric and/or the Poisson distribution.
3. Prove the expectation formula for non-negative continuous random variables give in remark (ii) page 14 of Part 3 of the lecture slides.
4. Derivation of the exponential distribution from the binomial distribution.
5. Derivation of the gamma distribution from the binomial distribution.
6. Computing the n-th moment of a standard normal random variable.
7. Prove Theorem 1 page 28 of Part 4 of the lecture notes.
8. Give a geometrical interpretation of the conditional expectation, and show that this geometrical interpretation coincides with standard definition given 34 of Part 4 of the lecture notes.
9. Prove the law of large numbers and the central limit theorem using moment generating functions.
10. Prove the asymptotic normal property of maximum likelihood estimators.
11. Derivation of the student-t distribution.

The remaining nine questions consist in exercises on any topic seen during the lectures. To be prepared for the final exam, students must be able to solve questions from the problem sheets 1 to 8, and questions from the three assignments.

15. Reading and Materials
The adaptation course is intended to fill the gaps in the student’s knowledge in probability theory. As such, we do not follow a particular textbook in this subject, but the student may find the following references useful

15.1. Recommended Reading


15.2. Course webpage

All material of the discipline are posted on [http://www.ami.hse.ru/ptms](http://www.ami.hse.ru/ptms)

Students are provided with links to the lecture notes, problem sheets and their solutions, assignments and their solutions, and additional readings.

16. Equipment

The course requires a laptop and projector.

Lecture materials, course structure and the syllabus are prepared by Geoffrey Decrouez.