Electoral Bargaining in Runoff Elections

An Analysis from Cooperative Game Theory and an Application to the French 2010 Regional Elections

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February 17, 2015

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Motivation

- An exploration in coalitional politics motivated by the French regional and municipal elections (but with a possibly larger scope of application)
- Elections of regional and municipal councils: Two round elections, proportional representation with a "winner premium", and *possibility of merging lists between the two rounds* (under some conditions).
- Our primary objective: Try and account for how parties negociate/bargain between the two rounds of such an election
- To this end, we build a model of bargaining in such a setting, where the analytical/structural framework we use is based on cooperative game theory, and derive its implications
- Constrast these predictions to actual outcomes observed in the 2010 French regional elections

(Related Literature on Coalitional Politics)

- **Pre-Electoral** Coalitional Politics
- **Post-Electoral** Coalitional Politics: Gamson, bargaining (Laver and Schofield (1998))
- Our paper is about **Interim** Electoral Coalitional Politics: we study how coalitions form between two rounds of an election. We are (we think) the first paper looking at that question. Nice setting because it allows us to

- Have good measure of bargaining powers (1rst round results)
- Observe detailed outcomes of the bargaining process (2nd round lists)

Road Map

- The Electoral Setting
- The Cooperative Game
- The French 2010 Regional Elections: Three examples
- WIP Further Empirical Developments (with Nicolas Sauger)

The Electoral Environment

- K : Number of seats (Regional Council, Municipal Council, ...)
- N : Number of voters.
- The electoral rule: A two round election, proportional representation with a "winner premium", and possibility of merging lists between the two rounds (under some conditions).

The first round

- P^1 lists compete. A ballot consists of an ordered list of K names. A voter cannot mix, which means that there are P^1 valid ballots in the first round.
- *If a list obtains the absolute majority of the valid ballots*, then the election is over.

- βK **seats are allocated proportionally** among the lists whose relative score (% of valid ballots) is larger than a given threshold $\underline{\alpha}$.
- **The remaining** $(1-\beta)K$ **seats are allocated as a premium** to the list with the highest number of votes.
- Attribution of seats within a list: If a list obtained k seats, **the seats are allocated to the top** k **candidates on the list**.

- *If no list obtains an absolute majority,* a second round is organized.

Between the two rounds

- **Can a 1st round list participate in the second round?** The rule is described by two thresholds $\underline{\alpha}$ and $\overline{\alpha}$, with $0 < \underline{\alpha} < \overline{\alpha} < 1$.

- *If its relative score below $\underline{\alpha}$:* NO
- *If its relative score between $\underline{\alpha}$ and $\overline{\alpha}$:* it cannot participate alone but can merge with other lists, if at least one of these lists has a score larger than $\overline{\alpha}$.
- *If its relative score above $\overline{\alpha}$:* YES (no other condition)

- Let M^2 be the number of lists with a score larger than $\underline{\alpha}$.
- **When two or more 1rst round lists merge together**, they must agree on an ordered list of K names, where the names have to come from 1rst round lists.

The second round

- Following possible alliances between the two rounds among the M^2 first-round lists with a score larger than $\underline{\alpha}$, P^2 (coalitional) lists participate to the second round.
- After the second round is held

- βK **seats are allocated proportionally** among the lists whose relative 2nd round score is larger than a given threshold $\underline{\alpha}$
- **The remaining** $(1-\beta)K$ **seats are allocated as a premium** to the list with the highest number of votes.
- Attribution of seats within a list: If a list obtained k seats, **the seats are allocated to the top** k **candidates on the list.**

Parameters

- French Regional Elections (2004, 2010): $\beta = 75\%$, $\underline{\alpha} = 5\%$, $\overline{\alpha} = 10\%$ (7% for Corsica)
- French Municipal Elections: $\beta = 50\%$, $\underline{\alpha} = 5\%$, $\overline{\alpha} = 10\%$.

Two questions

- **Question 1 (Coalition Formation):** Which coalitions among the M^2 players are going to form between the two rounds?
- **Question 2 (Sharing the Gains from Cooperation):** Within a coalition, what is the number of candidates and their rank on the coalitional list?

To answer these questions, we introduce a framework based on cooperative game theory

Several steps to construct the cooperative game

1. Define the admissible coalitions
2. Define the gains of a coalition
3. Make some assumptions about voters behavior in the second round
4. Construct the characteristic function of the game

Citations from Maskin

"That cooperative game theory should be in relative eclipse is regrettable because *this body of work offers us the opportunity to understand how coalitions behave*, i.e. how subsets of players bargain over their choice of actions. *Such bargaining seems basic to many aspects of economic and political life*" (2003)
"Mainstream economists make comparatively little use of cooperative game theory...*A shame, as cooperative game theory is a beautiful subject, potentially very important*...Most surprising thing about the absence of cooperative theory : coalitions play crucial role in many important real phenomena...For example, economic mergers,...even *political parties*. Non cooperative game theory can be used to study these phenomena too but then it is tied to a particular extensive or normal form. *Cooperative game theory has potential for a more general perspective*" (2004)
Whom is the author of these two assertions ?

Admissible Coalitions

- Set of players: the M^2 lists above $\underline{\alpha}$: $\{1, 2, ..., M^2\}$
- We assume that some coalitions might be infeasible due, among other things, to ideological barriers.
- We denote by $\mathbb{C}$ the set of admissible coalitions i.e. those considered by the players as conceivable.
- We assume the following particular structure: **There exists J political families such that: within each such family, any alliance is feasible and no alliance is feasible across different families.**
- Formally: There exists a partition $\mathbb{F} = \{F_1, ..., F_J\}$ of $\{1, 2, ..., M^2\}$ such that a coalition S is admissible iff there exists $j \in \{1, 2, ..., J\}$ such that $S \subset F_j$.
- Note in particular that in the case where $J = 1$, all coalitions are admissible.

Gains of a Coalition

- Let $S \in \mathbb{C}$ be an admissible coalition. It must evaluate its aggregate gain if it forms.
- We assume this gain is the number of seats allocated to the coalition.
- It will be denoted $V(S)$.The function V is the *characteristic function of the game*.
- To compute this value, coalition S needs to make some anticipations about what players outside S will do, and also on how voters are going to vote in the second round.

Assumptions about voter behavior in second round

We make the following assumptions about voters behavior: whatever the lists they face in the second round:

- Voters who abstained in the first round will abstain in the second round
- Those who voted for a list who is present in the second round (possibly through the participation to a coalitonal list) will vote for the same (coalitional) list
- Those who voted for a list which is absent in the second round will abstain.
  [Remark: we are currently working on an econometric estimation on these electoral mobility matrices]

Anticipated number of votes

- Under these assumptions about voter behavior, **the number of votes a coalition expects to receive in the second round does not depend on what lists outside this coalition do**.
- Let us denote by $N(S)$ the number of votes S will receive if it forms.
- If none of the members in S have reached the threshold of $\overline{\alpha}$, then $N(S) = 0$ (S cannot participate)
- If S can participate, $N(S)$ is simply the sum of the first round votes obtained by the members of S.
  Note that the anticipated number of votes of S does not depend on the partition π that forms out of S, but the number of seats S gets does.

The Partition Function

The coalition S can evaluate the number of seats $V(S,\pi)$ that it will get if some (admissible) partition $\pi = \{T_1, ..., T_L\}$ forms among other players:

- If $N(S) > N(T_l)$ for all $l = 1, ..., L$:

$$V(S,\pi) = \frac{N(S)}{N(S) + \sum_{l=1}^{l=L} N(T_l)} \beta K + (1-\beta)K,$$

- Otherwise, coalition S gets:

$$V(S,\pi) = \frac{N(S)}{N(S) + \sum_{l=1}^{l=L} N(T_l)} \beta K.$$

The Characteristic Function

- We now derive from this description of the game in partition form a description of the cooperative game in characteristic form, as typically done in cooperative game theory (after Aumann and others this form is often called the α form):
- **We define $V(S)$ as the smallest value of $V(S, \pi)$ with respect to π:**

$$V(S) = \min_{\pi \in \Pi(S)} V(S, \pi).$$

- Note that the worst case scenario for S is the situation where the maximal possible alliances form outside S. This allows us to compute the explicit formula for $V(S)$ (as a function of first round numbers of votes)

Superadditivity of the Game

Under our assumptions about voting behavior in the second round, one can check that **the game is superadditive**:
For all $S_1, S_2 \in \mathbb{C}$ such that $S_1 \cup S_2 \in \mathbb{C}$ and $S_1 \cap S_2 = \varnothing$:

$$V(S_1 \cup S_2) \geq V(S_1) + V(S_2).$$

Intuition: The characteristic function if the addition of two characteristic functions: the *proportional component* and the *premium component*:
For the premium component: straightforward.
For the proportional component: assume for simplicity $S_1 \cup S_2 = F_j$. If both S_1 and S_2 can present a list on their own, additivity; if only one of them can (say S_1):

$$\frac{N(S_1) + \sum_{m \in S_2} N_m^1}{N(S_1) + \sum_{m \in S_2} N_m^1 + \sum_{l \neq j} N(F_l)} > \frac{N(S_1)}{N(S_1) + \sum_{l \neq j} N(F_l)} + 0,$$

where N_m^1 is the nb of votes of 1st-round list m in the 1st-round.

Solutions

Question 1 (Coalition Formation): Our paper does not provide answer to coalition formation. We assume in each cell of $\mathbb{C}$, the greatest coalition forms.

Question 2 (Sharing the Gains from Cooperation): We will examine three solutions:

- *Bargaining solutions* :Shapley Value and Nucleolus
- *Gamson solution* : within a coalition S the gains are shared according to the number of votes N_m^1 obtained in the first round

The French Regional Elections of March 2010

- 1st round: March 14, 2010; 2nd round: March 21, 2010
- Main players in the first round (in the 21 French metropolitan regions):

Extreme Left (Front de Gauche:PCF+PG)
Greens (Europe Ecologie)
Socialist Party (PS+MRG+MDC)
Center (MODEM)
Right (UMP)
Extreme Right (Front National)

In some regions, pre-electoral alliances between PS and FG.

- The admissible coaltions: Given the French political landscape in 2010,

$$\mathbb{C} = \{\{PS, EE, FG\}, \{MODEM\}, \{UMP\}, \{FN\}\}.$$

A few Facts

Among the 21 French metropolitan regions

- In 17 regions, the three left parties presented separated different lists in the first round.
- In the remaining four regions, PS and FG concluded a pre-electoral coalitional agreement.
- In 1 region (Languedoc), PS, Greens and FG all got less than 10% (G. Frêche as a dissident). In all other regions, PS (and a possible preelectoral ally) was the biggest list on the Left in the first round.
- The grand coalition fails to form in 4 regions (Bretagne, Limousin, Languedoc, Picardie).

Narrative Evidence: Brittany

After the first round, FG below 5%, both PS and EE above 10%. Leaders Guy Hascoët (Greens) and Jean-Yves Le Drian (PS) have conflicting grievances (Le Télégramme, 03/17/10)

GH.—We have been humiliated by the president's arrogance, who offered us no more than 10 seats, while the proportionality rule entitled us to 14, even 15. This is thumbing your nose at democracy.

(...)

JYLD.—9 seats for EEB, this is the result after the allocation of the bonus of 25% of the seats to the leading list, meaning ours. No, it is out of the question to share the bonus. In cycling, the stage winner shares the bonus with his teammates, not with the teammates of the second.

(...)

JYLD.—During the negotiations, we spent about fifteen minutes on our project and they deemed it to be compatible as it was. The only issue were the seats, the seats, and again the seats!

Brittany: Facts

- $K = 83$
- *First Round:* $P^1 = 11$, $N = 2\ 332\ 945$.

In decreasing order:
PS: $N^1_{PS} = 408\ 551$ (37.19%),
UMP: $N^1_{UMP} = 260\ 731$ (23.73%),
EE: $N^1_{EE} = 134\ 161$ (12.21%),
FN (6.18%) and MoDem (5.36%), all other (including FG) below 5%.

- *Second round:* $M^2 = 5$ and $P^2 = 3$

PS: 50.27%, UMP: 32.36% and EE: 17.37%

- Observed Distribution of the 83 Seats: PS:52, UMP:20 and EE:11.

Brittany: Shapley and Nucleolus

Both PS and EE can participate on their own.
Given our assuptions about second round voting behavior, PS can get the premium even if no coalition is formed.

$$V(S) = \begin{cases} \beta \frac{N^1_{PS}+N^1_{EE}}{N^1_{PS}+N^1_{EE}+N^1_{UMP}} + (1-\beta) = 75,7\% \text{ if } S = \{PS, EE\}, \\ \beta \frac{N^1_{PS}}{N^1_{PS}+N^1_{EE}+N^1_{UMP}} + (1-\beta) = 63,1\% \text{ if } S = \{PS\}, \\ \beta \frac{N^1_{EE}}{N^1_{PS}+N^1_{EE}+N^1_{UMP}} = 12,5\% \text{ if } S = \{EE\}. \end{cases}$$

$$\begin{aligned} Sh(PS) &= Nu(PS) = V(\{PS\}) = 63,11\%, \\ Sh(EE) &= Nu(EE) = V(\{EE\}) = 12,5\%. \end{aligned}$$

Brittany: Gamson, Shapley and Nucleolus

Seats in percentage of the total number of seats obtained by PS-EE:

Party	Observed	Gamson	Shapley	Nucleolus
PS	82.5	75.3	83.5	83.5
EE	17.5	24.7	16.5	16.5

JYLD.—"No, it is out of the question to share the bonus. In cycling, the stage winner shares the bonus with his teammates, not with the teammates of the second."

A Typology of Bargaining situations

Our game identifies **two sources of potential superadditivity**:

- **Which lists need an alliance to participate in the second round?**
- **Which coalitions can obtain the premium?**

Two player games

- *PS, EE et FG on separate lists in the first round, FG below 5%*

Premium\2nd Round	Only PS above 10%	Both above 10%
PS gets the premium alone		Brittany Poitou-Charente
Coalition is necessary	Franche-Comté	Pays de la Loire
Coalition does not get the premium		Alsace

- *Preelectoral alliance between PS and FG*

Premium\2nd Round	Only PS above 10%	Both above 10%
PS gets the premium alone	Bourgogne Lorraine	Basse Normandie
Coalition is necessary	Champagne	

Three player games

- *PS, EE et FG run separate lists in the first round, all three of them above 5%*

Premium\2nd Round	1 above 10%	2 above 10%	3 above 10%
PS gets the premium alone	Aquitaine Haute-Normandie	Limousin Midi-Pyrénées Provence	Nord-PdCalais
PS only needs its weakest partner	Picardie	Centre Ile de France Rhône-Alpes	Auvergne

Remark: Out of the 17 possible patterns, only 6 show up.

Two extreme cases in the three player games

Hereafter, we focus on two regions:

- Aquitaine EE and FG both need PS to continue the race, and the PS does not need any of them to win the premium.
- Auvergne: EE and FG do not need the PS to continue the race, and the PS needs one of the two partners to win the premium.

Auvergne (Weak PS): Facts

- $K = 47$
- *First Round:* $P^1 = 8$ and $N = 994160$

In decreasing order:
UMP: $N^1_{UMP} = 137\ 232$ (28.72%),
PS: $N^1_{PS} = 133\ 925$ (28.02%),
FG: $N^1_{FG} = 68\ 146$ (14.26%),
EE: $N^1_{EE} = 51\ 106$.(10.69%)
FN (8.39%) and all the other below 5%.

- *Second round:* $M^2 = 5$ and $P^2 = 2$

Left coalition: 59.68% and UMP: 40.32%

- Seats: Left coalition: 33, UMP: 14 How the 33 seats have been distributed between the three members of the coalition?

Observed: PS: 17, EE:9, and FG:7.

Auvergne: Characteristic Function

$$V(S) = \begin{cases} \beta \frac{N^1_{PS}+N^1_{EE}+N^1_{FG}}{N^1_{PS}+N^1_{EE}+N^1_{FG}+N^1_{UMP}} + (1-\beta) = 73.6\% \text{ if } S = \{PS, EE, FG\} \\ \beta \frac{N^1_{PS}+N^1_{FG}}{N^1_{PS}+N^1_{EE}+N^1_{FG}+N^1_{UMP}} + (1-\beta) = 63.8\% \text{ if } S = \{PS, FG\} \\ \beta \frac{N^1_{PS}+N^1_{EE}}{N^1_{PS}+N^1_{EE}+N^1_{FG}+N^1_{UMP}} + (1-\beta) = 60.6\% \text{ if } S = \{PS, EE\} \\ \beta \frac{N^1_{EE}+N^1_{FG}}{N^1_{PS}+N^1_{EE}+N^1_{FG}+N^1_{UMP}} = 22.9\% \text{ if } S = \{FG, EE\} \\ \beta \frac{N^1_{PS}}{N^1_{PS}+N^1_{EE}+N^1_{FG}+N^1_{UMP}} = 25.7\% \text{ if } S = \{PS\} \\ \beta \frac{N^1_{FG}}{N^1_{PS}+N^1_{EE}+N^1_{FG}+N^1_{UMP}} = 13.1\% \text{ if } S = \{FG\} \\ \beta \frac{N^1_{EE}}{N^1_{PS}+N^1_{EE}+N^1_{FG}+N^1_{UMP}} = 9.8\% \text{ if } S = \{EE\} \end{cases}$$

Auvergne: Gamson, Nucleolus and Shapley

Party	Observed	Gamson	Shapley	Nucleolus
PS	51.5	52.9	57.6	68.9
FG	27.3	26.9	23.4	17.8
EE	21.2	20.2	19	13.3

Aquitaine (Strong PS): Facts

- $K = 85$
- *First Round:* $P^1 = 11$, $N = 2\ 280\ 634$

In decreasing order:
PS: $N^1_{PS} = 406\ 871$ (37.63%)
UMP: $N^1_{UMP} = 238\ 367$ (22.05%).
MoDem: 10.43%, EE 9.75, FN: 8.3%, FG: 5.95, all other below 5%

- *Second round:* $M^2 = 6$ and $P^2 = 3$

Left coalition: 56.33%, UMP: 28.01% and MoDem: 15.65%.

- Seats: Left coalition: 58, UMP: 17 and Modem: 10. How the 58 seats have been distributed between the three members of the coalition?

Observed: PS: 45, EE:10, and FG:3.

Aquitaine: Characteristic Function

$$V(S) = \begin{cases} \beta\frac{N^1_{PS}+N^1_{EE}+N^1_{FG}}{N^1_{PS}+N^1_{EE}+N^1_{FG}+N^1_{UMP}+N^1_{Modem}} + (1-\beta) = 71.6\% \text{ if } S = \{PS, EE, FG\} \\ \beta\frac{N^1_{PS}+N^1_{EE}}{N^1_{PS}+N^1_{EE}+N^1_{UMP}+N^1_{Modem}} + (1-\beta) = 69.5\% \text{ if } S = \{PS, EE\} \\ \beta\frac{N^1_{PS}+N^1_{FG}}{N^1_{PS}+N^1_{FG}+N^1_{UMP}+N^1_{Modem}} + (1-\beta) = 68.0\% \text{ if } S = \{PS, FG\} \\ 0\% \text{ if } S = \{EE, FG\} \\ \beta\frac{N^1_{PS}}{N^1_{PS}+N^1_{UMP}+N^1_{Modem}} + (1-\beta) = 65.3\% \text{ if } S = \{PS\} \\ 0 \text{ if } S = \{EE\} \\ 0 \text{ if } S = \{FG\} \end{cases}$$

Aquitaine: Gamson, Nucleolus and Shapley

Party	Observed	Gamson	Shapley	Nucleolus
PS	77.6	70.6	95.7	96
EE	17.2	18.3	2.7	2.5
FG	5.2.	11.2	1.6	1.5

Qualitative conclusion

- The Socialist party does not seem to use its strong bargaining position as much as it could
- Prevalence of the Proportionality Gamson Rule
- Still, evidence of some bargaining

- when the grand coalition fails to form
- even when it forms, moves from proportionality might be in the direction of the bargaining solutions

Work in Progress: Extend in several dimensions

- Study all regions for the 2004 and 2010 elections (in 1998, proportional rule)
- Take into account presidencies and vice-presidencies
- Do a better job when making assumptions about the transition matrix
- With Nicolas Sauger (Science Po Paris), and (probably) Thierry Magnac (TSE)
- Mathematics: Uncertainty, Integers, Partition form

Partition Function

Thrall et Lucas (1963)
Negative Externality
In the case of 3 players, the negative externality is relevant only for singletons. Consider the proportional game.Three cases are possible
If the three parties can continue the race no externality as the game is additive
If only the PS can continue the race, no externality either
If the two largest parties can continue the race without alliance, then the externality matters

Partition Function

$$V(S) = \begin{cases} 0 \text{ si } S = \{3\} \\ \frac{N_1}{N} \text{ si } S = \{1\} \text{ et } \pi_{-1} = \{\{2,3\}\} \\ \frac{N_1}{N-N_3} \text{ si } S = \{1\} \text{ et } \pi_{-1} = \{\{2\},\{3\}\} \\ \frac{N_2}{N} \text{ si } S = \{2\} \text{ et } \pi_{-1} = \{\{1,3\}\} \\ \frac{N_2}{N-N_3} \text{ si } S = \{2\} \text{ et } \pi_{-1} = \{\{1\},\{3\}\} \\ \frac{N_1+N_3}{N} \text{si } S = \{1,3\} \\ \frac{N_2+N_3}{N} \text{ si } S = \{2,3\} \\ \frac{N_1+N_2}{N-N_3} \text{ si } S = \{1,2\} \\ \frac{N_1+N_2+N_3}{N} \text{ si } S = \{1,2,3\} \end{cases}$$

Partition Function

What is the equivalent of the Shapley solution in the case of a game in partition form ?
Myerson (1977), Bolder (1989), Pham Do and Nolde (2007)...
Maskin (2003) has proposed an extension based on a non cooperative sequential bargaining game
A family of "Shapley like solutions has been introduced by De Clippel et Serrano (2008a, b). They demonstrate that if σ is a solution satisfying anonymity, efficiency and monotonicty, then for any player $i \in N$ $\sigma_i(V) \in [\mu_i(V), \nu_i(V)]$ where the bounds are values of two
linear programs. For three players, they show that:

$$\nu_i(V) = \sigma_i^*(V) + \frac{Max\{0, \varepsilon_i(V) - \varepsilon_j(V)\} + Max\{0, \varepsilon_i(V) - \varepsilon_k(V)\}}{6}$$

Partition Function

$$\mu_i(V) = \sigma_i^*(V) - \frac{\begin{array}{c} Max\{0, \varepsilon_j(V) - \varepsilon_i(V)\} + \\ Max\{0, \varepsilon_k(V) - \varepsilon_i(V)\} \end{array}}{6}$$

$$\varepsilon_i(V) = V(\{i\}, \{\{i\}, \{j, k\}\}) - V(\{i\}, \{\{i\}, \{j\}, \{k\}\})$$

is the externality index of player i and σ^* is the unique "Shapley value" satisfying marginality called "externality-free value".

Partition Function

They show that is the (ordinary) Shapley value of the cooperative game defined by the characteristic function $V^*(S) = V(S, \left\{S, \{j\}_{j \in N \setminus S}\right\}$.

$$\varepsilon_1(V) = \frac{N_1}{N} - \frac{N_1}{N - N_3} = -\frac{N_1 N_3}{N(N - N_3)}$$

$$\varepsilon_2(V) = -\frac{N_2 N_3}{N(N - N_3)}$$

$$\varepsilon_3(V) = 0$$

Partition Function

$$V^*(S) = \begin{cases} 0 \text{ si } S = \{3\} \\ \frac{N_1}{N-N_3} \text{ si } S = \{1\} \\ \frac{N_2}{N-N_3} \text{ si } S = \{2\} \\ \frac{N_1+N_3}{N} \text{si } S = \{1,3\} \\ \frac{N_2+N_3}{N} \text{ si } S = \{2,3\} \\ \frac{N_1+N_2}{N-N_3} \text{ si } S = \{1,2\} \\ \frac{N_1+N_2+N_3}{N} \text{ si } S = \{1,2,3\} \end{cases}$$

Partition Function

V^* is different from the V that we have constructed. We obtain:

$$
\begin{aligned}
\sigma_1^*(V) &= \frac{N_1}{N} + \frac{1}{6}\frac{N_3}{N}\left[1 + \frac{3N_1}{N - N_3}\right] \\
\sigma_2^*(V) &= \frac{N_2}{N} + \frac{1}{6}\frac{N_3}{N}\left[1 + \frac{3N_2}{N - N_3}\right] \\
\sigma_3^*(V) &= \frac{2N_3}{3N} - \frac{1}{2}\frac{N_3}{N}\left[\frac{N_1 + N_2}{N - N_3}\right]
\end{aligned}
$$

Partition Function

While in our paper, we obtain:

$$
\begin{aligned}
Sh_1(V) &= \frac{N_1}{N} + \frac{1}{6}\frac{N_3}{N}\left[1 + \frac{N_1 + N_2}{N - N_3}\right] \\
Sh_2(V) &= \frac{N_2}{N} + \frac{1}{6}\frac{N_3}{N}\left[1 + \frac{N_1 + N_2}{N - N_3}\right] \\
Sh_3(V) &= \frac{2N_3}{3N} - \frac{1}{3}\frac{N_3}{N}\left[\frac{N_1 + N_2}{N - N_3}\right]
\end{aligned}
$$

Partition Function

We observe that $Sh_3(V)$ corresponds to the upper bound of DCS cube while $Sh_1(V)$ corresponds to the lower bound. The value $Sh_2(V)$ is in between the two bounds. For all values, the third player recovers at most $\frac{2}{3}$of its share. In practice, the difference between the values turn to be very small.

Coalition Formation

Maskin (2003): Negative externality + three players+ superadditivity $\Rightarrow$ the grand coalition forms
Halafir (2007): more contrasted

Coalition Formation

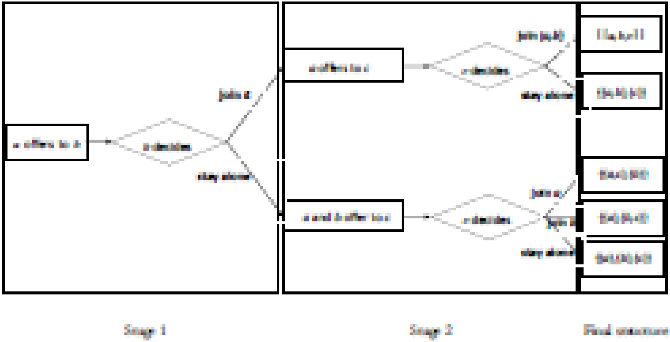

Figure 1: The Maskin bargaining model

The Partition Form of Limousin

$$V_{Limou}(S) = \begin{cases} 0 \text{ if } S = \{3\} \\ 0.448 \text{ if } S = \{1\} \text{ and } \pi_{-1} = \{\{2,3\}\} \\ 0.505 \text{ if } S = \{1\} \text{ and } \pi_{-1} = \{\{2\},\{3\}\} \\ 0.154 \text{ if } S = \{2\} \text{ and } \pi_{-1} = \{\{1,3\}\} \\ 0.173 \text{ if } S = \{2\} \text{ and } \pi_{-1} = \{\{1\},\{3\}\} \\ 0.562 \text{ if } S = \{1,3\} \\ 0.268 \text{ if } S = \{2,3\} \\ 0.679 \text{ if } S = \{1,2\} \\ 0.716 \text{ if } S = \{1,2,3\} \end{cases}$$

Maskin Limousin

Le Breton-Van Der Straeten	De Clippel-Serrano	Maskin
4.1145%	2.372 3%	4.416 7%

Shapley Value(s) of EE in the case of Limousin

THANK YOU!