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Marat Z. Kurbangaleev, Victor A. Lapshin,
Sergey N. Smirnov

STUDY OF CONSISTENCY OF BOND AND CDS QUOTES

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Marat Z. Kurbangaleev[1], *Victor A. Lapshin*[2], *Sergey N. Smirnov*[3]

Study of Consistency of Bond and CDS Quotes[4]

In this paper, we study the consistency of bonds and CDS quotes data within a widely accepted credit risk pricing framework, allowing for non-trivial term structures of risk-free interest rates and default intensities. We propose an approach to test this consistency and a procedure to deal with inconsistencies. Our approach is model-independent and does not rely on any term structure fitting method. It also allows us to assess the precision of constructing risk-free yield term structure can be estimated for a given bond and CDS quotes set. We apply the proposed approach to euro zone sovereign bond and CDS data, and demonstrate that, in general, bond/CDS quotes are typically inconsistent across issuers and require filtration. However, our findings suggest grouping the euro zone sovereign issuers according to the group-level internal consistency.

JEL Classification: G12, G15

Key words: feasibility band, data consistency, risk-free interest rates, term structure of credit spreads, sovereign default risk, euro zone, defaultable bond, credit default swap.

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[1] National Research University Higher School of Economics. Laboratory for Financial Engineering and Risk Management. Researcher; E-mail: mkurbangaleev@hse.ru

[2] National Research University Higher School of Economics. Laboratory for Financial Engineering and Risk Management. Researcher; E-mail: vlapshin@hse.ru

[3] National Research University Higher School of Economics. Laboratory for Financial Engineering and Risk Management. Director; E-mail: ssmirnov@hse.ru

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1 Introduction

Yields on government bonds have traditionally been considered as risk-free when dealing with corporate bonds within the same country. In particular, yields on US Treasury bonds were generally accepted worldwide as risk-free when dealing with dollar-denominated bonds. However, some recent events have raised doubts about this point of view. For instance, the debt-ceiling crisis in the US in August 2011, and repeatedly in October 2013, clearly demonstrated that even the US Treasuries can no longer be unconditionally considered risk-free. Another example is the recent European debt crisis, which caused concerns for the creditworthiness of European countries. Moreover, the last one was accompanied by a sovereign default: in 2012 the ISDA EMEA Determinations Committee[5] declared the Greek debt restructuring credit event. The declaration triggered the CDS credit event auction and the settlement of Hellenic[6] CDS contracts.

Over the last decades, several sovereign issuers actually defaulted on their debts. To name a few, Russia defaulted on its domestic (but not external) debt in 1998, whereas Argentina has defaulted three times since 1980. Two of Argentine defaults (1982 and 2002) involved external debt. Furthermore, collisions associated with the debt restructuring process launched in 2005 finally led the failure-to-pay credit event and triggered CDS on Argentina's debt in 2014.

One of the main reference rates in the euro zone, based on government bond quotes, is published by the European Central Bank (ECB). The corresponding ECB methodology (2014) prescribes the construction of the euro zone yield curve from AAA-rated government bonds. After France was downgraded to AA+ by Fitch in July 2013, the euro zone yield curve is substantially based on German bonds (though it is still based on French bonds for long maturities since there are no German bonds with sufficiently long terms to maturity). The credit risk of government bonds, especially in the long term, cannot be considered negligible even in the case of financially sound governments like that of Germany.

The credit risk in German bonds, however small, is certainly priced in by the market (German CDS are actively traded, and this interest in the market reflects existing concerns about the financial stability of the euro zone). Moreover, before the present crisis there were

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[5] The Credit Derivatives Determinations Committees were established in order to provide a transparent and credible quorum-based credit event determination process. EMEA Determinations Committee declares credit events which have occurred in respect to the debt issued in Europe, the Middle East and Africa.

[6] The official name of CDS written on Greek debt.

periods when German bond yields were not minimal with respect to government bonds of other euro zone countries.

The European debt crisis amplified an international interest in credit derivatives, particularly in sovereign credit default swaps (CDS)[7]. Here the question arises: can the use of CDS quotes (together with bond quotes) help to estimate the full term structure of risk-free interest rates? Credit default swaps are widely used to infer the credit risk priced in the bond credit spread. However, a naive approach to the problem fails, since it is impossible to extract credit spreads directly from CDS prices and then subtract them from risky bond yields to obtain 'risk-free' rates. To perform the extraction of credit spreads, one would need a market-consistent methodology for simultaneously extracting the term structure of risk-free interest rates and the term structures of issuer-specific hazard rates from bond and CDS quotes[8]. The term structure of risk-free rates should be regarded therefore as unobservable and determined implicitly.

Now, we will briefly overview the related literature on the joint pricing of bonds and CDS. The following studies are based on the paradigm of risk-neutral pricing[9]. Houweling and Vorst (2005) develop a simple reduced-form model for defaultable bonds and CDS pricing. Using different proxies for the risk-free rate and different specifications of the hazard rate, they calibrate their model to prices of corporate bonds with different maturities and apply the obtained estimate of hazard rates to CDS pricing. They find that the model-based approach outperforms the simple estimation of CDS spreads with the bond spreads, but the estimates are still biased. On the other hand, Longstaff et al. (2005) do the opposite. They calibrate a reduced-form model to CDS spreads and then price corporate bonds with its help. They report a significant non-default component in the bond spread, which they attribute to bond-specific and market liquidity factors. Buhler and Trapp (2009) extend previous models by incorporating an instrument-specific liquidity factor, which can be correlated with risk-free rates and credit risk.

Most of the papers in this field, including those mentioned above, consider corporate bonds and CDS, where there are usually few bonds and few liquid CDS, so that a reliable term structure can hardly be inferred from the data.

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[7] See Chapter 2 of the International Monetary Fund Global Financial Stability Report (2013).

[8] We adopt some simplifying assumptions about recovery rate, described below.

[9] The risk-neutral pricing paradigm is based on special assumptions about the asset pricing. Some of them, such as market completeness, can be debated. However, this paradigm is generally accepted, being fairly convenient for practical purposes, especially when dealing with interest rate or credit derivatives. In particular, the corresponding price dynamics is automatically arbitrage-free.

Longstaff et al. (2005) compare bond spreads and CDS-implied default intensities for US corporations and find that the hazard rates extracted from CDS have very low sensitivity to the risk-free interest rate used. Their study involves all corporate bonds grouped by credit rating, but regardless of term, with no term structure for hazard rates. Hull et al. (2004) estimate the effective risk-free rate, that is, the rate matching bond spreads to CDS-implied hazard rates (in the least squares sense). They found that it resided between the US Treasury yield and swap rate, but was much closer to the swap rate than the Treasury yield, thus supporting the findings of Houweling and Vorst (2002), who report that swap/repo rates perform significantly better than the Treasury yields when comparing CDS premiums with bond spreads. Both papers assume no term structure of either interest or default rates.

Unlike corporations, sovereign issuers usually have many bonds of different maturities and the corresponding CDS are actively traded for different tenors. At least since the beginning of the European debt crisis at the end of 2009 the trading activity in the sovereign CDS market has increased dramatically[10].

Sovereign bonds and CDS are studied by Adler and Song (2010). They use sovereign CDS and bonds in emerging markets and construct term structures of both in order to correct for the bias stemming from non-par traded bonds and CDS quoting in premiums different from bond coupons. However, after unwinding full term structures and correcting for all biases, they use all data in bulk, regardless of the term, to draw conclusions. They report that, for most of the countries, spreads are equal, but for Argentina, Brazil and Venezuela, which had very high probabilities of default at the time (the study is based on the 2001–2005 data) – CDS and bonds do not agree when characterizing default probabilities. They use US Treasury yields as the risk-free rates.

Arce et al. (2013) study euro zone bonds and CDS, but their study is limited to five year bonds, so there is no term structure, and they suppose German bonds to be risk-free.

CDS spreads are traditionally treated as an analogy of risky bond spreads. The seminal work of Duffie (1999) proves that in an arbitrage-free floating coupon bond market, the CDS spread should be equal to the spread of the defaultable bond rate over LIBOR. However, this result is impractical, since fixed coupon bonds generally dominate in real markets. Hull and White (2000) show that for a fixed coupon bond the equivalence between CDS and bond spreads holds only approximately.

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[10] Here we refer again to Chapter 2 of the International Monetary Fund Global Financial Stability Report (2013). The weekly data on CDS transactions are also provided by DTCC Deriv/SERV on the DTCC website.

The theoretically stated relation between the CDS and the bond spreads was tested in a series of empirical studies. The main question is how close CDS and bond spreads really are. Literature reveals the presence of significant discrepancies between bond and CDS spreads, called CDS – bond basis. For instance, Blanco et al. (2005) calculate corporate bond spreads over the government bond yields and the swap rate as a proxy of the risk-free rate. They find that spreads over government bond yields overestimate CDS spreads on average, whereas bond spreads over swap rates are more accurate but are still biased estimates of CDS spreads. The repo rates are considered as another proxy for the risk-free rate by Houweling and Vorst (2005), but they perform slightly worse than the swap rates. Houweling and Vorst also find that the magnitude of the CDS – bond basis varies with the credit ratings of bond issuers. Zhu (2006) investigates corporate bond and CDS. In contrast to the work by Houweling and Vorst, this work does not find any significant relation between CDS – bond bases and the credit quality of bond issuers. However, liquidity and bond short sales restrictions are found to be significant determinants of the CDS – bond basis for corporate reference entities.

The joint dynamics of the CDS and bond spreads is also intensively examined in the literature. Theoretically, the time series of the CDS and the bond spreads should be cointegrated with the vector [1; -1]. Most of the studies confirm the fact of cointegration, but in every particular case the cointegration vector is different from what the theory predicts; spreads may deviate significantly from the long-term equilibrium in the short run.

Furthermore, the CDS market typically leads the bond market in the price discovery process, and this effect is more pronounced in the US market than in Europe. Here we refer to the papers by Blanco et al. (2005); Zhu (2006); Norden and Weber (2009). While these three papers use time-invariant models, Dötz (2007) analyzes the dynamic aspects of the price discovery process in the European corporate bond market. Dötz concludes that the CDS market significantly contributes to the price discovery and slightly dominates the bond market, but in periods of market stress the bond market usually outperforms the CDS market in price discovery. Palladini and Portes (2011) come to a fairly similar conclusion, although they analyze sovereign bonds and CDS.

Calice et al. (2013) incorporate liquidity in their study of interaction between bond and CDS markets. The authors measure the liquidity of both markets by the quoted bid-ask spread and use a time-varying vector autoregression framework to investigate the credit and liquidity interactions in the euro zone sovereign bond and CDS markets from 2009–2010. They find that the credit quality of a sovereign issuer is definitely associated with the liquidity of the sovereign bond and CDS markets, but also found that the patterns of transmission effect vary between maturities and across countries. The European Central Bank working paper by Fontana and Scheicher (2010) studies the CDS and bond relationship in the euro zone using the notion of a cross-market spread; the difference between bond and CDS spreads. The authors use only one tenor, 10 years, discarding the term structures, and regard the swap rate as risk-free.

As the literature overview shows, although the joint pricing framework for bonds and CDS is widely used, there is also an understanding that the prices of the bond and CDS referred to the same entity may differ in terms of information content. However, to the best of our knowledge, the accuracy and reliability of the estimates of pricing factors, especially the term structure of risk-free rates based on such a framework is still unknown. In this paper we make an attempt to fill this gap. To do so, we investigate the internal consistency of the data within a widely accepted credit risk pricing framework. Here we consider the bond and CDS quotes consistent if there exists a non-empty set of risk-free yield and default intensity curves which ensure the value of those instruments lying between bid and ask quotes First, we check whether the instruments referenced to a single issuer are internally consistent. Then we check whether several issuers provide consistent information on the risk-free term structure. Since the answer to both questions turns out to be negative, we propose a means of dealing with it by filtering and adjusting the data, which we call consistency adjustment. We also introduce the tightness factor, which can be considered as a measure of the precision inherent in the problem of fitting the risk-free term structure to the data in question. Finally, we apply the proposed approach to the euro zone sovereign bond and CDS data.

The remainder of this paper is organized as follows. Section 2 presents the pricing framework for defaultable bonds and CDS. Section 3 describes the data we study. Section 4 reports the empirical findings, and Section 5 concludes.

2 Pricing Framework

In this paper we follow the general practice of describing interest rates in terms of spot rates and instantaneous forward rates. We denote the spot rate prevailing at time t as r_t and the zero-coupon yield curve prevailing at time t as $r_t(\cdot)$. Using continuous compounding, we can express the corresponding discount function $D_t(\cdot)$ as follows:

$$D_t(s) \quad = \quad \exp\left[-s\cdot r_t(s)\right] \quad = \quad \exp\left[-\int_0^s f_t(\tau)d\tau\right] \quad = \quad E_t\left(\exp\left[-\int_t^{t+s} r_x dx\right]\right), \quad (1)$$

where $r_t = r_t(0) = f_t(0)$ is the instantaneous spot rate, $f_t(x)$ is the instantaneous forward rate prevailing at time t for the future time $t+x$, and E_t denotes the conditional expectation given available at time t information[11] with respect to the risk-neutral probability measure.

Numerous models have been developed for interest rate modelling[12]. Most models assume that the interest rate in question is the risk-free rate, i.e. that the bonds are not subject to default risk, whereas in reality bonds bear the issuer's default risk.

We also follow general practice describing credit risk, namely reduced form models. This approach is convenient for introducing credit risk in the model in a way similar to the interest rate mechanics described above. Default is treated as a random event that is completely unpredictable[13], so we can focus solely on the default probability distribution.

Let $Q(s)$ be the risk-neutral probability of no default[14] up to time $t+s$ conditional on no default at time t. Under mild regularity conditions there is a predictable process λ_t, the spot default rate, which is generally termed the spot hazard rate, such that

$$Q_t(s) = E_t\left(\exp\left[-\int_t^{t+s} \lambda_x dx\right]\right). \quad (2)$$

Continuing the analogy with interest rates, we can introduce the notions of hazard rate to maturity $\Lambda_t(s)$ and instantaneous hazard rate $\lambda_t(s)$ by the following relation:

$$Q_t(s) \quad = \quad \exp\left[-s\cdot\Lambda_t(s)\right] \quad = \quad \exp\left[-\int_0^s \lambda_t(x)dx\right] \quad = \quad E_t \exp\left[-\int_t^{t+s} \lambda_x dx\right]. \quad (3)$$

Here, the spot hazard rate λ_t is analogous to the future instantaneous spot rate r_t, $\lambda_t(\cdot)$ plays the role of $f_t(\cdot)$, and $\Lambda_t(\cdot)$ is similar to $r_t(\cdot)$.

From now on, we drop the subscript t always assuming the present moment of time (i.e. we always set $t=0$), since we have no intention of considering the dynamics in what follows. So the only relevant parameters are maturities and tenors.

A number of hazard rate models can be devised from existing interest rate models[15]. For

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[11] Formally such an information can be described in a standard mathematical language in terms of σ-algebras (namely as a filtration).

[12] See the books by James and Webber (2000) and Andersen and Peterbarg (2010) for a comprehensive review.

[13] From mathematical point of view the default time can be assumed to be a totally inaccessible stopping time.

[14] It is also called the survival probability.

[15] Please refer to the book by Brigo and Mercurio (2006) for the details and the analogies between interest

joint modelling of interest rates and hazard rates, models can be introduced just like interest rate models rewritten for new variables. These models can be naturally combined together into a double stochastic model. In order to do it in the simplest way possible, we adopt a simplifying assumption (generally accepted, but not always realistic): the independence of hazard rates and the interest rates[16].

Now let $k = 1, ..., K$ denote different issuers, then $F_{i,j,k}$ is the promised cash flows for the bond j of the issuer k at time $t_{i,j,k}, i = 1, ..., n^{bond}_{j,k}$. Cash flows $F_{i,j,k}$ are calculated according to the bond-specific coupon schedule parameters, including coupon structure, day count convention, and coupon payment frequency. The CDS premium payment times for the contract j on the issuer k maturing in $T^*_{j,k}$ are denoted by $T_i, i = 1, ..., n^{CDS}_{j,k}$ (note that the premium payment times are standard across different contracts). Let the risk-free spot forward rate be denoted by $f(\cdot)$, the risk-free discount function by $D(\cdot)$, the spot hazard rate for the issuer k by $\lambda_k(\cdot)$ and the corresponding survival probabilities by $Q_k(\cdot)$.

The theoretical present value of the bond j of the issuer k is

$$P^{bond}_{j,k} = \sum_{i=1}^{n^{bond}_{j,k}} F_{i,j,k} D(t_{i,j,k}) Q_k(t_{i,j,k}) + 1 \cdot D(t^*_{j,k}) Q_k(t^*_{j,k}) + \\ + \int_0^{t^*_{j,k}} \left[(1 - L^{bond}_k) D(x)\right] d(1 - Q_k(x)), \quad (4)$$

where $t^*_{j,k}$ is the time of maturity for the bond j of the issuer k and L^{bond}_k is the (fixed) loss given default (LGD) fraction of the issuer k.

The theoretical price of the CDS j for the issuer k is

$$P^{CDS}_{j,k} = \int_0^{T^*_{j,k}} L^{CDS}_k D(x)\, d(1 - Q_k(x)) - \\ - S \left[\sum_{i=1}^{n^{CDS}_{j,k}} D(T_i)(T_i - T_{i-1}) Q_k(t_i) + \int_0^{T^*_{j,k}} D(x) \left(T_{I(x)} - x\right) d(1 - Q_k(x)) \right], \quad (5)$$

where S is is the standard CDS premium, L^{bond} is the (fixed) loss given default fraction assumed by CDS prices, and $I(x)$ is the index of the last premium payment before the

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and hazard rates.

[16] Introducing correlations between interest rates and default intensities into the model would be a challenging problem; however, we remind that one of the main goals of the present work is to achieve tractability reasonable for practical purposes together with model independence (within some set of standard assumptions, one of which is the independence of default and interest rates). If we drop the independence assumption, the model independence cannot be achieved; the tractability also suffers considerably.

time[17] x. The first term in this equation represents the present value of the credit protection granted by the CDS seller, whereas the second term is the present value of the premium flow paid by the CDS buyer.

In practice, for pricing purposes the LGD is typically set expertly at some particular value for all debt of a given seniority irrespective of its duration. An alternative approach is estimating the expected LGD from the market data. Note that the LGD values assumed in pricing bonds and CDS may be different due to the perceptions of credit risk on different markets.

We can refer to the LGD values set by ISDA CDS Standard Model[18] which is considered the market standard of CDS pricing. For instance, for sovereign senior unsecured debt, ISDA CDS Standard Model 2014 assumes an LGD of 60% (recovery rate of 40%). Based on this widely recognized market standard, we set $L^{CDS} = 0.6$ for all sovereign issuers in our sample, except Greece for which we set L^{CDS} equal to 0.8, as its credit quality was highly distressed within the time period considered. In contrast to CDS, we are not aware of any agreed LGD standard for pricing bonds. However, we can determine the LGD for bonds from the regulatory prospective and the best practice of risk management. Based on the Basel 2 rules[19] we set $L^{bond} = 0.45$ in the bond pricing formula.

In the described framework the risk-free interest rates could be, in principle, estimated using data only on a single sovereign issuer. Individual CDS and bonds of a specific issuer are subject to price fluctuations due to liquidity issues and idiosyncratic noise, so one can hardly expect to get a quality estimate. The use of the data on several issuers (subject to preliminary selection) would average out the individual discrepancies and result in a more robust estimate. Therefore we consider the risk-free term structure as a characteristic of the whole fixed-income and CDS market – as a kind of market index.

In order to obtain an estimate of the term structure of risk-free rates, one need to specify a method for fitting the risk-free forward rate curve and the hazard rate curve. We leave the fitting problem outside of the scope of this paper, but rather focus on the question whether there exists any estimate consistent with the data.

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[17] Here and throughout the article we refrain from considering different day counting conventions and assume that all time variables are expressed in year fractions. Typically, day counting conventions would influence the exact value of the year fractions.

[18] The ISDA CDS Standard Model is administrated by Markit and is used by market participants for CDS pricing and for the conversion of CDS spreads into the up-front payments and vice-versa. For the rigorous overview of ISDA CDS Standard Model see the paper by White (2013).

[19] Under the foundation IRB approach, Basel 2 accord prescribes fixed LGD ratios for certain classes of unsecured exposures, in particular, for senior claims on corporates, sovereigns and banks not secured by recognized collateral, a 45% LGD is applied (subordinated claims correspond to a 75% LGD level).

3 Data

In this paper we study the euro zone sovereign bond and CDS markets. Over the last 15 years the euro zone members have been actively issuing euro-denominated debt in the form of bonds with several benchmark issues and different maturities. The financial crisis has significantly increased the trading activity in the sovereign segment of the CDS market, especially in the euro zone. The global CDS market reform has fostered contract standardisation, transparency, and liquidity. Note that several large financial information services such as Reuters, Bloomberg, Markit, and CMA provide fairly credible aggregated market pricing data on both bonds and CDS on a regular basis.

The bond data is provided by Markit and includes the euro zone sovereign euro-denominated zero- and fixed-coupon bonds constituting the Markit Iboxx EUR Index family. The index-eligible sovereign bond issues are those attributed by the maturity over one year, the amount outstanding over 2 bln. euro and the issuer's investment grade credit rating. The data set includes daily bid and ask quotes, full specifications and amount outstanding for bonds of 12 euro zone countries namely Austria, Belgium, Germany, Greece, Ireland, Italy, Finland, France, the Netherlands, Portugal, Slovakia and Spain. Ot covers the period from January 2006 to January 2012.

The Iboxx EUR Indices are rebalanced on a monthly basis and the issues not complying with inclusion criteria are to be suspended, therefore the information for some countries is incomplete in our dataset. In particular, during the European debt crisis Greek and Portuguese bonds were excluded from the index due to the downgrade below investment grade level in June 2010 and in July 2011, respectively. The bid and ask quotes represent quotes provided by major dealers, aggregated by Markit according to its pricing rules. We adjust the data for accrued interest to get dirty prices, as required by our methodology. For European bonds, the coupon-bearing issues pay coupons once a year – the only exception being Italian bonds, which pay semi-annually. For all coupon-bearing bonds in the sample, the standard ICMA actual/actual day count convention is used.

The CDS data comes from Reuters. The sample consists of daily upfront prices for CDS figures written on the bonds issued by each of the chosen euro zone sovereign issuers and covers the period from August 2010 to December 2011. The term structure of CDS prices is not available for every euro zone sovereign issuer, therefore we limit our sample to Austria, Belgium, Germany, Greece, Ireland, Italy, France, Netherlands, Portugal and Spain. For each of those countries, although not on each trading day, we have 10 standard tenors (6 month, from 1 to 5 years, 7, 10, 20 and 30 years). The prices in the data set are essentially the Reuter's composite index, which the agency calculates for single-name contracts of each of ten standard tenors (6 month, from 1 to 5 years, 7, 10, 20 and 30 years) from quotes posted by multiple dealers at the end of each day. The prices are calculated for the following contract specification: reference obligation is the entity's senior bullet bond denominated in euros; the restructuring clause is Modified–Modified; the standard quarterly paid CDS premium is 100 basis points (bp); the premium is paid in euros on the standard dates (20th March, 20th June, 20th September, 20th December); the day count convention is Act/360; and if a credit event occurs, the CDS buyer pays the part of the premium accrued from the last payment. As with bond prices, we also adjust the CDS upfront prices for the accrued premium.

Since there are no Finish and Slovakian CDS in our data sample, we had to exclude these countries from consideration. Due to the short observation period and high frequency of missing data, we also exclude Ireland, Portugal and Greece. As a result, we are left with around 330 daily 'bond-CDS' observations from August 2010 – January 2012 in seven countries (Austria, Belgium, France, Germany, Italy, Netherlands, Spain). On each observation date we have ten CDS and from fifteen up to fifty bond issues of different maturities for each country. The considered period includes the peak of the euro zone sovereign debt crisis.

An important technical remark has to be made here. Prior to April 2009, the CDS premium was set so that the contact was worth nothing for both counterparties. The market innovations[20] re-established the structure of the contract by fixing the periodically paid premium at one of several standard levels and setting up a variable upfront amount exchanged at the moment of the deal. Although all payments on a contract are settled according to the upfront + fixed premium' convention, dealers still quote CDS contracts in terms of the implied spread which makes the contract value zero. This spread is disseminated through trading platforms and/or informational systems, such as Bloomberg or Reuters, and is termed the conventional spread.

The conventional spread is an indicative value and is derived from the quoted upfront amount via the Standard ISDA Model (see the ISDA web site) transformation. Therefore, if one wants to estimate the term structure of default intensities from conventional spreads of different tenors, these spreads should be initially transformed back to upfront amounts,

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[20] See ISDA Big Bang Protocol (2009a) and Small Bang Protocol (2009b).

accordingly. Note that the Standard ISDA Model also adopts a simplifying assumption of independence between interest and hazard rates together with the constant and deterministic recovery rate.

4 Empirical Study

Now we turn to examining data consistency and start with the consistency of single issuer's bond quotes. In order to do this, we employ the notion of the arbitrage bounds introduced by Jaschke (1998). We briefly introduce it here for the sake of completeness.

Suppose we have N coupon bonds indexed by $j = 1..N$ on the market with promised cash flows $F_{i,j}$ at times t_i, $\quad i = 1..n$, which may be considered common for all bonds by introducing zero cash flows when necessary. Let B_j^{bond} and A_j^{bond} be the dirty bid and ask quotes of the j-th bond. Then on the arbitrage-free market the bond discounted cash flow must satisfy the following inequality

$$B_j^{bond} \leq \sum_{i=1}^{n} F_{i,j} d_i \leq A_j^{bond}, \quad j = 1..N, \tag{6}$$

where $d_i = D(t_i)$ is the (risky) discount factor for the term t_i subject to the usual discount factor constraints:

$$1 \geq d_1 \geq d_2 \geq ... \geq d_n \geq 0. \tag{7}$$

Now one can posit the following problem: what are the possible discount functions satisfying (6)–(7). It can be formalized by a series of linear programming problems:

$$\begin{cases} d_i \to \min, \max \\ B_j^{bond} \leq \sum_{i=1}^{n} F_{i,j} d_i \leq A_j^{bond}, \quad j = 1..N, \\ 1 \geq d_1 \geq d_2 \geq ... \geq d_n \geq 0. \end{cases} \tag{8}$$

The solution of these LP problems gives an upper and a lower bound for the possible (risky) discount function values at times t_i. The bounds in intermediate points can be inferred from (7). Note that these bounds are model-independent. The only assumptions needed are those of a standard liquid arbitrage-free market of infinitely divisible bonds and no trading restrictions. If constraints in (8) are infeasible, theoretically it means that quotes are inconsistent and arbitrage opportunities exist.

In fact, inconsistent quotes in our dataset do not necessarily represent an arbitrage trading opportunity, because available quotes are without commitment and not synchronous (the quotes should be regarded as indicative and possibly as averaged). Fitting inconsistent data might yield economically unsound results. However, arbitrage-like considerations for bounds in (8) are still meaningful. They yield the region between these bounds, reflecting the possible values of interest rates for different maturities, which we call the feasibility band. Regardless the fact that arbitrage, in strict sense, is not exploitable in these conditions, the feasibility band reflects the precision at which the problem of constructing the risk-free yield term structure can be solved for the quotes given. Note that the same approach, in principle, may be used to derive the bounds and feasibility band for the survival probabilities of different issuers. We shall say that the problem is feasible if the feasibility band is non-void.

However, in our study it turns out that the problem (8) is usually infeasible for any single country. This may suggest that the actual bond pricing is not performed using the discounted cash flow approach. Bond liquidity is almost surely taken into account by market participants, as well, as on-the-run / off-the-run property. In the absence of the trading restrictions, this would mean arbitrage opportunities on the market. However, these inconsistencies may also be caused by a technical reason, because the bounds are sensitive to small perturbations in quotes of bonds with similar times to maturity. Since the data at our disposal is pre-aggregated by the vendor (Markit), we are unable to ensure consistency of quotes for similar bonds across different dealers. In any case, since our approach implies an arbitrage-free market and we don't account for the trading restrictions, we need to adjust our data by either removing some of the bonds or widening the bid-ask spreads. Peeking ahead, we do both, but we start with removing some of the bonds. Usually yield curve calculations are based not on a full set of traded bonds, but on a subset called benchmark bonds. As we deal with government bonds, the first candidates for benchmarks are key issues regularly renewed and maintained by issuers. Unfortunately, it turns out to be hard to retrospectively identify the key issues even for actively borrowing countries. Therefore, a formal algorithm based on some reasonable criteria has to be developed. In this paper we apply a selection procedure in the spirit of the Markit Iboxx Benchmark Determination Guide 2014. Our algorithm starts with setting adjoined bands for bond terms. Each band is an interval around the ten key terms, coinciding with the ten CDS tenors. Each of the issuer's bonds is assigned to one of the bands. Bonds in each band are ranked according to the amount outstanding and issuance date. The greater amount outstanding and the younger the issue, the higher the rank of the bond in the band. Benchmarks are then picked from the highest-ranked issues in each band ensuring that there is at least six months distance between the maturity dates of different benchmark bonds.

This procedure leaves us with 10 bonds for each issuer instead of the former 15–50 on each observation date in our sample, and the bounds for the risky rates are consistent for each issuer. The next step is to check the consistency of CDS quotes. Unfortunately, there is no way of inferring the hazard rates from CDS quotes without knowing the risk-free rates, so testing the consistency of CDS quotes alone is problematic.

So we start with checking the consistency of bond and CDS quotes for a single issuer within our framework. We use the notation and pricing formulas for bonds and CDS introduced earlier.

The arbitrage-like considerations yield the following sets of inequalities for a single issuer k:

$$B_{j,k}^{bond} \leq P_{j,k}^{bond} \leq A_{j,k}^{bond}, \tag{9}$$

$$B_{j,k}^{CDS} \leq P_{j,k}^{CDS} \leq A_{j,k}^{CDS}. \tag{10}$$

Now we can state the optimization problem for the no-arbitrage bounds for the risk-free discount factors[21] at moments $t_{i,k}$, corresponding to payment times of bonds and CDS of the k-th issuer:

$$\begin{cases} D(t_{i,k}) \to \min, \max, \\ B_{j,k}^{bond} \leq P_{j,k}^{bond} \leq A_{j,k}^{bond}, j = 1..N_k^{bond}, \\ B_{j,k}^{CDS} \leq P_{j,k}^{CDS} \leq A_{j,k}^{CDS}, j = 1..N_k^{CDS}, \\ D(0) = 1, D(t) \geq 0, D(\cdot) \text{ decreases}, Q_k(0) = 1, Q_k(t) \geq 0, Q_k(\cdot) \text{ decreases}, \end{cases} \tag{11}$$

for every single issuer k. This can easily be generalized to include data from several issuers by allowing k to vary.

We should note that the problem (11) is not completely model-independent. In addition to the assumptions stated in Section 2 above, it depends on the functional family, parametric or non-parametric, for $D(\cdot)$ and $Q_k(\cdot)$, over which the optimization is to be performed.

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[21] Due to the nature of expressions (4) and (5) for $P_{j,k}^{bond}$ and $P_{j,k}^{CDS}$, the problem should be stated in terms of the risk-free discount function $D(\cdot)$ and survival probability functions $Q_k(\cdot)$, rather then the risk-free instantaneous forward rate f and default intensities λ_k.

This is no longer an LP problem, but a linear optimization problem with bilinear inequality constraints presenting a reasonable extension of the feasibility band approach to our framework.

Since (8) is feasible at this point, the possible infeasibility of (11) would mean inconsistencies between the bond and CDS markets' perception of the default probabilities. In fact the solution of (11) gives the bounds for the euro zone risk-free curve implied by the bond and CDS quotes of a single issuer. It turns out that the feasibility band is always non-empty, proving that the bonds and CDS are consistent with each other on a single-issuer level.

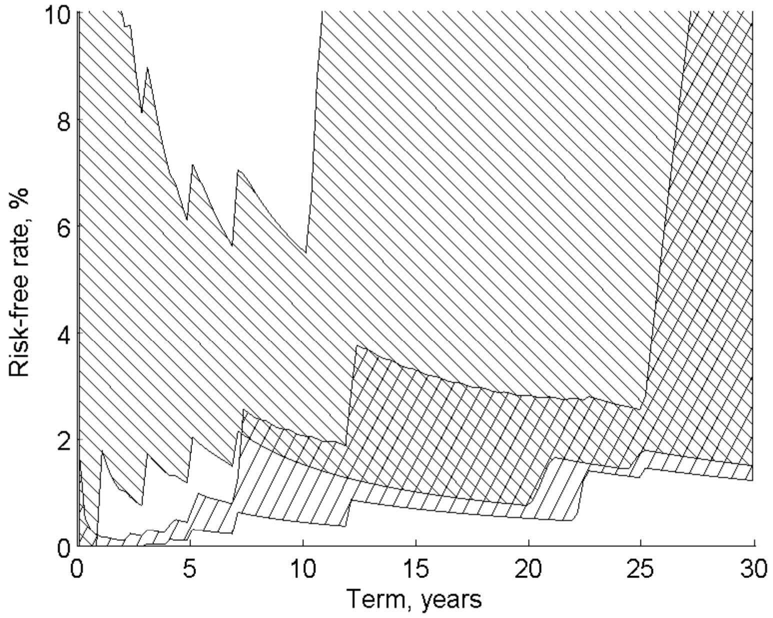


Figure 1: Feasibility bands for the risk-free curve implied by bonds and CDS of Germany (lower) and Italy (upper) for 9 Nov 2011.

Although the feasibility bands are non-empty in our data sample, they vary significantly across countries and observation dates. As Figure 1 shows, Italian and German bands for risk-free curve on 9 November 2011 are inconsistent with each other. Figure 1 is limited to only two issuers for the reader's convenience. However, it turns out that Italy and Spain usually provide significantly different feasibility bands to other issuers. It makes us suspect that the prices of Italian and Spanish bonds and/or CDS may reflect some features which are irrelevant for other countries, therefore the nature of Italian and Spanish instruments prices, maybe due to their economic problems, is somewhat different in comparison with the other countries in our data sample. We also understand that Austria, Belgium and the Netherlands may be different from Germany and France; the latter two being the major euro zone borrowers, as well. In order to monitor the implications of such data fragmentation, in what follows, we divide the countries in our sample into subgroups. We start with Germany and France – large borrowers with high credit quality. The next group is Austria, Belgium and the Netherlands, also high quality issuers with smaller amounts outstanding. Finally come Italy and Spain, which are major borrowers, but facing financial problems during the period considered.

Now we turn to testing data consistency across different issuers using our risk-free approach. Problem (11) turns out to be infeasible for almost any cross-subgroup combination of issuers, with only several exceptions (see Table 1).

Issuer Group(s)	N of consistent days
Germany, France	116 (36%)
Italy, Spain	339 (77%)
Netherlands, Austria, Belgium	176 (54%)
Germany, France, Netherlands, Austria, Belgium	7 (2%)
Germany, France, Italy, Spain	0 (0%)
Germany, France, Netherlands, Austria, Belgium, Italy, Spain	0 (0%)

Table 1: Quote consistency statistics

One example of a consistent feasibility band for five issuers is given on Figure 2. We can also see that the swap curve (dashed line) is significantly higher than the feasibility band, hence any possible risk-free curve is lower than the swap curve. In other words, the swap curve can hardly be regarded as a data-consistent estimate of risk-free yield term structure.

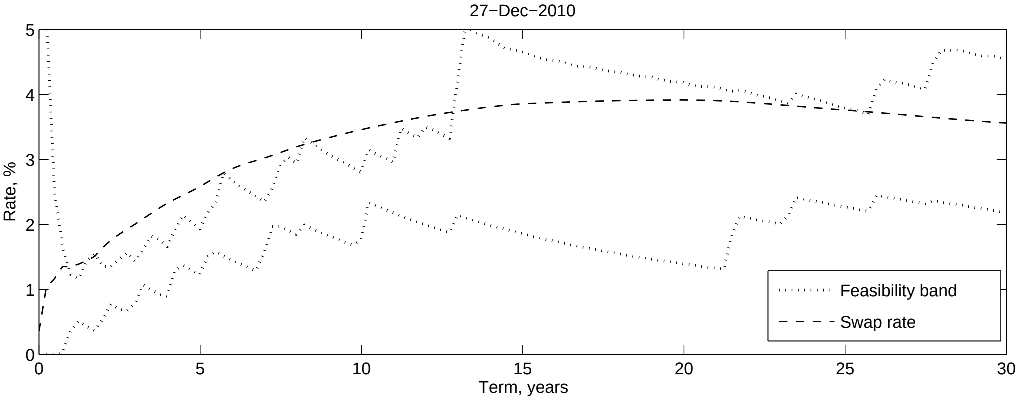


Figure 2: Germany, France, Netherlands, Belgium, Austria: feasibility band.

Now we quantify the discrepancies in bond and CDS pricing across the issuers. To do so, we introduce the notion of the tightness factor. The market tightness factor Θ is the least factor, by which one needs to widen the bid-ask spreads to make problem (11) feasible. It is formalized via the following problem:

$$\begin{cases} \Theta \to \min\limits_{D(\cdot), Q_k(\cdot)}, \\ B_{j,k}^{bond} - \Theta \frac{A_{j,k}^{bond} - B_{j,k}^{bond}}{2} \leq P_{j,k}^{bond} \leq A_{j,k}^{bond} + \Theta \frac{A_{j,k}^{bond} - B_{j,k}^{bond}}{2}, j = 1..N_k^{bonds}, k = 1..K, \\ B_{j,k}^{CDS} - \Theta \frac{A_{j,k}^{CDS} - B_{j,k}^{CDS}}{2} \leq P_{j,k}^{CDS} \leq A_{j,k}^{CDS} + \Theta \frac{A_{j,k}^{CDS} - B_{j,k}^{CDS}}{2}, j = 1..N_k^{CDS}, k = 1..K, \\ D(0) = 1, D(t) \geq 0, D(\cdot) \text{ decreases}, \\ Q_k(0) = 1, Q_k(t) \geq 0, Q_k(\cdot) \text{ decreases}, k = 1..K. \end{cases} \tag{12}$$

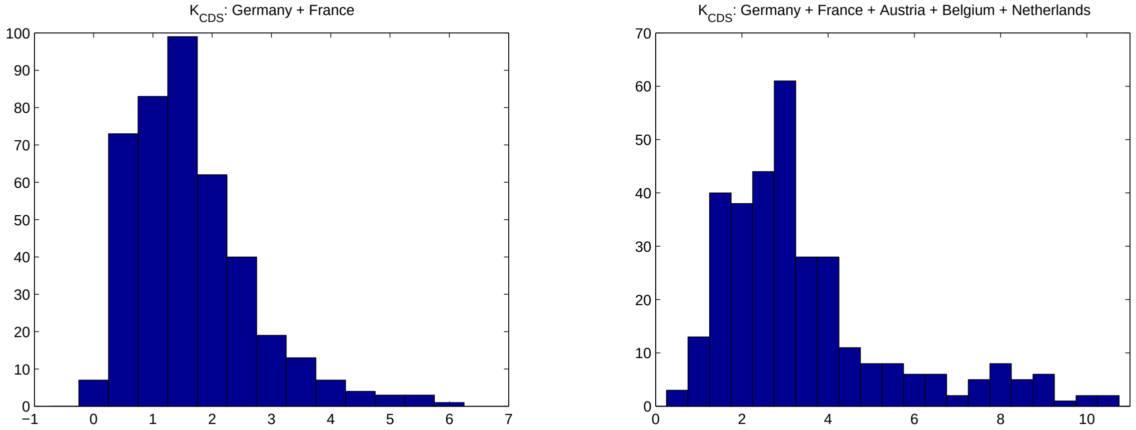
For example, $\Theta = 2$ means that the bid-ask spreads have to be twice as wide for the inequality constraints in (11) to be consistent on the day in question.

Widening the spreads proportionally means that we assign equal credibility to all quotes. If we have reason to believe that some quotes are more credible than others, the spreads can be widened according to some specified rule of 'consistency adjustment'. For example, we can leave bond spreads fixed while widening CDS spreads or vice versa. The choice of this rule can be motivated, in particular, by studying the price discovery process in these markets. Or we can assign greater credibility to quotes of some issuers, which will result in lesser changes in these quotes. We can also interpret the inconsistency of bond and CDS quotes across the issuers as a sign of a latent factor, which may be significant for some individual countries or group(s) of countries, and which is left out in our methodology. Apart from liquidity, these additional factors might include, for example, the possibility of that particular country dropping out of the euro zone. In this case, the tightness factor can be regarded as a measure of the precision inherent in the risk-free curve estimation problem from the given data.

As for each single country its bond and CDS quotes are consistent – the additional issuers used in the risk-free curve estimation process can only increase the tightness factor. So starting with Germany and France, which have the tightest individual bounds, we incrementally add other issuer subgroups and examine how it affects tightness factor. Table 2 presents the statistics while Figure 3 shows histograms for the tightness factor for different issuer groups in our data sample.

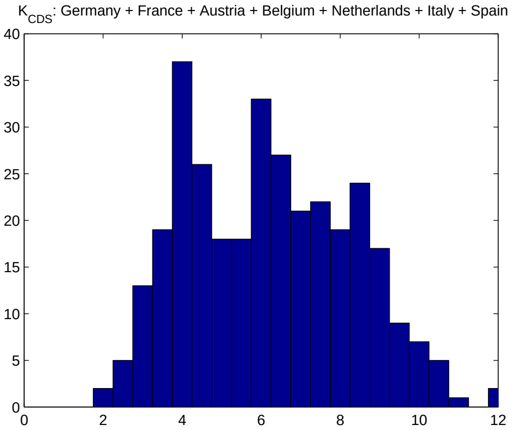
	Min	Max	Mean	Std	Quantile 95%
Germany, France	0.0	5.9	1.7	1.0	3.6
+ Austrian, Belgium, Netherlands	0.3	10.4	3.4	2.0	8.2
+ Italy, Spain	2.1	12.0	6.2	2.1	9.6

Table 2: Descriptive statistics of tightness factor.



(a) Germany, France

(b) +Austria, Belgium, Netherlands



(c) +Italy, Spain

Figure 3: Distribution of the tightness factor for different issuer groups.

As Table 2 and Figure 3 show, the distribution of the tightness factor for Germany and France is quite narrow and skewed: on average the bid-ask spreads have to be increased by 70%, while the factor of 3.6 makes quotes of German and French instruments consistent in 95% of observations. Adding Austria, Belgium and the Netherlands shifts the distribution to the right and makes it more heavy-tailed: the 95% quantile increases faster than the standard deviation. This may be due to the greater sensitivity of added countries to the market stresses during the peak of the euro zone crisis. Including Italy and Spain shifts the distribution further to the right and makes it more dispersed, but more symmetric. In this we see a sign of the determinants of Italian and Spanish instruments being fundamentally different from those of the other countries in our sample.

5 Conclusion

In this paper we proposed an approach to testing the consistency of bond and CDS quotes. The approach is based on the widely accepted pricing framework. The quotes data is considered consistent if there exists a non-empty set of risk-free yield and default intensity curves, which ensure that the model prices of the instruments lay between bid and ask quotes.

The application of the proposed approach to the euro zone sovereign bond and CDS data shows that, in general, the full sample of bond quotes for an individual issuer is inconsistent. We argue in favor of the preliminary selection of the benchmark bonds in order to ensure the homogeneity and consistency of bond data. We also document that, although CDS and benchmark bond quotes are consistent for each single issuer, they are typically inconsistent across issuers, but to some extent the issuers may be grouped according to consistency within a group. We identified the following groups: 1) Germany and France; 2) Austria, Belgium and the Netherlands; 3) Italy and Spain.

We also proposed a consistency adjustment procedure, which quantifies the discrepancies in bond and CDS pricing across the issuers. This procedure is based on the notion of tightness factor; that is, the least factor by which one needs to widen the bid-ask spreads of bonds and CDS to make the feasibility band for the risk-free curve estimate across the issuers non-empty. We concluded that the group consisting of Italy and Spain contributes the most to the magnitude of the tightness factor. It makes us think the price determinants of Italian and Spanish instruments might be fundamentally different from those of the other countries in our sample. The inconsistency of bond and CDS quotes across the issuers may also be a sign of a latent factor left out in our methodology. In this case, the tightness factor may be interpreted as a measure of the precision of the risk-free curve estimation problem for the given data. We were thus able to assess the possible solution precision of constructing a euro zone risk-free curve.

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Contact Author:
Victor Lapshin, National Research University Higher School of Economics (Moscow, Russia).
Laboratory for Financial Engineering and Risk Management. Researcher;
E-mail: vlapshin@hse.ru