Profit sharing contracts and quality improvements in competing supply chains

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Abstract

This paper contributes to the hold-up problem resolving, quality improvement in supply chains, and the contract theory. We aim to investigate the effects of competitive environment on manufacturer's incentive to change the quality of product and the efficiency of the profit sharing contracts. Our results show that the stronger competition motivates the quality improvement and under particular parameters profit sharing contract is an efficient mechanism of coordination within the supply chain.

JEL classification: D24, D43, L14

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Introduction

The problem of coordination within a supply chain has got a lot of attention in both economics and management literature. In this article we consider the supply chain consisting of two firms: the supplier, who produces an intermediate good, and the manufacturer, who uses the intermediate good to produce a final product. The quality of the intermediate good (which may be either high or low) determines the quality of the final product. To produce the intermediate good of high quality the supplier must undertake the costly investments. We assume that the quality of the intermediate good is observable, but not contractible. Thus, the payment from manufacturer to supplier cannot be conditional on the quality level. In such situation neither a simple linear pricing contract nor a two-part tariff may serve as efficient mechanism of coordination. We assume the manufacturer holds all the bargaining power and uses a profit sharing contract to motivate the supplier to undertake the desired level of the investments. We assume that the supply chain competes with the other firms that produce non-perfect substitutes.

Our main purposes are (i) to investigate the efficiency of the profit sharing contracts for achieving coordination within the supply chain, and (ii) to study how does the market environment (in particular, the strength of competition measured by the number of competing firms) affects the equilibrium chain's profit, the manufacturer's incentive to produce the high/low quality product and change the parameters of the optimal contract. With the presence of a competitive fringe under several assumptions we argue that the strength of competition affects the supply chain internal coordination.

An investigation of the effect of market competition on the optimal within chain decision is the main novelty of a proposed model. Up to now, most of authors consider the interaction in a single chain or at most the interaction between two supply chains. Wu et al. (2009) consider two competing chains in the presence of the demand uncertainty and compare three organizational forms (full integration, bargaining over wholesale price and Stackelberg type of interaction) to determine equilibrium market structure. Ai et al. (2012) use similar setup to analyze the role of the full return policy on market outcome. Anderson and Bao (2010) analyze a price competition between two chains and obtain results for an arbitrary number of firms. However, their analysis is limited to the cases of a full integration and complete.

Also our article links three different fields of literature: the hold-up problem resolving, the quality improvement in supply chains, and the contract theory. The classical papers of Grossman and Hart (1986), Hart and Moore (1990), and Williamson (1975, 1985) demonstrate that non-contractibility of investments leads to underinvestment and investigate the efficient property rights allocation. Several other solutions of the hold-up problem have been also proposed: vertical integration (Williamson, 1979), shifting control rights (Aghion and Bolton, 1992), and authority relationship (Aghion and Tirol, 1997). Chung (1991), Aghion et al. (1994), Noldeke and Schmidt (1995) show efficiency of the ex-post renegotiation when one party has the whole bargaining power. Acemoglu et al. (2007) analyze the relationship between contractual incompleteness and technology adoption.

Xie et al. (2011) consider two supply chains competing in quality of products produced and compare two possible structures of each chain: full integration and decentralization. Chao et al., (2009) focus on not contractible investments in quality by supplier in the single supply chain and show that a cost sharing contracts may lead to the efficient outcome. Cachon and Lariviere (2005) assert that revenue-sharing contracts coordinate a supply chain with retailers competing in quantities. Inderst and Wey (2007) use the rent-sharing contracts considering bargaining power with the cooperative game framework in their model of strategic oligopoly. Van der Veen and Venugopal (2005) specified the hold-up problem according to movie studio and video rental store and claim that revenue-sharing contract can optimize the chain as a partnership but is easier to implement.

Thus, we investigate the impact of competitive fringe on firms' relations within chain, namely, on firms' incentives for profit sharing and investing to resolve the hold-up problem. Moreover, up to now, competition between chains has been considered (with few exceptions) in a simple setup of duopoly. Due to difficulties arising when a fully strategic behavior is analyzed, authors employ only simple structures of interactions within and among chains. The monopolistic competition framework allows us go further and provides more insights on both types of interactions. In particular, we obtain that (i) market competition affects an efficiency of a within-chain interaction, (ii) for some range of parameters, efficiency may be restored, and (iii) stronger competition may lead to higher chain's profit. In other words, we have an example of "profit increasing competition".

The paper is structured as follows. The next section is devoted to the formal description of our model of supply chain operated in the competitive market. Futher, in Section 3 we find an equilibrium of the model and the parameter of the profit sharing contract. In Section 4 we disscus the equilibrium and efficiency under sertain assumptions. We summarize our findings in Section 5.

The Basic Model and Preliminary Results

Consider a supply chain consisting of two firms: the supplier (*S*), who produces the intermediate good, and the manufacturer (*M*), who uses the intermediate good to produce the final product. The intermediate good may be of a high or a low quality and the quality level of the intermediate good determines the quality of the final product, which may be also either high or low, $\upsilon = \{H, L\}$. We assume that the quality of the intermediate good is observable, but not contractible. Thus, the payment from the manufacturer to the supplier cannot be conditional on the quality level. We assume that the intermediate product of low quality may be produced at marginal cost c_L, which we normalize to zero. To produce the intermediate good of high quality the supplier must bear the cost $c_H > c_L$. The manufacturer transforms one unit of intermediate product to one unit of final good at the marginal cost $c_M \geq 0$.

We consider the following interaction within the chain. At stage one, the manufacturer requires the suppler to produce Q units of the intermediate good and commits to the share $\alpha \in (0, 1)$ of splitting the operational profit. At stage two, the supplier chooses the level of quality of the intermediate product $\upsilon = \{H, L\}$. At the third stage, the manufacturer sells Q units of the final product of quality υ. Simultaneously, each competing firm $i \in (0, N]$ sells its output q_i to maximize its profit.

The chain competes at the market with a competitive fringe. Namely, there is a mass N of firms producing heterogenous products, which are not-perfect substitutes for the manufacturer's product. The representative consumer' utility function is quasi-linear with a quadratic subutility (Ottaviano et al., 2002):

$$U = \int_0^N \left[A^i q_i - B^i q_i^2/2 - G^i q_i \int_0^N q_j dj \right] di + E,$$

where $A^0 = A(\upsilon), B^0 = B(\upsilon), G^0 = G(\upsilon)$ are parameters that characterize the utility from a consumption of the chain's product (and determine the demand on the manufacturers' product depending on its quality), $q_0 = Q$ is a consumption of the manufacturer's product; $A^i = A^f, B^i = B^f, C^i = C^f$ for all $i \in (0, N]$ are parameters that determine the demand for the differentiated products produced by the firm i from the competitive fring, and q_i is its output, and E is the consumer's expenditures for outside products. The representative consumer maximizes her utility choosing $Q,$ $\{q_i; i \in (0, N]\},$ and E subject to a budget constraint $\int_0^N p_i q_i + E \leq I,$ where I is the consumer's income. We assume that I is sufficiently high, such that in equilibrium $E > 0$.

The utility function corresponds to the following system of demands:

$$p_0 = P(\upsilon) = A(\upsilon) - B(\upsilon)Q - G(\upsilon)\int_0^N q_j dj,$$
$$p_i = A^i - B^i q_i - G^i \int_0^N q_j dj.$$

The profit of each competitive firm is as follows:

$$\pi_i = (p_i - c^f)q_i.$$

Selling Q units of quality υ , the manufacturer gets the following operational profit $(P(\upsilon) - c_M)Q$.

Suppose the manufacturer uses a profit sharing contract in its relation with the supplier. That is, the manufacturer commits that the supplier will get the share α of the operational profit. Then, the final manufacturer's and supplier's profits are as follows, respectively:

$$\pi_M(\upsilon, Q, \alpha) = (1 - \alpha)(P(\upsilon) - c_M)Q,$$
$$\pi_S(\upsilon, Q, \alpha) = \alpha(P(\upsilon) - c_M)Q - c_S(\upsilon)Q.$$

Thus, the supplier bears the full cost of producing the product of quality υ , while it receives the share α of the operational profit.

Characterization of equilibrium

Competitive fringe. We start with analysis of the optimal behavior of competing firms. While all competitors and the supply chain are monopolists for their product, due to the small size relative to market (more precisely each firm is of measure zero), actions of any single firm have no impact on the market aggregates. Thus, the optimality condition for an output of a competing firm, $d\pi_i/dq_i = 0$ is given as follows:

$$A^f - 2B^f q_i - G^f \int_0^N q_j dj - c^f = 0.$$

Hereinafter we assume that $q_i = q$ for all $i \in (0, N]$. In such a symmetric equilibrium we have the following:

$$q = \frac{A^f - c^f}{2B^f + G^f N}.$$

Then the total output of the competitive fringe is as follows:

$$Nq = \frac{(A^f - c^f)N}{2B^f + G^f N},$$

and it monotonically increases in N and bounded by the level $z \equiv (A^f - c^f)/G^f$. The profit of each competitive firm $\pi = (A^f - c^f)^2/(2 + G^f N)^2 > 0$ for any N.

The chain: the supplier's decision. Let denote $P^H \equiv P(H), P^L \equiv P(L)$ and π_S^H and π_S^L be the supplier's profit when the quality is high and low, respectively. Given the output level Q and the profit share α, the supplier prefers the high quality if and only if

$$\begin{aligned} \pi_S^H &= \alpha(P^H - c_M)Q - c_H Q \\ &> \alpha(P^L - c_M)Q = \pi_S^L. \end{aligned}$$

This is equivalent to

$$\alpha([A^H - A^L] - [B^H - B^L]Q + [G^L - G^H]Nq) > c_H.$$

The impact of the quality level on the demand is determined by $A^H - A^L, B^H - B^L, G^L - G^H$. Given this impact and the cost of producing the high quality good c_H, the supplier chooses to produce the high quality if its share of the operational profit is sufficiently large: $\alpha > \alpha^*$, with α^* given as follows

$$\alpha^* \equiv \frac{c_H}{[A^H - A^L] - [B^H - B^L]Q + [G^L - G^H]Nq}.$$

Note, α^* describes the minimum level of the profit share which motivates the supplier to produce high quality good. As the output of the competitive fringe Nq increases in N , and $G^H < G^L$, we have that α^* decreases in N . The parameters of the model are such that $c_H < [A^H - A^L] - [B^H - B^L]Q + [G^L - G^H]Nq$ for the α to be less then 1. Otherwise, there is no contract, which could ensure the production of high quality.

The chain: the manufacturer's decision. If the manufacturer decides to produce the low quality product, then setting $\alpha = 0,$ it may obtain the profit

$$\max_{Q^L} \pi_M^L = (P^L - c_M)Q^L.$$

Setting $\alpha = \alpha^*$, the manufacturer may obtain the final profit

$$\max_{Q^H} \pi_M^H = (1 - \alpha^*)(P^H - c_M)Q^H.$$

Model solution

Special case I

To simplify an exposition of the model, let's assume that the manufacturer meets the capacity contrarian $Q^\upsilon \leq 1$, whereas the optimal Q^{*L} and Q^{*H} solving Eqs. (QL) and (QH) are not less than unity. Effectively, this assumption prescribes the fixed level of outputs for each quality level $Q^L = Q^H = 1.$ The sufficient condition for this is

$$Q^{*L} = \frac{A^L - G^L Nq - c_M}{2B^L} \geq 1.$$

Then, the level of the profit share α^* , which is sufficient to motivate the supplier to produce the high quality good, may be written as follows:

$$\alpha^*(N) \equiv \frac{c_H}{D + [G^L - G^H]Nq},$$

where $D = [A^H - A^L] - [B^H - B^L] > 0.$ Note, that α^* decreases in N , $\alpha^* = c_H/D$ for

$N = 0$, and

$$\lim_{N\to\infty} \alpha^* = \frac{c_H}{D + [G^L - G^H]z},$$

so $0 < c_H < D$ in order to satisfy the condition $0 < \alpha < 1$.

Thus, for $N = 0$ the manufacturer has no incentives to produce the high quality product if $\pi_M^H < \pi_M^L$ or

$$c_H > \frac{D^2}{A^H - B^H - c_M}.$$

Now, let's consider the case when N is large enough for the manufacturer's profit to be higher if it sells the high quality product. The condition $\lim_{N\to\infty} \pi_M^H > \lim_{N\to\infty} \pi_M^L,$ is equivalent to the following:

$$c_H < \frac{(D + [G^L - G^H]z)^2}{A^H - B^H - G^H z - c_M}.$$

In other words, whenever the conditions (left cond 1) and (right cond 1) hold there exists unique $\widetilde{N}$ such that for all $N \geq \widetilde{N}$ it holds that (*i*) the supplier chooses to produce the intermediate good of the high quality ($H \succ_s L$) and (*ii*) the manufacturer profit is large comparing to the low quality of $\pi_M^H \geq \pi_M^L$. Clearly, for all $N \geq \widetilde{N}$ the manufacturer's profit is maximized when $\alpha = \alpha^*(N)$. For all $N < \widetilde{N}$, we have $\pi_M^H < \pi_M^L$ with the share $\alpha^*(N)$, which is sufficient to motivate the supplier to produce the high quality good. We can summarize this in the following proposition.

Proposition *Suppose* $Q^L = Q^H = 1$ *and conditions (left cond 1) and (right cond 1) hold. Then, (i) if the competitive strength is low (* $N \leq \widetilde{N}$ *), the supply chain produces low quality product and the optimal profit share* $\alpha^* = 0$ *, (ii) if the competitive strenght is high (* $N \geq \widetilde{N}$ *), the supply chain produces high quality product, and the optiml profit share* $\alpha^* > 0$ *is determined by Eq. (a*1).*

Supply chain's profits. For any $N < \widetilde{N}$, we have

$$\pi_M^L = A^L - B^L - G^L Nq - c_M,$$
$$\pi_S^L = 0.$$

For any $N \geq \widetilde{N}$ the profits given as follows:

$$\pi_M^H = (1-\alpha^*)(A^H - B^H - G^H Nq - c_M),$$
$$\pi_S^H = \alpha^*(A^H - B^H - G^H Nq - c_M) - c_H.$$

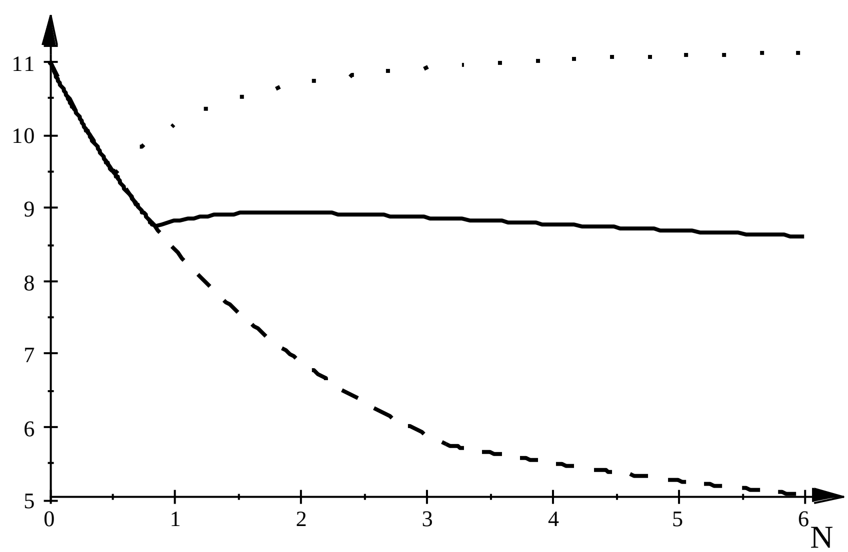
Using Eq. (a*1) we have

$$\pi_S^H = \left(\frac{A^L - B^L - G^L Nq - c_M}{D + [G^L - G^H]Nq}\right) c_H.$$

Thus, $\pi_S^H > 0$ for any $N \geq \widetilde{N}$. Morever, π_S^H decreases in N. It is worth noting, that this is the analog of information rent in a principal-agent problem. !!!!!!!!!!

As α^* decreases in N, and the operational profit, $A^H - B^H - G^H Nq - c_M$, decreases in N, the impact of changes in N on the manufacturer's profit is ambiguous. We illustrate it by the following example.

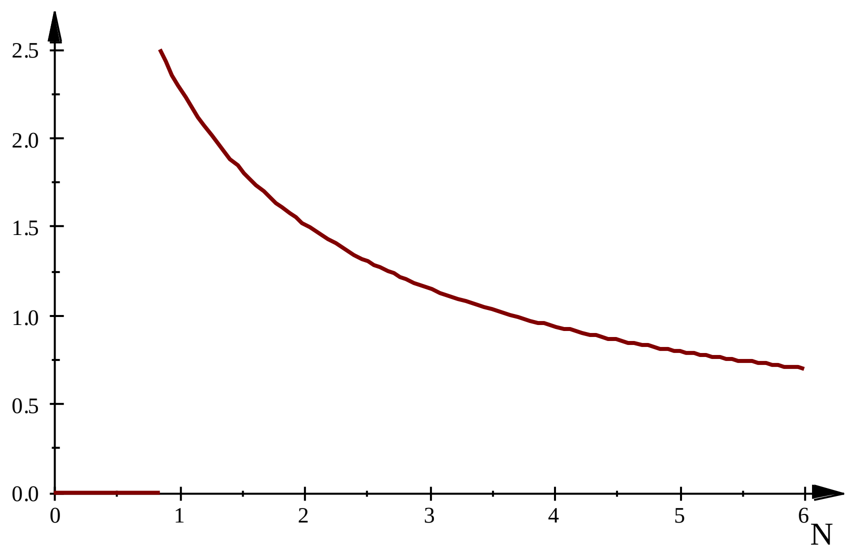
Example 1. The Figures 1 depicts the plots of manufacturer's profit with the same parameters of $A^H = 14$, $A^L = 12$, $A^f = 10$, $c_M = c^f = 0, G^L = G^f = 0.7$, $B^H = 0.8$, $B^L = B^f = 1$, $c_H = 1$, and different parameters of $G^H = 0.06$ for dots line, $G^H = 0.3$ for solid line, and of $G^H = 0.6$ for dash line. Thus, here we can see an increasing, non-monotonic and decreasing manufacturer' profit, respectively.

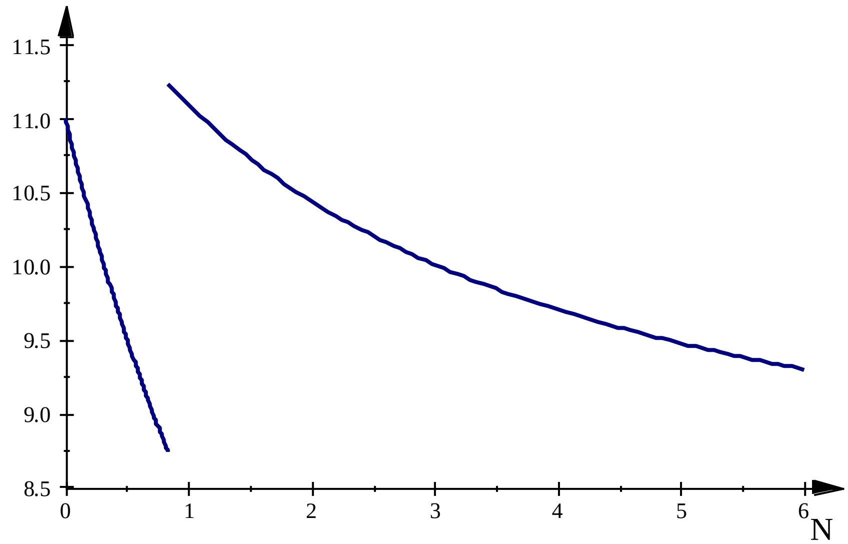


As $\widetilde{N}$ is such that $\pi_M^L(\widehat{N}) = \pi_M^H(\widehat{N})$ and the supplier's profit jumps from zero to some positive level, we have that the profit of the chain (that is the sum of the manufacturer's and the supplier's profit, $\Pi \equiv \pi_M + \pi_S$) aslo has a positive jump at $\widetilde{N}$. This leads to the following statement.

Proposition *Suppose* $Q^L = Q^H = 1$. *Then an increase in the number of competitive firms from* $\widetilde{N} - \epsilon$ *to* $\widetilde{N} + \epsilon$ *results in a strictly positive increase in the chain's profit* Π .

The Figures 2-3 depict the plots of manufacturer's profit, the supplier's profit and the chain profit for the parameters $A^H = 14,$ $A^L = 12,$ $A^f = 10,$ $c_M = c^f = 0, G^L = G^f = 0.7,$ $G^H = 0.3,$ $B^H = 0.8,$ $B^L = B^f = 1,$ $c_H = 1$.





Efficiency

Proposition *Suppose* $Q^L = Q^H = 1$ *and conditions (left cond 1) and (right cond 1) hold. Then, as* $N \geq \tilde{N}$ *the profit sharing contract maximizes the chain's profit.*

Let us prove this proposition further. Consider the problem of maximization of the chain's profit under capacity constraint:

$$\max_{\upsilon} \Pi = [A(\upsilon) - B(\upsilon)] - G(\upsilon)Nq - c_{\upsilon}.$$

The optimal $\upsilon = H$ if and only if

$$[A^H - B^H] - G^H Nq - c_H \geq [A^L - B^L] - G^L Nq,$$

or

$$D + [G^L - G^H]Nq \geq c_H,$$

or

$$Nq \geq \frac{c_H - D}{G^L - G^H}.$$

As $c_H < D,$ it is always optimal for a chain to set $\upsilon = H$ for any N . Therefore, the profit sharing contract leads to efficient outcome (maximizes the chain's profit) for $N \geq \tilde{N}$. Otherwise, the outcome is not optimal from the chain's point of view (however, it is optimal for the manaufacturer).

Special case II

Here we relax the assumption about the capacity constraint but we assume $B^H = B^L = B$ to simplify futher calculations. As is well known, the parameter $B^i > 0$ in the quadratic utility function (U) exhibits love for variety. Fixing this parameter we reduce the utility to a symmetric dispersion in the consumption of varieties.

Solving Eqs. (QL) and (QH) the optimal Q^{*L} and Q^{*H} are

$$Q^{*H} = \frac{A^H - G^H Nq - c_M}{2B},$$
$$Q^{*L} = \frac{A^L - G^L Nq - c_M}{2B}.$$

Then, the level of the profit share α^* , which is sufficient to motivate the supplier to produce the high quality good, may be written as follows:

$$\alpha^*(N) \equiv \frac{c_H}{A^H - A^L + [G^L - G^H]Nq}.$$

Note, that α^* decreases in N as in the previous case, $\alpha^* = c_H/[A^H - A^L]$ for $N = 0$, and

$$\lim_{N\to\infty} \alpha^* = \frac{c_H}{A^H - A^L + [G^L - G^H]z},$$

so $0 < c_H < [A^H - A^L]$ in order to satisfy the condition $0 < \alpha < 1$.

Otherwise, the manufacturer may obtain the final profit

$$\pi_M^H(Q^{*H}) = \frac{(A^H - G^H Nq - c_M)^2 (A^H - A^L - (G^H - G^L)Nq - c_H)}{4B(A^H - A^L - (G^H - G^L)Nq)}.$$

Thus, for $N = 0$ the manufacturer has no incentives to produce the high quality product if $\pi_M^H < \pi_M^L$ or

$$c_H > \frac{(A^H - A^L)^2 (A^H + A^L - 2c_M)}{(A^H - c_M)^2}.$$

Now, let's consider the case when N is large enough for the manufacturer's profit to be higher if it sells the high quality product. The condition $\lim_{N\to\infty}\pi_M^H > \lim_{N\to\infty}\pi_M^L,$ is equivalent to the following:

$$c_H < \frac{(A^H - A^L + [G^L - G^H]z)^2 (A^H + A^L - [G^L + G^H]z - 2c_M)}{(A^H - G^H z - c_M)^2}.$$

In other words, whenever the conditions (left cond 2) and (right cond 2) hold there exists unique $\widetilde{N}$ such that for all $N \geq \widetilde{N}$ it holds that (*i*) the supplier chooses to produce the intermediate good of the high quality ($H \succ_s L$) and (*ii*) the manufacturer profit is large comparing to the low quality of $\pi_M^H \geq \pi_M^L$. Clearly, for all $N \geq \widetilde{N}$ the manufacturer's profit is maximized when $\alpha = \alpha^*(N)$. For all $N < \widetilde{N}$, we have $\pi_M^H < \pi_M^L$ with the share $\alpha^*(N)$, which is sufficient to motivate the supplier to produce the high quality good. We can summarize this in the following proposition.

Proposition *Suppose* $B^H = B^L = B$ *and conditions (left cond 2) and (right cond 2) hold. Then, (i) if the competitive strength is low* $N \leq \widetilde{N}$ *, the supply chain produces low quality product and the optimal profit share* $\alpha^* = 0$ *, (ii) if the competitive strenght is high (* $N \geq \widetilde{N}$ *), the supply chain produces high quality product, and the optiml profit share* $\alpha^* > 0$ *is determined by Eq. (a*2).*

Firms' profits. For any $N < \widetilde{N}$, the manufacturer decides to produce the low quality product, and we have

$$\pi_M^L = \frac{(A^L - G^L Nq - c_M)^2}{4B},$$
$$\pi_S^L = 0.$$

For any $N \geq \widetilde{N}$ the profits given as follows:

$$\pi_M^H = \frac{(A^H - G^H Nq - c_M)^2 (A^H - A^L + (G^L - G^H)Nq - c_H)}{4B(A^H - A^L - (G^H - G^L)Nq)},$$
$$\pi_S^H = c_H\left[\frac{(A^H - G^H Nq - c_M)^2}{4B(A^H - A^L + (G^L - G^H)Nq)} - 1\right].$$

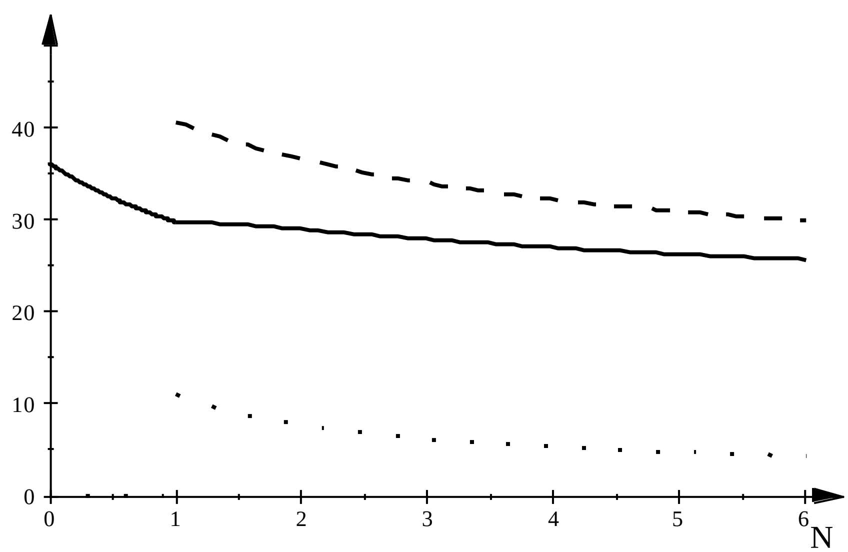
The joint profit of the manufacturer and the supplier in this case is

$$\pi_M^H(Q^{*H}) + \pi_S^H(Q^{*H}) = \frac{(A^H - G^H Nq - c_M)(A^H - G^H Nq - 2c_H - c_M)}{4B}.$$

As the supplier's profit jumps from zero to some positive level at $\pi_M^L(\widetilde{N}) = \pi_M^H(\widetilde{N})$, we have that the profit of the chain (that is the sum of the manufacturer's and the supplier's profit, $\Pi \equiv \pi_M + \pi_S$) aslo has a positive jump at $\widetilde{N}$. This leads to the following statement.

Proposition *Suppose* $B^H = B^L = B$. *Then an increase in the number of competitive firms from* $\widetilde{N} - \epsilon$ *to* $\widetilde{N} + \epsilon$ *results in a strictly positive increase in the chain's profit* Π .

Further, we provide here a particular example that demonstrates the effect of the external competitive environment on the internal manufacturer and supplier interaction in supply chain. The Figure 4 depicts the manufacturer's profit (solid line), the supplier's profit (dots line) and the joint profit (dash line) for the parameters $A^H = 14$, $A^L = 12$, $A^f = 10$, $B^f = B^H = B^L = B = 1$, $G^L = G^f = 0.7$, $G^H = 0.3$, $c_H = 1$, $c_M = c^f = 0$.



Efficiency

Proposition *Suppose* $B^H = B^L = B$ *and conditions (left cond 2) and (right cond 2) hold. Then, as* $N \geq \widetilde{N}$ *the profit sharing contract maximizes the chain's profit.*

Let us prove this proposition further. Consider the problem of maximization of the chain's profit with no capacity constraint but fix parameter $B = B(\upsilon)$:

$$\max_{Q(\upsilon),\upsilon} \Pi = Q(\upsilon)[A(\upsilon) - BQ(\upsilon) - G(\upsilon)Nq - c_M - c_\upsilon].$$

The optimal Q^{*L} and Q^{*H} are

$$Q^{*H} = \frac{A^H - G^H Nq - c_M - c_H}{2B},$$
$$Q^{*L} = \frac{A^L - G^L Nq - c_M}{2B}.$$

Then the optimal $\upsilon = H$ if and only if

$$A^H - G^H Nq - c_H \geq A^L - G^L Nq,$$

or

$$Nq \geq \frac{c_H - [A^H - A^L]}{G^L - G^H}.$$

As $c_H < [A^H - A^L]$, it is always optimal for a chain to set $\upsilon = H$ for any N . Therefore, the profit sharing contract leads to efficient outcome (maximizes the chain's profit) for $N \geq \widetilde{N}$. Otherwise, the outcome is not optimal from the chain's point of view (however, it is optimal for the manaufacturer).

General case

Solving Eq. (QL) we obtain the optimal Q^{*L} and the manufacturer's profit $\pi_M^L(Q^{*L})$:

$$Q^{*L} = \frac{(2B^f + G^f N)(A^L - c_M) - (A^f - c^f)G^L N}{2B^L(2B^f + G^f N)},$$
$$\pi_M^L = \frac{((2B^f + G^f N)(A^L - c_M) - (A^f - c^f)G^L N)^2}{4B^L(2B^f + G^f N)^2}.$$

and solving Eq. (QH) we can obtain the optimal Q^{*H} and the manufacturer's profit $\pi_M^H(Q^{*H})$. Under particular conditions on parameters of the model there exists unique $\widetilde{N}$ such that for all $N \geq \widetilde{N}$ it holds that (*i*) the supplier chooses to produce the intermediate good of the high quality ($H \succ_s L$) and (*ii*) the manufacturer profit is large comparing to the low quality of $\pi_M^H \geq \pi_M^L$.

Clearly, for all $N \geq \widetilde{N}$ the manufacturer's profit is maximized when $\alpha = \alpha^*(N)$. For all $N < \widetilde{N}$, we have $\pi_M^H < \pi_M^L$ with the share $\alpha^*(N)$, which is sufficient to motivate the supplier to produce the high quality good. In general case we can claim this based on the two special cases above. We can summarize this in the following proposition.

Proposition *Under certain conditions on parameters of the model, (i) if the competitive strength is low* $N \leq \widetilde{N}$ *, the supply chain produces low quality product and the optimal profit share* $\alpha^* = 0$ *, (ii) if the competitive strenght is high (* $N \geq \widetilde{N}$ *), the supply chain produces high quality product, and the optiml profit share* $\alpha^* > 0$ *.*

Supply chain's profits. For any $N < \widetilde{N}$, we have

$$\pi_M^L = \frac{((2B^f + G^fN)(A^L - c_M) - (A^f - c^f)G^LN)^2}{4B^L(2B^f + G^fN)^2},$$

$$\pi_S^L = 0.$$

For any $N \geq \widetilde{N}$ the profits given as follows:

$$\pi_M^H = (1 - \alpha^*)(A^H - B^HQ^{*H} - G^HNq - c_M)Q^{*H},$$
$$\pi_S^H = \alpha^*(A^H - B^HQ^{*H} - G^HNq - c_M)Q^{*H} - c_H.$$

Thus, $\pi_S^H > 0$ for any $N \geq \widetilde{N}$. Morever, π_S^H decreases in N . As $\widetilde{N}$ is such that $\pi_M^L = \pi_M^H$ and the supplier's profit jumps from zero to some positive level, we have that the profit of the chain (that is the sum of the manufacturer's and the supplier's profit, $\Pi \equiv \pi_M + \pi_S$) aslo has a positive jump at $\widetilde{N}.$ This leads to the following statement.

Proposition *An increase in the number of competitive firms from* $\widetilde{N} - \epsilon$ *to* $\widetilde{N} + \epsilon$ *results in a strictly positive increase in the chain's profit* Π *.*

Efficiency

Consider the problem of maximization of the chain's profit:

$$\max_{Q(\upsilon),\upsilon} \Pi = Q(\upsilon)[A(\upsilon) - B(\upsilon)Q(\upsilon) - G(\upsilon)Nq - c_M - c_\upsilon].$$

In general case the optimal $\upsilon,$ and $Q^*(\upsilon)$ for the chain differ from the optimal $\upsilon,$ and $Q^*(\upsilon)$ for the manufacturer obtained from the Eqs. (QL) and (QH). Therefore, the profit sharing contract does not always lead to efficient outcome, and thence, the following statement is true:

Proposition *The profit sharing contract does not maximizes the chain's profit in general case.*

Conclusion

We analyze the role of profit sharing contracts for the coordination within the supply chain when quality improvement is associated with the hold-up problem. We show that the strength of market competition affects the firms' relations within the chain in a several ways. Firstly, we find that market competition affects an efficiency of the within-chain interaction. Under a weak competition the manufacturer may prefer to keep all profit; thus other partners have no incentives for costly quality improvement. However, the stronger competition increases the role of quality improvement and results in the profit sharing that motivates costly investments in quality improvement. Secondly, we find that for some range of parameters, the profit sharing contract serves as efficient mechanism of the coordination within the supply chain. Finally, we find that the stronger competition may lead to the higher chain's profit.

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