# **Syllabus for Further Calculus**

A course for the undergraduate students on specialization Mathematics and Economics

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**Course description**

Further Calculus is a one-semester course for the fourth year students studying at ICEF specializing in Mathematics and Economics. It is based on the Further Calculus (MT2176) course of the University of London (UoL) with further expansions into selected topics from real analysis, theory of functions and functional analysis.

The central topic of the course is the theory of integration in real analysis. The emphasis is not on the acquiring the techniques and methods (it was done in previous years), but on the theoretical ideas and constructions behind them. Starting by recalling the Riemann integral for real-values functions of one real variable (a topic discussed in the 1st year Calculus course), we then move on to the definite and indefinite integrals of functions of several variables, Laplace transforms and their applications, initial topics from measure theory and Lebesgue integration. If time permits we can discuss the differentiable manifolds and integration of the differential forms on them. We also plan to demonstrate some very recent applications of these classical theories.

During the course the students should acquire a solid understanding of how various integration constructions work, how the classical definitions of Riemann and Lebesgue can be generalized to domains other than subsets of real line or real plane.

Upon completion of this course the students will have to take the University of London (UoL) exam at the end of the seventh semester of their studies at ICEF.

**Learning objectives**

Having taken this course you should

* demonstrate knowledge of the subject matter, terminology, techniques and conventions covered in the subject;
* demonstrate an understanding of the underlying principles of the subject;
* demonstrate the ability to solve problems involving an understanding of the concepts.

**Teaching Methods**

The course program consists of:

* lectures,
* classes,
* regular self-study based on class problem sets, regular homework assignment problem sets and extra problem sets.

**Assessment and grade determination**

There are the following forms of control:

* written home assignments posted and turned in every week;
* written mock exam at the end of module 1.
* University of London exam by the end of module 2 on Further Calculus.

The cumulative final grade is comprised of:

* average grade for the home assignments (30%);
* mock exam at the end of module 1 (21%);
* UoL external exam (49%).

**Reading**

Recommended by UoL Further Calculus syllabus:

1. Binmore, K. and J. Davies, *Calculus: concepts and methods*. (Cambridge University Press, 2002) second revised edition.
2. Ostaszewski, A. *Advanced mathematical methods*. (Cambridge University Press, 1991).
3. Adams, R.A. and C. Essex Calculus: A complete course. (Toronto: Pearson, 2009), seventh edition.
4. Wrede, R. C. and M. Spiegel Schaum's outline of advanced calculus. (London: McGraw-Hill, 2010) third edition.

Additional reading:

1. Zorich V. A., *Mathematical Analysis*, vol.1, 2 (MCCME, 2012).
2. Kolmogorov A.N., Fomin S.V., *Elements of the Theory of Functions and Functional Analysis* (any Russian or English edition).
3. Warner, Frank W., *Foundations of Differentiable Manifolds and Lie Groups*. (Springer, 1983).

**Course outline**

1. The Riemann integral
   1. Definition and basic properties.
   2. Fundamental Theorem of Calculus.
2. Improper integrals
   1. Definitions and basic properties
   2. Tests for convergence and divergence
3. Double integrals
   1. Definitions and basic properties
   2. Fubini theorem
   3. Change of variables techniques
   4. Improper double integrals
4. Integrals of functions of several real variables
   1. Triple integrals.
   2. Integrals in higher dimensions.
5. Manipulation of integrals
   1. Manipulation of proper integrals
   2. Manipulation of improper integrals
6. Laplace transform
   1. Definitions and basic properties
   2. Solving ODEs with constant coefficients
   3. Convolutions
7. Measure theory and Lebesgue integral
   1. Lebesgue measure on real line
   2. Measurable sets
   3. Definition and properties of the integral
   4. Integration for general measures
   5. Radon-Nykodim theorem
   6. Product measures
8. Integration of differentiable forms on differentiable manifolds\*

Note: topics marked with \* will be only touched at the introductory level in the additional homework assignments. They are not included in any of the internal exams.