

XXIV R-S-P seminar, Theoretical Foundation of Civil Engineering (24RSP) (TFoCE 2015)

Asymptotic solution for deep bed filtration with small deposit

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Abstract

Filtering the suspension in a porous soil is important for long-term evaluation of soil strength in the construction of underground and hydrotechnical structures. A size-exclusion model of solid particle capture for a flow of suspension in a porous media is considered: particles pass freely through the large pores and get stuck at the inlet of small pores whose diameter is less than the particle size. The asymptotic solution for the concentrations of suspended and retained particles is constructed under the assumption that the limit deposit is small.

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Peer-review under responsibility of organizing committee of the XXIV R-S-P seminar, Theoretical Foundation of Civil Engineering (24RSP)

Keywords: Suspension; Porous media; Deep bed filtration; Filtration coefficient; Size exclusion; Asymptotic solution.

1. Introduction

Filtration of suspension in a porous media is an important problem of the underground fluid mechanics, which is often found in nature and technology [1-3]. The design and construction of tunnels, underground and hydraulic structures must take into account the movement of groundwater and its influence on the structure and strength of the soil. In various wastewater and liquid industrial waste cleaning systems solid particles are retained by porous filters [4, 5].

Capture of particles in a porous media can be determined by different physical mechanisms: the electrostatic and mechanical forces, viscosity, diffusion, etc. The paper deals with mechanical-geometric model of particle capture by the filter pores [6, 7]. It is assumed that the particles pass freely through the large pores and get stuck in the entrance of pores smaller than the particle size. One small pore is clogged by a single particle; the retained particle can not leave the pore inlet to join the fluid flow (Fig. 1).

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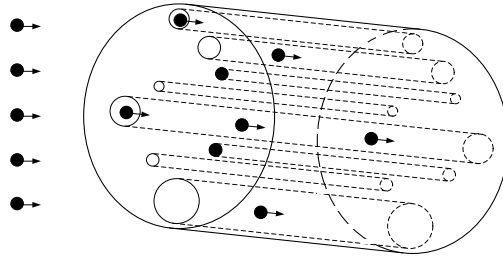


Fig. 1. Suspended and retained particles of suspension in a porous media.

Classical stochastic model of the suspension transport in porous media includes balance equation of suspended and retained particles and the kinetic equation for the retained particles. In this paper we consider the system of equations obtained by macro-averaging of micro-stochastic equations for particles of the same size and pores of different sizes [8]. After a long period of filtration all small pores are clogged by deposit while the large pores are free and pass the suspended particles, so the maximum deposit limit is proportional to a number of small pores. Assuming that the deposit limit is small the asymptotic solution of the filtration problem is constructed.

2. Governing Equations

In the domain $\Omega = \{0 < x < 1, t > 0\}$ concentrations of suspended $C(x, t)$ and retained $S(x, t)$ particles satisfy quasilinear hyperbolic system of equations ([9])

$$\frac{\partial(a(S)C + S)}{\partial t} + \frac{\partial(b(S)C)}{\partial x} = 0; \quad \frac{\partial S}{\partial t} = \Lambda(S)C; \tag{1}$$

where $\Lambda(S)$ is a filtration coefficient, $a(S)$, $b(S)$ are given functions dependent on the porosity and permeability of the media, $\Lambda(S)$, $a(S)$, $b(S)$ are positive for $S \geq 0$.

The boundary conditions for the system (1) are placed at the filter inlet $x = 0$ and at the initial moment $t = 0$

$$C(x, t)|_{x=0} = p, \quad p > 0; \tag{2}$$

$$C(x, t)|_{t=0} = 0; \quad S(x, t)|_{t=0} = 0. \tag{3}$$

At the initial moment the porous media does not contain any particles. During the filtration process the concentration front of suspended and retained particles moves through the filter with a speed $b(0)/a(0)$ along the characteristic line $t = \alpha x$, $\alpha = a(0)/b(0)$. Ahead of the front the two concentrations are zero.

The initial conditions (3) can be replaced by one condition on the characteristic line

$$S(x, t)|_{t=\alpha x} = 0. \tag{4}$$

In the domain $\Omega' = \{0 < x < 1, t > \alpha x\}$ the solutions of the problems (1) - (3) and (1), (2), (4) are identical [10].

A simple system with constant coefficients $a(S) = a_0$, $b(S) = b_0$ and linear filtration coefficient $\Lambda(S) = \lambda(S_M - S)$ has an exact solution [11]

$$C(x, t) = \begin{cases} 0, & t < \alpha x \\ \frac{pe^{\lambda p(t-\alpha x)}}{e^{\lambda p(t-\alpha x)} + e^{kx} - 1}, & t > \alpha x \end{cases}; \quad S(x, t) = \begin{cases} 0, & t < \alpha x \\ \frac{S_M(e^{\lambda p(t-\alpha x)} - 1)}{e^{\lambda p(t-\alpha x)} + e^{kx} - 1}, & t > \alpha x \end{cases}; \quad k = \frac{\lambda S_M}{b_0}, \tag{5}$$

where S_M is a limit concentration of the deposited particles at $t \rightarrow \infty$.

In the general case, the solution of (1) - (3) can not be expressed analytically. In [10] an asymptotic solution is constructed near the concentration front, an asymptotic solution at the filter inlet is found in [12]. Below an asymptotic solution in the form of a series in powers of a small parameter S_M is constructed.

3. Asymptotic Solution

Let the functions $a(S)$, $b(S)$, $\Lambda(S)$ are regular in a neighborhood of $S = 0$ and

$$a(S) = a_0 + a_1 S + a_2 S^2 + \dots; \quad b(S) = b_0 + b_1 S + b_2 S^2 + \dots; \quad \Lambda(S) = \lambda_1 (S_M - S) + \lambda_2 (S_M - S)^2 + \dots \quad (6)$$

In Ω' for $S_M \rightarrow 0$ the asymptotics can be found in the form

$$S(x, t) = s_1(x, t) S_M + s_2(x, t) S_M^2 + \dots; \quad C(x, t) = p + c_1(x, t) S_M + \dots \quad (7)$$

Substituting (6), (7) into (1), (2), (4) and equating the expressions of the same powers S_M we obtain recurrence equations for the terms of the asymptotic expansions (8). Finally,

$$S(x, t) = (1 - e^{-\lambda_1 p(t - \alpha x)}) S_M + (e^{-\lambda_1 p(t - \alpha x)} - e^{-2\lambda_1 p(t - \alpha x)}) \left(\frac{\lambda_2}{\lambda_1} - \frac{\lambda_1 \alpha_1}{b_0} x \right) S_M^2 + \dots; \quad (8)$$

$$C(x, t) = p - e^{-\lambda_1 p(t - \alpha x)} \frac{\lambda_1 \alpha_1 p}{b_0} x S_M + \dots, \quad (9)$$

where $\alpha_1 = p(a_1 - \alpha b_1) + 1$.

The figures below show the graphs of the suspended and deposited particle concentrations for $a(S) = 1 + 3S$, $b(S) = 1 + 2S$, $\Lambda(S) = S_M - S$ when $S_M = 0.2$, $p = 1$ at the outlet of the filter $x = 1$ (fig. 2 a, b) and at the moments $t = 0.5$ and $t = 2$ (fig. 3 a, b). Each figure shows the numerical calculations (solid line), linear (points) and quadratic (dotted line) asymptotic expansions.

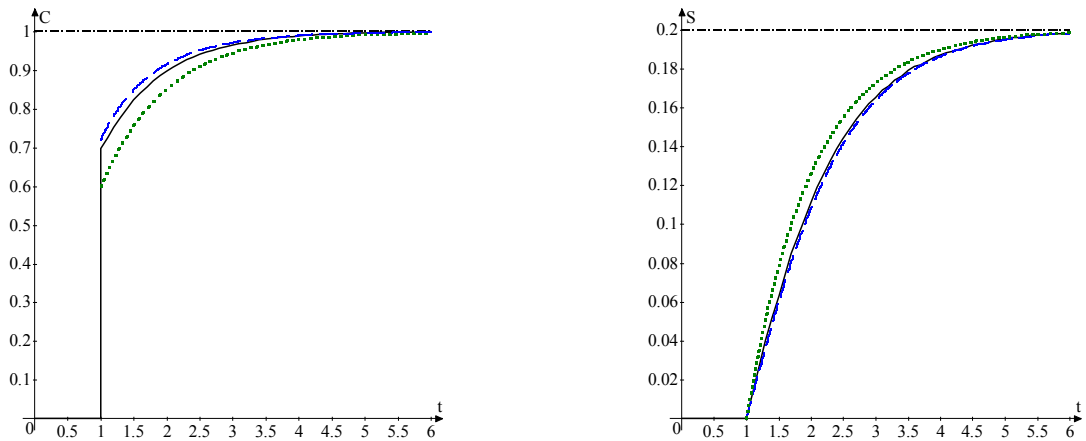


Fig. 2. (a) Concentration of the suspended particles $C(1, t)$;

(b) Concentration of the retained particles $S(1, t)$.

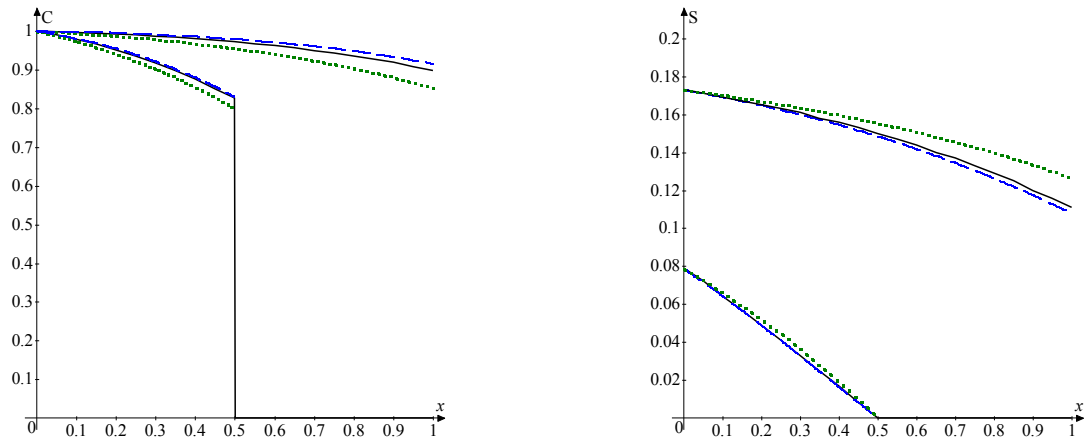


Fig. 3. (a) Concentrations of the suspended particles $C(x, 0.5)$, $C(x, 2)$; (b) Concentrations of the retained particles $S(x, 0.5)$, $S(x, 2)$.

4. Conclusions

The asymptotics (8), (9) has the following important properties.

- The asymptotic solution of the problem (1), (2), (4) is applicable in the entire domain Ω' and is in good agreement with numerical calculations.
- For $t \rightarrow \infty$ the concentration limits are equal to the given values

$$\lim_{t \rightarrow \infty} C(x, t) = p; \quad \lim_{t \rightarrow \infty} S(x, t) = S_M; \quad 0 \leq x \leq 1. \quad (10)$$

- For a simple system asymptotic solution (8), (9) is an expansion of the exact solution (5).
- At the filter inlet $x = 0$ the asymptotics (8) is an expansion of the exact solution [9].
- At the concentration front $t = \alpha x$, $0 < x < 1$ the asymptotics (9) is an expansion of the exact solution [10].

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