Правительство Российской Федерации

Федеральное государственное автономное образовательное учреждение

высшего профессионального образования   
"Национальный исследовательский университет   
"Высшая школа экономики"

Факультет Математики

Программа дисциплины

**Дифференциальная топология**

для направления 010100.68 «Математика» подготовки магистра

магистерская программа «Математика» (англоязычная)

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# Scope of use and legal references

This course program sets minimal requirements to the knowledge and skills of students and determines the contents and kinds of lectures and reporting. The program is intended for lecturers teaching this course, course assistants and students of 010100.68 specialization «Mathematics» who study the course “Differential Topology”.

* The program has been elaborated in accordance with the Educational standard of HSE for training area 010100.68 «Mathematics» (Master level);
* In accordance with the working studying plan of the university for training area of 010100.68 specialization «Mathematics» (Master level), master program “Mathematics”, approved in 2012.

# Learning objectives

Studying the course «Differential Topology » aims at making the students familiar with the main examples, constructions and techniques of the theory of smooth manifolds. In particular, the students will learn basics of the Morse theory and the Surgery Theory. The culmination of the course is proofs of two fundamental results in Differential Topology: the h-cobordism theorem of Smale (which implies, in particular, the Poincare conjecrure in dimension greater than 4) and the Thom theorem, which describes the cobordism ring.

Course goals:

* To make students familiar with the language of differential geometry ( in particular, with the notions of smooth manifold, vector field, Lie derivative, vector bundle).
* To learn foundational results: The Frobenius Theorem, Sard’s Lemma, Whitney embedding Theorem.
* To study the Morse Theory and its applications to Algebraic Topology.
* To understand the h-cobordism theorem and its role in the classification of higher dimensional manifolds.
* To learn basics of the Cobrdism Theory and its application to the proof of the Signature formula.

# Learning outcomes

By the end of the course student is supposed to

* Know: The basic concepts of differential topology.
* Be able to: use the Morse Theory to compute cohomology of manifolds, use characteristic classes to construct obstructions to the existence of linearly independent vector fields, embeddings. Given a CW complex X understand whether there exists a smoth compact manifold homotopy equivalent to X.

# Place of Discipline in MA program structure

This course is a professional one. This is an elective course for the “Mathematics” specialization.

This course is based on knowledge and competences that were provided by the following disciplines:

* Алгебраическая топология (Algebraic Topology)
* Алгебра (Algebra)

# Course plan

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| № | Название раздела | Total hours | Contact hours | | | Independent student’s work |
| Lectures | Seminars |  |
| 1  2  3  4 | Foundation of Diferential Topology  Proof of h-cobordism theorem  Characteristic classes, cobordisms  Novikov’s conjecture | 32  40  40  18 | 6  10  10  4 | 6  10  10  4 |  | 20  20  20  10 |
|  | Total: | 185 | 30 | 30 |  | 125 |

# Requirements and Grading

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Type of grading | Type of work | 1 year | | | | Characteristics |
| 1 | 2 | 3 | 4 |
| Running  (week) | test | 1 | 1 |  |  | written, 60 min |
| Homework | 7 | 7 | 8 |  | oral |
| Running | Test |  |  |  |  |  |
| Final | Examination |  | 1 |  |  | wtitten, 240 min |

## Knowledge and skills grading criteria

All work is graded on the scale from 1 to 10.

## Calculating the grades for the course

The resulting grade for the running check evaluates the results of student’s work and is calculated according to the following formula:

Отекущий = 0.3·Ок/р + 0.7·Одз ;

The resulting grade for the final evaluation in the form of examination is calculated according to this formula (where Оэкзамен is the evaluation of the student’s performance at the exam itself):

Оитоговый = 0,4·Оэкзамен + 0,6·Отекущий

This grade which is the resulting grade for the course is written down in the student’s certificate (diploma).

# Content of the subject

Section 1. Introduction to smooth manifolds

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| № | Topic | Total hours | Lectures | Seminars | Independent student’s work |
| 1 | Introduction. Smooth manifolds and maps. Statement of the h-cobordism theorem. Derivation of the Poincare Conjecture | 8 | 2 | 2 | 4 |
| 2 | Partition of unity. Derivations, tangent vectors | 8 | 2 | 2 | 4 |
| 3 | Vector bundles, tangent bundle | 6 | 1 | 1 | 4 |
| 4 | Flow of a vector field, commutator of vector fields, Lie derivative, distributions, Frobenius theorem. Sard’s Lemma, Whitney embedding theorem | 8 | 1 | 1 | 6 |
|  | Total: | 26 | 6 | 6 | 18 |

Section 2. Morse theory and proof of the h-cobordism theorem

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| № | Topic | Total hours | Contact hours | | | Independent student’s work |
| Lectures | Seminars |  |
| 1 | Introduction to Morse theory | 10 | 2 | 2 |  | 4 |
| 2 | Cancellation Lemma | 10 | 2 | 2 |  | 4 |
| 3 | Whitney trick | 10 | 3 | 3 |  | 4 |
| 4 | Proof of the h-cobordism theorem. The Poincare conjecture | 12 | 3 | 3 |  | 6 |
|  | Total: | 38 | 10 | 10 |  | 18 |

Section 3. Applications of the h-cobordism theorem to classification of manifolds

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| № | Topic | Total hours | Contact hours | | | Independent student’s work |
| Lectures | Seminars |  |
| 1 | Given a CW complex X when there is a smooth compact manifold homotopy equivalent to X? | 4 | 1 | 1 |  | 2 |
| 2 | Novikov’s conjecture | 4 | 1 | 1 |  | 2 |
| 3 | Cobordism hypothesis (after Lurie) | 4 | 1 | 1 |  | 2 |
| 4 | Smooth structures on spheres | 4 | 1 | 1 |  | 2 |
|  | Total: | 16 | 4 | 4 |  | 8 |

Section 4. Thom’s theorem on the cobordism ring

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| № | Topic | Total hours | Contact hours | | | Independent student’s work |
| Lectures | Seminars |  |
| 1 | Statement of Thom’s Theorem | 9 | 2 | 2 |  | 5 |
| 2 | Pontryagin-Thom construction | 9 | 2 | 2 |  | 5 |
| 3 | Steendrod algebra | 9 | 2 | 2 |  | 5 |
| 4 | Proof of Thom’s Theorem | 9 | 2 | 2 |  | 5 |
| 5 | Oriented cobordisms. The signature formula | 9 | 2 | 2 |  | 5 |
|  | Total: | 45 | 10 | 10 |  | 25 |

# Grading estimation for the running check and the final assessment of students

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## Topics for Current Control

Question for the test:

1. Classify vector bundles on the two-dimensional sphere. Prove that there are no nowhere vanishing vector fields on the two-dimensional sphere.
2. Show that the two-dimensional complex projective space is not diffeomorphic the boundary of a smooth manifold.

## Questions for evaluating student’s performance

* + 1. Show that given a Lie group any subalgebra of its Lie algebra can be integrated to a quasi-subgroup.
    2. Construct a Morse function on the complex projective space and use it to compute the cohomology of the projective space.

# Readings and materials for the course

## Fundamental textbooks

J.M. Lee, Introduction to Smooth Manifolds. Graduate Text in Mathematics (2013)

## Required reading

A.A. Kosinski, Differential Manifolds. Pure and applied Mathematics, volume 138 (1992)

J. Milnor, Lectures on the h-cobordism theorem. Princeton University press (1965)

## Further reading

J. Lurie, On the Classification of Topological Field Theories. Available electronically at http://www.math.harvard.edu/~lurie/

Novikov conjectures, Index Theorems and Rigidity. London Mathematical Society Lecture Notes 226 (1993)