Complexity of verification of fuzzy multi-agent systems

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A multi-agent system (MAS) is a system of interacting agents controlled by their intelligent components.

Agents have different (changing in time) *states* and communicate with other agents sending and receiving *messages* through communication channels.

Many different architectures of agents and MAS. *V.Subrahmanian* et al. (a book, 2000) IMPACT.

The behavior of MAS can be very complicate, and there is a strong necessity to verify that it satisfies certain dynamic restrictions (properties) such as
“any agent answers ultimately to all requests of other agents”.

This problem we call **MA-BEHAVIOR.**
More formally, it is:
to verify that a formula of a temporal logic is satisfied on the tree of trajectories of a MAS with a start state.

So, MA-behavior problem is a generalization of the model checking problem which is actively studied for transition systems since 80-es (*Clarke, Emerson, Vardi* and many other).
The first results on MA-behavior (begin of 2000-es, *Benerecetti-Giunchiglia, DDV*, *Wooldridge* et al.): for MAS with agents operating with a complete and certain view of the world.

In the general case agents operate in conditions of incomplete determinacy: states and results of actions can be uncertain, and communication channels can be unreliable.

Two variants of modeling such situations:

-probabilistic,

-fuzzy.

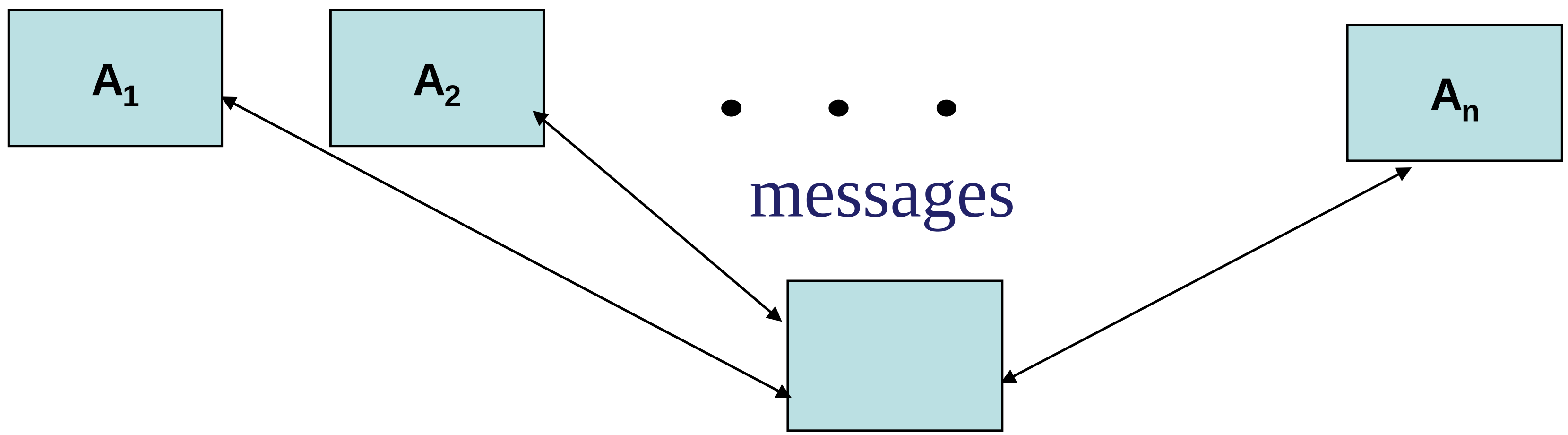
The middle of 2000-es: we described a method of simulating probabilistic MAS (sequential or nondeterministic) by Markov systems (Markov chains or decision systems).

This gives a possibility to obtain algorithms for verifying dynamic properties of probabilistic MAS and to establish their computational complexity relying on the known results for probabilistic transition systems (Vardi, Courcoubetis-Yannakakis and others).

In this talk we propose an analogous approach to *fuzzy MAS*.

But in this case (in contrast to the probabilistic one) we had to investigate verification problem for *fuzzy Markov systems*, too, since there was not such a work earlier.

A multi-agent system with probabilistic (fuzzy) channels and actions



Probabilistic (or fuzzy) communication channels

Message: msg*(Sender, Receiver, Msg): $p_{Sender,Receiver}(t)$

p_{AB} - a finite probabilistic (fuzzy) distribution (on a time interval $[0,t_{AB}]$)

$p_{AB}(t)$ – *probability* (possibility) of receiving a message during t time instants through the channel CH_{AB}

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Architecture of fuzzy agents

Agent A

- dynamic data (local state) **:**

IDB I_A	$MsgBox_A$

An example of IDB element: *cause(shortage):0,3*

- static data:

a) A base of actions $\boldsymbol{AB_A}$ (a set of possible actions which update states of IDB and send messages to other agents),

b) A fuzzy logic program $\boldsymbol{P_A}$ (determines a set of permitted now for execution actions with their possibilities)

c) A selection operator **Sel**$_A$ (selects ultimately actions to execute from the set of permitted ones). Can be nondeterministic.

The common possibilistic execution of selected actions by all the agents of the system ***A*** determines a nondeterministic-possibilistic transition relation between global states of ***A.***

- So, a MAS **A** determines a probabilistic (fuzzy) transition system over the set of its global states.
  p(S,S`)*** – probability (possibility ) of transition from ***S*** to ***S`.

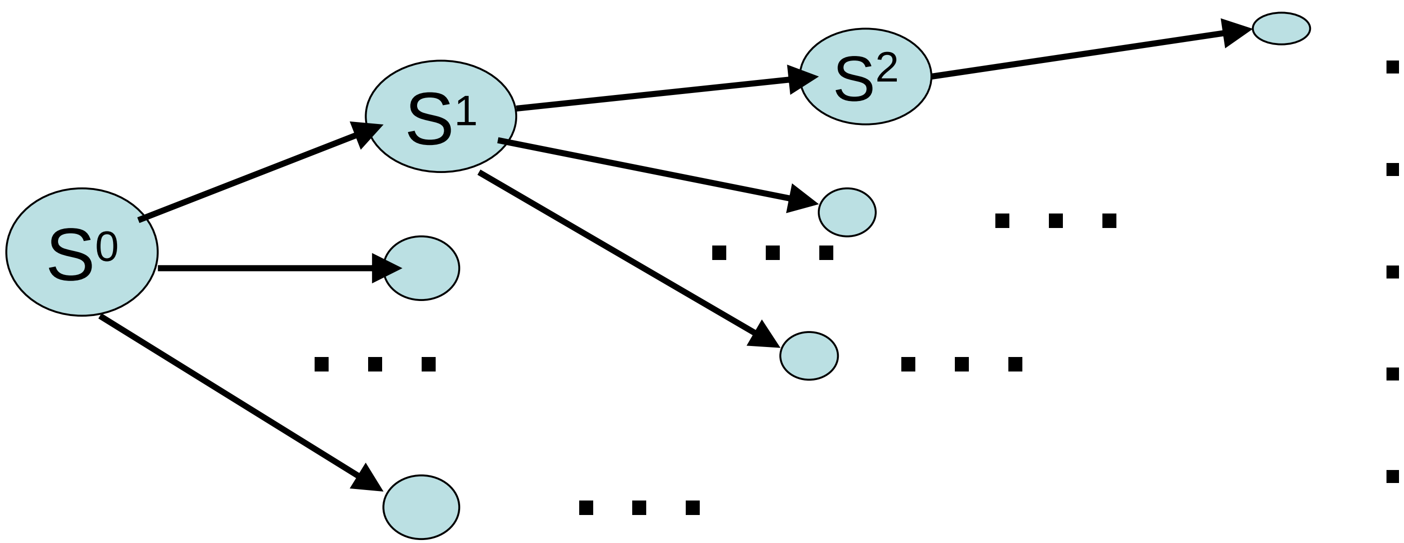
- In general case: nondeterministic (FNMAS).
  Sequential MAS (FSMAS): select operators are deterministic

The behavior of FNMAS $\mathbf{A} = \{ A_1, \dots , A_n; P \}$

A start state $S^0 = \langle (I^0_{A1}, MsgBox^0_{A1}), \dots, (I^0_{An}, MsgBox^0_{An}), I^0_P \rangle$
$(I^0_P = \varnothing, MsgBox^0_{Ai} = \varnothing , I = 1, \dots , n)$

$\mathbf{T} = \mathbf{T}_{\mathbf{A}}(S^0)$ – the tree of infinite trajectories

$\tau = (S^0 \Rightarrow_{\mathbf{A}} S^1 \Rightarrow_{\mathbf{A}} \dots S^t \Rightarrow_{\mathbf{A}} S^{t+1} \Rightarrow_{\mathbf{A}} \dots)$



We can consider the *probability (possibility)* of satisfying different properties of trajectories of the MAS.

For nondeterministic systems: *the problem of finding optimal policies to reach a goal needed*.

Probabilistic MAS are similar to Markov systems (chains or decision processes), and fuzzy MAS to fuzzy variants of Markov systems.

But there are two essential differences making MAS more complicate and expressive:

- states of MAS have a structure of databases;
- transitions are not fixed, they are computed in dependence of instant situation.

Nevertheless, we can show: **there is a method of simulating probabilistic (fuzzy) MAS by Markov systems (fuzzy Markov systems).**

Unfortunately, for basic MAS it leads in the worst case *to exponential increasing set of states*, and for non-ground MAS to *double exponential one*.
But in many cases such a “blow-up” can be avoided.

For the non-ground case it is also compensated with the fact that non-ground systems are used in the situations where they are more economical than the ground ones.

Proposed simulation method permits to use the known results of *Vardi, Courcoubetis-Yannakakis* and others on verification of probabilistic transition systems to verification of *probabilistic MAS*.

To apply the simulation method to fuzzy MAS we considered first the verification problem for fuzzy variants of Markov systems.

For this we had to introduce fuzzy variants **FCTL** and **FCTL*** of probabilistic temporal logics **PCTL** and **PCTL*** used to formally describe dynamic properties of ordinary Markov systems.

In particular, these logics contain formulas **[f]**$_J$ expressing that *the possibility of trajectories satisfying the linear time formula **f** lies in the interval **J**.*

The results for fuzzy Markov systems are in some extent similar to results for usual Markov systems, but there are also some subtle differences (especially, for fuzzy decision processes).

Here are some of many results on complexity of verification of fuzzy MAS obtained as corollaries of results on verification of fuzzy Markov systems combined with the simulation method.

Theorem 1. ***There exists algorithm verifying satisfiability of a FCTL-formula F on a state of a ground FSMAS or FNMAS A in time exponential on size of A (for FNMAS with higher basis of the exponent) and linear on size of F.***

Theorem 2. ***There exists algorithm verifying satisfiability of a FCTL*-formula F on a state of a ground FSMAS A in time exponential on size of A and linear on size of F.***

Theorem 3. ***There exists an algorithm verifying satisfiability of a FCTL*-formula F on a state of a ground FNMAS A in time exponential on size of A and double exponential on size of F.***

For non-ground systems the dependence on size of the system is double exponential .

These upper bounds of complexity are true for the worst case. It seems that most of them cannot be essentially improved.

However, in the practice of constructing a Markov system simulating MAS, most of global states occur to be *unacceptable* or *unreachable*; this permits to strongly decrease the complexity of verification of MAS.

An experimental system for verification of probabilistic MAS was implemented by *P.Lebedev*, a postgraduate student of Tver State University.

Now the work on extending this system to fuzzy MAS is going on.

EXAMPLE. We consider a fuzzy MAS **SB** which simulates a simplified fuzzy version of alternative model of negotiation proposed by *Rubinstein.*

In this model two agents **S**(eller) and **B**(uyer) take it in turns to make an action. The action can be either *(i) put forward a proposal (offer),* or *(ii) accepts the most recent proposal*.

For an agent, the decision on the opponent's offer is a *fuzzy* one, which depends on the offered *amount* and the *market situation*. Two kinds of market situations: *prices are going up* ***(u)*** and *prices are going down* ***(d)***.

The value of agent's **a** *i*-th offer is defined by functions $\boldsymbol{of_{u,a}(i)}$ and $\boldsymbol{of_{d,a}(i)}$, which depend on the market situation.

Acceptance Possibility functions, $\boldsymbol{AP_{u,a}(x,i)}$ and $\boldsymbol{AP_{d,a}(x,i)}$, and *Rejection Possibility functions* $\boldsymbol{RP_{u,a}(x,i)}$ and $\boldsymbol{RP_{d,a}(x,i)}$ return possibility values of the actions depending on the offer price $\boldsymbol{x(i)}$ and time (step number) ***i***.

All the functions in the example are defined by the tables.

Suppose that we are interested in *what is the possibility degree that the negotiations terminate with the deal price 80*?

This question can be stated as the question about the possibility degree of LTL-formula

$$f = \mathit{true}\ \boldsymbol{U}\ \mathit{deal}(80)$$

An analysis of the system **SB** allows to obtain the following bounds on the possibility degree of **f**.

i) Formula **f** has the maximal possibility degree Pmax(**f**) = 0.4.

ii) Formula **f** has the minimal possibility degree Pmin(**f**) = 0.1.

There is a much work to be done further:

- *to study possibilities to decrease the verification complexity of fuzzy MAS;*
- *to find interesting classes of fuzzy MAS with a feasible verification complexity;*
- *to study verification complexity of formulas from more expressive logics (e.g. for the fuzzy variant of mu-calculus); ad so on.*

Thanks for attention!