Random graphs: models and asymptotic characteristics

M.E. Zhukovskii

For some large scale networks, there are random graphs which asymptotic properties are close to the properties of these networks. However, characteristics of such random graphs are much simpler to analyze. In particular, preferential attachment models are used to analyze Web and can be applied for the problem of Web pages ranking. For example, for some random graphs, estimations of the expected values of PageRank are known. In the talk, we review the results on PageRank distributions in random graph models.

Another application of the theory of random graphs, is the probabilistic method. Solutions of some combinatorial problems follow from the fact that probabilities of random graphs (commonly, in the classic binomial and uniform Erdős-Rényi models) to have the considered properties are positive. In the talk, we give a brief survey of random graph models and focus on asymptotic probabilities of the classic Erdős-Rényi model.

To the best of our knowledge, the class of first-order properties of graphs is the most widely studied class of graphs in a sense of asymptotic probability. The random graph $G(n,p)$ *obeys Zero-One Law* if for each first-order property its probability tends to 0 or tends to 1. In 1988, S. Shelah and J. Spencer showed that if α is an *irrational* positive number and $p(n) = n^{-\alpha+o(1)}$, then $G(n,n^{-\alpha})$ obeys Zero-One Law.The random graph $G(n,p)$ *obeys Zero-One k-Law* if for each first-order property which is expressed by a first-order formulae with quantifier depth at most k its probability tends to 0 or tends to 1.

We present the results on Zero-One Law and Zero-One k-Law for the random graph $G(n,n^{-\alpha})$ (for the most non-trivial case) and the distributions of positive rational numbers $\alpha < 1$ such that Zero-One k-Law for $G(n,n^{-\alpha})$ does not hold (in this case, we say that α is in *k-spectrum*). In particular, we found bounds on minimal and maximal limit points of k-spectrum. Moreover, we prove that the minimal k such that k-spectrum is infinite is either 4 or 5.