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THE VALUE OF PUBLIC INFORMATION IN A TWO-REGION MODEL

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THE VALUE OF PUBLIC INFORMATION IN A TWO-REGION MODEL

Economic literature is far from having a consensus about the social value of public information. Nevertheless, most studies agree that strategic complementarity increases the weight of public signals in private actions. In our paper we show that this result is not general. In a two-region model we relax the autarky assumption, common to previous studies, and suppose that strategic complementarity is present both inside and between the regions. When strategic complementarity is strengthened, the agents redistribute increased weight of public information between the signals from different regions. If the weight of the domestic public signal is sufficiently high, an increase in strategic complementarity may lower it. In this paper we study the welfare properties of this information structure and show that transparency in our model may be detrimental only if strategic complementarity is weak. Furthermore, we compare equilibrium information policies with the social optimum and show that policymakers in small regions tend to be too transparent, while policymakers in large regions tend to be too opaque.

JEL: D82, E61

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1 Introduction

Economic literature suggests different opinions about the value of public information. Morris and Shin (2002) show that in a model of beauty contest with strong strategic complementarity, provision of public information may be undesirable. Strategic complementarity forces private agents to put excessive weight on public signals, and can provoke excessive volatility if the quality of public information is low. Svensson (2006) questions the main conclusion of Morris and Shin (2002) and claims that this result can only be achieved under unrealistic assumptions about the quality of public information. James and Lawler (2011) debate the criticism of Svensson (2006) and find that transparency is always detrimental in the beauty-contest model if the policymaker governs the economy with both public signals and standard policy instruments. Angeletos and Pavan (2004) agree that transparency may lower social welfare in environments with strong complementarity and multiple equilibria. Hellwig (2005) and Roca (2010) study the welfare effects of public information in models with imperfectly-informed monopolistically competitive firms, and claim that public information is always welfare-improving. Nevertheless, Walsh (2013) shows that transparency may be detrimental in a New-Keynesian model with aggregate supply and demand shocks, while Myatt and Wallace (2008) argue that neither transparency nor opacity are optimal in a world without purely public signals.

Angeletos and Pavan (2007) explain this diversity of views within a general linear-quadratic framework. They show that, while strategic complementarity always contributes to an increase in the weight of public information, the welfare effect of this increase depends on the relation between equilibrium and efficient degrees of coordination. In economies which are efficient under full information, transparency may be detrimental only if the equilibrium degree of coordination exceeds the efficient degree. In inefficient economies, the value of information also depends on the gap between efficient and full-information weights of the signals. When this gap is sufficiently high, transparency is always beneficial; when the gap is small, transparency may be detrimental.

Although the literature on the value of public information examines different environments, all these environments represent autarky: private payoffs are determined by fundamentals and the strategic coordination effect inside the economy, without any recourse to the foreign sector. Even in the two-region model of Arato and Nakamura (2013), there is neither correlation between fundamentals nor strategic complementarity between the two regions. Private actions, therefore, depend only on expected fundamentals and average actions inside the domestic region. Consequently, in this environment, public signals from one region do not affect private actions in another region.

Meanwhile, there is growing evidence that private actions respond to foreign signals. A number of studies reveal significant impacts of US news on foreign financial markets (Kim and Sheen (2000) for Australian markets, Bredin, Gavin and O’Reilly (2005) for Irish markets, Hausman and Wongswan (2011) for 49 different countries). Ehrmann and Fratzscher (2005) investigate spillovers between the European Union and the US and find that macroeconomic news affects financial markets both domestically and abroad. Büttner, Hayo and Neuenkirch (2012) and

In order to fill the gap between the two strands of literature, we modify the two-region model of Arato and Nakamura (2013) by allowing cross-region strategic complementarity. Cross-region strategic complementarity forces private agents to take foreign average actions into account. This, in turn, creates information spillovers between regions: public signals influence not only domestic agents, but also agents abroad. Furthermore, we add to the model informative priors, while keeping the assumption of zero cross-region correlation between fundamentals.

The contribution of our paper is threefold. Firstly, we demonstrate that an increase in strategic complementarity may lead to a decrease in the weight of the domestic public signal, which is a novel result in economic literature. Secondly, we show that in our model, transparency may be detrimental, but only under weak strategic complementarity. This is in a sharp contrast with the studies which show that transparency may be detrimental only if strategic complementarity is sufficiently strong. Thirdly, we detect a strategic effect of cross-region complementarity at the regional level. Due to this effect the information policy in small regions tend to be too transparent, while the policy in large regions tend to be too opaque.

The paper is organized as follows. The model is presented in the next Section. The equilibrium private actions are computed in Section 3, while Section 4 determines the properties of the social optimum. Section 5 compares the choice of policymakers with the social optimum. Section 6 concludes.

2 Model

Consider an economy with a unit mass of population which is divided between two regions, labeled $H$ and $F$. The relative size of region $H$ is equal to $n \in (0, 1)$, so agents with index $i \in I_H = [0, n]$ live in region $H$ and agents with index $i \in I_F = (n, 1]$ live in region $F$. The fundamentals of region $j$ ($j \in \{H, F\}$) are described by the variable $\theta_j$ which is independently and normally distributed with zero mean and variance $\frac{1}{\phi_j}$. To put it differently, $\phi_j$ represents the precision of common prior information about $\theta_j$.

A private agent $i$ chooses action $a_i$, which minimizes his loss $l_i$:

$$l_i = (1 - r)(a_i - \theta_j)^2 + r(L_i - \bar{L}),$$

where $j = H$ if $i \in I_H$ and $j = F$ if $i \in I_F$. The measure $L_i = \int_0^1 (a_k - a_i)^2 \, dk$ represents the average distance between the action of the agent and the actions of all other private agents. The term $\bar{L} = \int_0^1 L_k \, dk$ measures the variance of private actions in the economy.

The private agent has two objectives. His first goal is to minimize the distance between his action and the domestic fundamentals, $(a_i - \theta_j)^2$. The second goal is to minimize the average distance to others’ actions, $L_i$. This gives rise to a coordination motive and strategic
complementarity in private actions; higher average action induces the agent to increase his action $a_i$. The relative strength of strategic complementarity depends on the coefficient $r \in (0, 1)$. If $r$ is equal to zero, the only goal of the agent is to mimic the fundamentals $\theta_j$ and the coordination motive disappears. If $r = 1$, the only incentive is “to do what others do”.

The social loss in this economy is simply the sum of private losses:

$$L^W \equiv \int_0^1 l_i(a, \theta) \, di = (1 - r) \left( L^W_H + L^W_F \right)$$

(2)

where $L^W_j = \int_0^1 (a_i - \theta_j)^2 \, di$ represents the average distance between private actions and the fundamentals of region $j$. Thus, the social loss does not reflect any coordination effect.

The policymaker $P_j$ aims to minimize the sum of private losses inside region $j$. These losses, $L^P_j$, are defined by two terms. The first term represents the distance between private actions and fundamentals inside the region; the second term describes the relative performance of the region in comparison to the whole economy:

$$L^P_j \equiv \int_0^1 l_i \, di = (1 - r)L^W_j + r \Delta_j$$

(3)

where $\Delta_j = \overline{L}_j - n_j \overline{L}$, the term $\overline{L}_j = \int_0^1 l_i \, di$ represents the loss from the coordination motive in region $j$, and $n_j$ is the size of region $j$: $n_j = \begin{cases} n & \text{if } j = H \\ 1 - n & \text{if } j = F \end{cases}$. In other words, $\Delta_j$ represents the regional strategic effect which arises due to the coordination motive at the private level.

We assume that neither policymakers nor private agents know the true value of fundamentals. The policymaker $P_j$ receives a noisy private signal $y_j$ about $\theta_j$:

$$y_j = \theta_j + \eta_j,$$

(4)

where $\eta_j \sim N(0, \frac{1}{\alpha_j})$ and $\alpha_j$ is the precision of this signal. We assume that the policymaker does not receive any private information about fundamentals in the other region. The policymaker can disclose his private information, $y_j$, via the public signal $s_j$ which can be costlessly observed by all the agents in the economy. This public signal is equal to the sum of $y_j$ and some additional noise $v_j$, which is normally distributed with zero mean and variance $\frac{1}{\gamma_j}$:

$$s_j = y_j + v_j$$

(5)

Precision of the public signal, $\mu_j$, depends both on the precision of the policymaker’s private information, $\alpha_j$, and on the precision of the additional noise, $\gamma_j$:

$$\mu_j = \frac{\alpha_j \gamma_j}{\alpha_j + \gamma_j}.$$  

(6)

By choosing different values of $\gamma_j$, the policymaker can influence the precision of his public signal,
When the policymaker is transparent and does not add any noise, the precision of his public signal coincides with the precision of his private information, \( \mu_j = \alpha_j \). When the policymaker is opaque, the precision of his public signal drops to zero. This means that the policymaker does not disclose any information. In what follows, we assume that the precision of his public signal is the only instrument of the policymaker.

In addition to public signals, every private agent observes a private signal \( x_i \) about the true value of domestic fundamentals:

\[
x_i = \theta_j + \varepsilon_i, \tag{7}
\]

where \( \varepsilon_i \sim i.i.d. N(0, \frac{1}{\beta_j}) \) is the noise of this private signal and \( \beta_j \) stands for its precision. We suppose that agents have no private information about the fundamentals in the foreign region. After receiving all the signals, private agents simultaneously define their actions. In the next Section, we derive equilibrium private strategies for a given information structure and show the effect of information policies on the behavior of private agents.

### 3 Equilibrium private strategies

Minimization of the private loss (1) yields the optimal private action as a weighted sum of expectations of domestic fundamentals, \( E_i(\theta_j) \), and the average private action, \( E_i(\bar{a}) \):

\[
a_i = (1 - r)E_i(\theta_j) + rE_i(\bar{a}) \quad \forall i \in I_j \tag{8}
\]

On the other hand, Morris and Shin (2002) demonstrate that the equilibrium strategy of private agents is linear over all received signals:

\[
a_i = \kappa_{H,j}s_H + \kappa_{F,j}s_F + \kappa_{x,j}x_i \quad \forall i \in I_j, \tag{9}
\]

where \( \kappa_{x,j} \) is the weight of private signals in region \( j \), \( \kappa_{H,j} \) and \( \kappa_{F,j} \) are the weights of public signals \( s_H \) and \( s_F \) in region \( j \). The average private action \( \bar{a} \) is:

\[
\bar{a} = \kappa_{H}s_H + \kappa_{F}s_F + \kappa_{x,H}\int_{I_H} x_i \, di + \kappa_{x,F}\int_{I_F} x_i \, di, \tag{10}
\]

where \( \bar{K}_j = n\kappa_{H,j} + (1 - n)\kappa_{F,j} \) is the average weight of the signal \( s_j \) in private actions.

An agent \( i \) forms his expectations about the average action:

\[
E_i(\bar{a}) = \kappa_{H}s_H + \kappa_{F}s_F + \kappa_{x,H}E_i\theta_H + \kappa_{x,F}(1 - n)E_i\theta_F \tag{11}
\]

Expectations about fundamentals are computed using all the information available to private agents. Any agent in region \( j \) applies two relevant signals to forecast the domestic fundamental \( \theta_j \). These are the public signal \( s_j \) and the private signal \( x_i \). Any agent in the other region can use only
the public signal $s_j$ to forecast $\theta_j$. Hence, the expected value of the fundamental $\theta_j$ is computed according to:

$$E_i\theta_j = \begin{cases} 
\frac{\mu_j s_j + \beta_j x_i}{\mu_j + \varphi_j}, & \text{if } i \in I_j \\
\frac{\mu_j s_j}{\mu_j + \varphi_j}, & \text{otherwise}
\end{cases}$$

(12)

As stated in (12), the weights of the signals depend positively on their precisions and negatively on the precision of corresponding fundamental. By substituting (11) and (12) into the first-order condition (8), we compute the equilibrium weights of the signals in private action:

$$\kappa_{j,j} = \frac{\mu_j}{\beta_j t_j} \left( t_j (1 - r n_{-j}) + n_j n_{-j} r^2 \right) \left( t_j - n_j r + 1 \right),$$

(13)

$$\kappa_{-j,j} = \frac{\mu_{-j} t_{-j}}{\beta_{-j} t_{-j}},$$

(14)

$$\kappa_{x,j} = \frac{1 - r}{t_j - n_j r + 1},$$

(15)

where we use the following notation for the relative precision of public information in region $j$:

$$\tau_j = \frac{\mu_j + \varphi_j}{\beta_j}.$$ 

(16)

As we showed earlier, $\mu_j$ can vary from 0 to $\alpha_j$. Thus, the minimal value of $\tau_j$ is equal to $\tau_j^O = \frac{\varphi_j}{\beta_j}$ and is achieved under full opacity of the policymaker $P_j$. The maximum value of $\tau_j$ is equal to $\tau_j^T = \frac{\alpha_j + \varphi_j}{\beta_j}$ and is achieved under full transparency.

Equations (13-15) demonstrate that an increase in $\mu_j$ forces private agents to put more weight on the public signal $s_j$. At the same time, agents in region $j$ decrease the weight of their private information. The reasoning is straightforward. When precision $\mu_j$ increases, the public signal $s_j$ becomes a better predictor of the fundamental $\theta_j$. As a result, agents from region $j$ substitute their private information with this signal, since they have an incentive to keep their actions close to the fundamental $\theta_j$. Consequently, the value of the signal $s_j$ becomes a better predictor of private actions in region $j$. Due to the coordination motive, agents from the other region increase the weight of the signal $s_j$ in their actions. This, in turn, leads to an increase in the weight of this signal in region $j$. The process continues until a new equilibrium is achieved.

An increase in $n_j$ leads to an increase in the weights of both private signals in region $j$ and the public signal $s_j$ in the other region. On the other hand, the weight of private signals in region $-j$ and the weight of the signal $s_{-j}$ in region $j$ drop. When region $j$ becomes larger, the higher ratio of average actions is explained by behavior of agents who live in this region. Thus, the coordination motive forces agents from the other region to substitute their private information with the public signal $s_j$ which is a better predictor of private actions in region $j$. For the same reason, agents from region $j$ substitute foreign public signal $s_{-j}$ with their private signals $x_i$.

The influence of $n_j$ on the weight of $s_j$ in region $j$ depends on region size $n_j$, strategic complementarity $r$ and the relative precision of public information, $\tau_j$. An increase in $n_j$ leads
to a decrease in the weight of this signal in region $j$ if and only if the following condition holds:

$$\frac{\partial \kappa_{j,j}}{\partial n_j} < 0 \iff \left[ n_j > \frac{1}{2} \lor r < \frac{2n_j - 1}{n_j^2} \lor \tau_j < \hat{\tau}_j \right],$$

(17)

where $\hat{\tau}_j = n_j r - 1 + \sqrt{1 - r} > 0$.

If the coordination motive is absent, agents do not use public signals from the foreign region and $\kappa_{-j,j} = 0$. An increase in strategic complementarity $r$ implies strengthening of the coordination motive. In this case, private agents become more anxious about predicting what the others are going to do, and public signals from the other region bring some information about others’ actions. On the other hand, private information, which is not known by others, is not useful in forming expectations about average actions. As a result, private agents substitute their private information with the public signal from the foreign region.

The influence of $r$ on the weight of the public signal in the domestic region, $\kappa_{j,j}$, is far from trivial. This influence is positive only if the relative precision of public information is low:

$$\frac{\partial \kappa_{j,j}}{\partial r} > 0 \iff \tau_j < \hat{\tau}_j,$$

(18)

where $\hat{\tau}_j = \frac{1 - 2(1-n_j)(1-n_j) + \sqrt{1 - 4n_j(1-n_j)(1-r)}}{2(1-n_j)} > 0$. If the relative precision of public information is higher than the threshold $\hat{\tau}_j$, an increase in strategic complementarity lowers the weight of the domestic public signal. This result is novel in economic literature, as most studies claim that strategic complementarity always increases the weight of public signals. In our model private agents redistribute increased weight of public information between two signals. If the relative precision of domestic public information is relatively high, the weight of the domestic public signal is already sufficiently high and its further increase cannot improve prediction of domestic fundamentals. In this case, a private agent may prefer, in part, to substitute foreign public signals for domestic public signals, because foreign information is a good predictor of average actions abroad. The threshold value $\hat{\tau}_j$ depends positively on the region size. When $n_j$ goes to zero, private agents in region $j$ do not rely on their domestic public signal and $\lim_{n_j \to 0} \hat{\tau}_j = 0$. An increase in $n_j$ strengthens the coordination motive inside the region and weakens the coordination motive between regions; therefore, $\hat{\tau}_j$ increases. If $n_j = 1$, $\hat{\tau}_j$ tends to infinity and the weight of the domestic public signal increases with strategic complementarity for any precision of public information, $\tau_j$. This case corresponds to the findings of most previous studies.

In sum, strategic complementarity affects the use of information in three distinct ways. First of all, private agents put too little weight on their private information due to the coordination motive. Secondly, agents use public signals from the foreign region, although these signals are not correlated with the domestic fundamentals. Finally, agents put too much weight on the domestic public information when its precision is relatively low and too little weight when its precision is high. These three effects define the role of information policy in the model. In the case of low precision of public information, both $\mu_j$ and $r$ drive the weights in the equilibrium strategy (9) in the same direction. As a result, transparent policy aggravates distortions created by the coordination motive. Only if the precision of public information is relatively high, can
transparency, at least partially, balance the influence of strategic complementarity. The overall effect of transparency on social welfare is analyzed in the next section.

4 Social value of public information

Taking into account the equilibrium private strategy (9), the distance between private actions and the fundamentals in region $j$ is defined by:

$$L^W_j = n_j \left[ \frac{\kappa_{j,j}^2}{\mu_j} + \frac{\kappa_{x,j}^2}{\beta_j} + \left(1 - \frac{\kappa_{j,j} - \kappa_{x,j}}{\varphi_j}\right)^2 \right] + n_j \kappa_{-j,j}^2 \left( \frac{1}{\mu_{-j}} + \frac{1}{\varphi_{-j}} \right)$$

(19)

The sum in squared brackets in (19) represents the loss which arises due to imperfect information about the fundamental $\theta_j$. This loss comes from the noise in the public signal $s_j$, the private signal $x_i$ and prior information about $\theta_j$. The term $n_j \kappa_{-j,j}^2 \left( \frac{1}{\mu_{-j}} + \frac{1}{\varphi_{-j}} \right)$ stands for the loss which arises from the use of information about $\theta_{-j}$. According to (14), agents from region $j$ give a positive weight $\kappa_{-j,j}$ to the public signal $s_{-j}$. This signal does not correlate with the fundamental $\theta_j$, and its use raises the average distance between private actions and fundamentals in region $j$.

According to (2), the social loss $L^W$ is the weighted sum of average distances between private actions and fundamentals in both regions. Appendix A1 shows that, depending on relative region size and strategic complementarity, the social loss function is either decreasing in $\tau_j$, takes inverted U-shape, or has two local extrema; these forms are presented in Figure 1.

For any form of function $L^W(\tau_j)$, the social loss is decreasing for sufficiently large values of $\tau_j$. In other words, there always exists $\tau^W_j \geq 0$ such that $\frac{\partial L^W(\tau_j)}{\partial \tau_j} < 0$ for all $\tau_j > \tau^W_j$. This means that transparency is always beneficial if the precision of prior information is high in comparison to the precision of private signals, so that $\tau^O_j \geq \tau^W_j (n_j, r)$. In this case, prior information predicts the true value of fundamentals better than private signals. Thus, a decrease in the use of private signals due to strategic complementarity does not lead to a significant increase in losses. The possible positive effects of opacity, discussed in Section 3, are negligible for any precision of
policymaker information. Consequently, $\tau_j^O \geq \tau_j^W(n_j, r)$ is a sufficient condition for transparency to be optimal. Opacity or intermediate transparency can be optimal only if $\tau_j^O < \tau_j^W(n_j, r)$.

Full opacity is socially desirable if the prior and the policymaker’s information are both uninformative. For the case described in Figure 1b, this takes place if $\tau_j^O < \tau_j^W$ and $\tau_j > \tau_j^T$, where $L^W(\tau_j^O) = L^W(\tau_j)$. In this case, $L^W(\tau_j^O) < L^W(\tau_j^T)$ and full opacity is optimal. The underlying reasoning is as follows: if the distance $\tau_j^T - \tau_j^O = \alpha_j$ is low, the publication of a public signal cannot considerably improve the quality of forecasts. At the same time, the coordination motive forces private agents to substitute their private information with the public signal of low precision. This leads to an increase in expected distance between private actions and fundamentals; therefore, the social loss increases.

An intermediate level of transparency may be optimal if the function $L^W(\tau_j)$ has the form depicted in Figure 1c and the quality of private information is very low, so that $\tau_j^O < \tau_j^*$. In this situation $\frac{\partial L^W}{\partial \tau_j} \Big|_{\tau_j=\tau_j^O} < 0$ and full opacity is never optimal. For intermediate values of public signal precision ($\tau_j^* < \tau_j^O < \tau_j$, where $L^W(\tau_j)= L^W(\tau_j^*)$), an intermediate $\tau_j = \tau_j^*$ is optimal.

Figure 2 presents the threshold level $\tau_j^W$ for different $n_j$ and $r$:

Figure 2: The social threshold for the relative precision $\tau_j^W$

Figure 2 shows that $\tau_j^W$ is positive for small values of $r$. This comes from the negative externality of public information about region $j$. For large $n_j$, an increase in $n_{-j}$ exacerbates this externality and $\tau_j^W$ goes up. When $n_j$ is sufficiently small, this negative externality diminishes because the agents from region $-j$ become less interested in taking information from region $j$ into account. For relatively low values of $n_j$, its further decrease leads to a decrease in $\tau_j^W$ for low $r$.

It is obvious that for every $n_j$ there exists a unique $\tau_j^W \leq 1$ such that $\tau_j^W(n_j, r) = 0$ if $r \geq \tau_j^W$. This means that $L^W$ is decreasing over $[\tau_j^O, \tau_j^T]$ if $r > \tau_j^W$. In other words, $r > \tau_j^W(n_j)$ is a sufficient condition for transparency to be beneficial, and the value of $\tau_j^W$ represents the maximum extent of strategic complementarity for which transparency can be detrimental. When the coordination motive is strong and $r > \tau_j^W$, private agents put high weights on foreign public information, irrespective of policy in their domestic region. Opacity of the policymaker $P_j$ does not prevent private agents from coordination, but reduces the accuracy of their forecasts. Thus, for high $r$, opacity is never optimal.
This conclusion diverges from many studies which show that transparency may be detrimental only if strategic complementarity is strong. In all these studies, private agents do not observe any public signal which is not related to the fundamentals of their region. Thus, the only positive effect of opacity is to re-balance the use of private and public signals by private agents inside the region. This effect is sufficiently strong only if strategic complementarity is high and the initial distortion between $\kappa_{x,j}$ and $\kappa_{j,j}$ is substantial. On the contrary, in our model private agents do have access to foreign public information. The policymaker cannot prevent private agents from the use of this information. Consequently, the policymaker cannot do much with the private coordination motive when $r$ is high. In this case, the positive effect of opacity is negligible, while possible losses from the difference between average actions and fundamentals become higher. As a result, transparency is socially optimal under strong complementarity.

![Figure 3: Social optimum](image)

Function $\tau_{W}(n_{j})$ is represented in Figure 3 which also shows the optimal policies for different combinations of $r$ and $n_{j}$. If strategic complementarity is high and $r > \tau_{W}$, $\frac{\partial L_{W}}{\partial \tau_{j}}$ is negative for all $\tau_{j}$ (Figure 1a) and transparency is beneficial for all possible values of $\tau_{j}^{O}$ and $\alpha_{j}$. When strategic complementarity is weak and $r \leq \frac{1-\sqrt{1-n_{j}}}{n_{j}}$, $L_{W}$ has inverted U-form (Figure 1b). In this case, transparency can be detrimental for $\tau_{j}^{O} < \tau_{j}^{W}$ and sufficiently low precision of the signal received by the policymaker $P_{j}$. When both $n_{j}$ and $r$ are high, $L_{W}(\tau_{j})$ has two extrema (as in Figure 1c) and an intermediate level of transparency may be optimal.

After discussing the socially optimal information policy, we now turn to the analysis of the equilibrium information policies. In the next section we show that the equilibrium policies in our model can differ significantly from the social optimum.

### 5 Information policy

The policymaker $P_{j}$ conducts his information policy in order to minimize the loss of region $j$. Accordingly to (3), this loss consists of the terms $L_{j}^{W}$ and $\Delta_{j}$. The former represents the average distance between private actions in region $j$ and the economic fundamental $\theta_{j}$. The latter represents the average performance of the region in comparison to the whole economy. The influence of
information policies on $L^W_j$ was discussed in previous sections. To analyze the influence of information policy on $\Delta_j$, we rewrite this term by using the definition of $\bar{L}$:

$$\Delta_j = L_j - n_j L = L_j - n_j(L_j + L_{-j}) = (1 - n_j)\bar{L}_j - n_j\bar{L}_{-j}$$  \hspace{1cm} (20)$$

Equation (20) shows that $\Delta_j$ consists of two parts. Firstly, each policymaker is interested in decreasing the relative loss in his domestic region, $\bar{L}_j$. Secondly, he tries to maximize the relative loss of the foreign region, $\bar{L}_{-j}$. The relative weights of the two incentives correspond to the relative sizes of the regions. If the ratio of population in region $j$ increases, the policymaker $P_j$ becomes more conscious about increasing the losses of agents with $i \in I_{-j}$ and less conscious about lowering the relative losses of agents with $i \in I_j$.

For further calculations, we rewrite the average loss $\bar{L}_j$ in the following way:

$$\bar{L}_j = \int_{I_j} \int_{I_j} (a_k - a_i)^2 \, dk \, di + \int_{I_j} \int_{L_{-j}} (a_k - a_i)^2 \, dk \, di = \bar{L}^{\text{in}}_j + \bar{L}^{\text{between}}_j,$$  \hspace{1cm} (21)$$

where $\bar{L}^{\text{in}}_j$ stands for the quadratic distance between private actions inside region $j$ and $\bar{L}^{\text{between}}_j$ represents the quadratic distance between the regions. Then we substitute (21) into (20) to obtain:

$$\Delta_j = [(1 - n_j)\bar{L}^{\text{in}}_j - n_j\bar{L}^{\text{in}}_{-j}] + (1 - 2n_j)\bar{L}^{\text{between}}_j$$  \hspace{1cm} (22)$$

According to (22), the relative performance $\Delta_j$ depends on the variance of private actions inside and between the regions. The variance of private actions inside the region comes from heterogeneity of private signals. The policymaker $P_j$ cannot influence the use of private signals abroad, so he cannot affect the variance of private actions inside the foreign region, $\bar{L}^{\text{in}}_{-j}$. Nevertheless, he can influence the variance of private actions inside region $j$. When the policymaker becomes more transparent, domestic private agents put less weight on their private signals. As a result, the variance of private actions inside the region, $\bar{L}^{\text{in}}_j$, drops. Moreover, the policymaker can influence the average difference between the two regions, $\bar{L}^{\text{between}}_j$. When the precision of the public signal increases, the weight of the signal increases in both regions. As a result, the difference between the regions diminishes and $\bar{L}^{\text{between}}_j$ decreases.

The overall influence of transparency on the relative performance of region $j$ depends on $n_j$. If the region is small ($n_j < 1/2$), the relative performance $\Delta_j$ increases in both the variances inside and between the regions. The two variances depend negatively on the precision of public signals, so the policymaker $P_j$ has incentives to be transparent. When the region is large ($n_j > 1/2$), the relative performance $\Delta_j$ decreases in $\bar{L}^{\text{between}}_j$, and the policymaker has the incentive to increase the difference between the regions. Thus, he is less disposed to be transparent.

The optimal information policy minimizes the sum of the relative effect and the average distance to fundamentals inside the region. Appendix A2 shows that the loss of region $j$ is either decreasing in $\tau_j$ (Figure 1a) or has an inverted U-shape (Figure 1b). In both cases, there exists a threshold $\tau^{\text{opt}}_j(n_j, r)$ such that $\frac{dL^W_j(\tau_j)}{d\tau_j} < 0$ for all $\tau_j > \tau^{\text{opt}}_j$: 

\[
\tau^P_j = \begin{cases} 
0, & \text{if } n \leq \frac{1}{2}, \\
\max\{0, \arg \max \tau^P_j L^P_j\}, & \text{otherwise.}
\end{cases}
\] (23)

This threshold represents a sufficient condition for the policymaker to choose transparency. If the precision of prior information is relatively low and \(\tau^O_j \geq \tau^P_j(n_j, r)\), the policymaker \(P_j\) chooses transparency even if his private information is imprecise. The policymaker \(P_j\) can choose opacity only if \(\tau^O_j < \tau^P_j(n_j, r)\) and \(L^P_j(\tau^O_j) \leq L^P_j(\tau^T_j)\).

The first line in (23) implies that the policymaker from the region with \(n_j < \frac{1}{2}\) never chooses opacity. According to the previous section, \(L^W_j\) is strictly decreasing in the precision of public information if the region is small. Moreover, in this case, the policymaker is going to keep \(L^\text{between}\) at the lowest possible level. As \(L^\text{between}\) is also decreasing in \(\tau_j\), the policymaker has no incentives to be opaque.

For \(n_j > \frac{1}{2}\), the threshold \(\tau^P_j\) can be positive. Figure 4 represents \(\tau^P_j(r)\) for different values of \(n_j\):

![Figure 4: The threshold relative precision \(\tau^P_j\)](image)

According to Figure 4, \(\tau^P_j\) is inverted U-shaped function of \(r\). To a certain level, an increase in region size causes an increase in \(\tau^P_j\) for small extents of strategic complementarity. This means that the incentive to be opaque strengthens. Starting from some point, a further increase in \(n_j\) leads to a decrease in \(\tau^P_j\) for low \(r\). When the region is sufficiently large, possible benefits of opacity are low if strategic complementarity is not very strong.

For high values of \(r\), the threshold \(\tau^P_j\) is equal to zero, which means that \(L^P_j\) is decreasing over \([\tau^O_j, \tau^T_j]\) and the policymaker chooses transparency. Let \(\pi^P(n_j)\) denote the maximum level of strategic complementarity, such that \(\tau^P_j = 0\) for all \(r \geq \pi^P\). Then for all \(r \geq \pi^P\), the policymaker is always transparent. To explain this result, we need to recall that agents put significant weights on all available public information when the coordination motive is strong. Thus, opacity cannot prevent private agents from using prior information to coordinate. Consequently, opacity can not significantly affect \(L^\text{between}\) and \(L^m_j\), but has a considerable negative impact on \(L^W_j\). Thus, policymakers have less incentives to be opaque when \(r\) is high. When the coordination motive is weak and \(r < \pi^P\), the policymaker chooses opacity if his private information is relatively imprecise and \(L^P_j(\tau^O_j) \leq L^P_j(\tau^T_j)\).
Figure 5 represents the threshold $r^P$ which can be computed with the following formula:

$$r^P = \begin{cases} \frac{2n_j - 1}{n_j}, & \text{if } n_j \geq \frac{1}{2}, \\ 0, & \text{otherwise.} \end{cases}$$  \tag{24}$$

The policymakers never choose an intermediate level of transparency in the equilibrium, even if it is socially optimal. Moreover, it is obvious that for $n_j \geq \frac{3}{4}$, $r^P$ is higher than $r^W$. This means that for any $r \in [r^W, r^P]$, there exist $\tau_j^O$ and $\alpha_j$ such that policymaker chooses opacity, although transparency is socially optimal. For $n_j \in \left[\frac{1}{2}, \frac{3}{4}\right]$, the threshold $r^P$ is lower than $r^W$; therefore, when $r \in [r^P, r^W]$, the policymaker chooses transparency even if this policy is socially detrimental. In other words, policymakers in relatively small regions tend to be too transparent and policymakers in large regions tend to be too opaque.

**Conclusion**

This paper examines the equilibrium and the social optimum in a two-region model with strategic complementarity both inside and between the regions. Due to strategic complementarity between the regions, private agents react to foreign news even if this news is not correlated with the domestic fundamentals. Thus, our model allows to study cross-regional information spillovers, which are usually neglected by the literature on public information, but find a growing support in empirical studies of financial markets.

Cross-regional spillovers render an interesting effect on the use of public information. Under certain conditions, an increase in strategic complementarity may lead to a decrease in the weight of the domestic public signal in private actions. Strengthening of strategic complementarity always leads to a decrease in the weight of private information, but agents can redistribute increased weight of public information between the two public signals. If the cross-borders coordination motive is sufficiently strong, private agents put much more weight on the foreign public signal and the weight of the domestic public signal drops. This, in turn, gives rise to peculiar properties of the
The model can be extended in several ways. First of all, in this paper, we assume that the extent of strategic complementarity inside the region is equal to the cross-regional extent of strategic complementarity. The assumption of lower complementarity between the regions would probably make the picture more realistic. Secondly, in our model, private agents have the identical access to both public signals. More natural environment would suppose some additional noise which distorts the signal on its cross-border way. Furthermore, the fundamentals in our model are uncorrelated, and the regional losses are additively separable in the signal precisions. If we relax this assumption, we will obtain the optimal policies in the form of reaction functions. This would provide a useful benchmark to analyze informational policy interactions.

**Appendix**

**A1. Social loss**

After substituting the coefficients (13-15) into the social loss (19), we easily obtain that the partial derivative of $L^W$ over $\tau_j$ is equal to:

$$\frac{\partial L^W}{\partial \tau_j} = (1-r)n_j P_W(\tau_j, n_j, r) \frac{B_W}{2} \beta_j \tau_j^2 \left( \tau_j - n_j r + 1 \right)^3,$$

where $P_W(\tau_j, n_j, r) = A_W \tau_j^3 + B_W \tau_j^2 + C_W \tau_j + D_W$ and $A_W = -(1 - (1 - n_j)r^2) < 0$, $B_W = 3n_j^2 r^3 - 3n_j r(r^2 + r - 1) + r^2 - 1$, $C_W = n_j r(1 - n_j)(n_j r^2 - 2r + 1)$, $D_W = n_j r(1 - n_j)(n_j r^2 - 2r + 1)$.

As $\frac{C_W}{D_W} = \frac{1}{1 - n_j r} > 0$, the signs of $C_W$ and $D_W$ coincide. Moreover, the sign of $\lim_{\tau_j \to 0} \frac{\partial L^W}{\partial \tau_j}$ coincides with the sign of $D_W$. We can conclude that $D_W > 0$ if $r < \tilde{r} = \frac{1}{\sqrt{1 - n_j}}$. In this case, $L^W$ is increasing in $\tau_j$ when $\tau_j$ is small. When the extent of complementarity is high, so that $r > \tilde{r} = \frac{1}{\sqrt{1 - n_j}}$, $D_W$ is negative and $L^W$ is decreasing in $\tau_j$ when $\tau_j$ is small.

The polynomial $P_W$ can have at most three rational roots which correspond to Vieta’s formulas:

$$\sum \tau_k = -\frac{B_W}{A_W} \quad \text{and} \quad \prod \tau_k = -\frac{C_W}{A_W},$$

where $\tau_k$ is the k-th root of $P_W$. Thus, if $D_W$ is positive, $\prod \tau_k > 0$ and $P_W$ has an uneven number of positive roots. Suppose that there are three positive roots. Then $\frac{\partial P_W}{\partial \tau} = 3A_W \tau^2 + 2B_W \tau + C_W = 0$ has two positive roots. The smallest root of this expression is equal to $\frac{B_W - \sqrt{B_W^2 - 4A_W(C_W)}}{2}$. This value is negative because $A_W < 0$ and $C_W > 0$. Thus, there is only one positive root, $\tau^*$, and $L^W(\tau_j)$ has inverted-U shape irrespectively of the presence of complex roots (Figure 1a).

If $D_W$ is negative, $\prod \tau_k < 0$ and $P_W$ has either none or two positive roots. There are two necessary conditions for the existence of two positive roots: the discriminant of $P_W$, $\Omega_W$, should be positive and $\frac{\partial P_W}{\partial \tau} = 3A_W \tau^2 + 2B_W \tau + C_W = 0$ should have one positive root. The largest root of $\frac{\partial P_W}{\partial \tau} = 0$ is equal to $\frac{B_W + \sqrt{B_W^2 - 4A_W(C_W)}}{2A_W}$. Given that $C_W < 0$ and $A_W < 0$, the value of this root is positive only if $B_W > 0$. We can show that these three conditions ($D_W < 0$, $B_W > 0$ and
$\Omega_W > 0$) hold only if $r \in [\hat{r}, r^*)$, where $r^*$ is a feasible solution for $\Omega_W = 0$, different from $\hat{r}$. These conditions hold for the dark gray part in Figure 3. In this case, $L_W$ is represented in Figure 1c. If these conditions do not hold, $P_W$ has no positive root. As $D_W < 0$, $L_W$ is decreasing in $\tau_j$ and Figure 1a illustrates this situation.

**A2. Information choice**

Similar to A1, we compute the partial derivative of $L^P_j$ over $\tau_j$, which is defined by the following expression:

$$(A2) \frac{\partial L^P_j}{\partial \tau_j} = \frac{(1 - r)n_j P_P(\tau_j, n_j, r)}{\beta_j \tau_j^2 (\tau_j - n_j r + 1)^3},$$

where $P_P(\tau_j, n_j, r) = A_P \tau_j^3 + B_P \tau_j^2 + C_P \tau_j + D_P$ and $A_P = -(n_j^2 r^2 - 2n_j^2 r - nr^2 + 3n_j r - r + 1) < 0$, $B_P = 3n_j^3 r^3 - 6n_j^3 r^2 - 3n_j^2 r^3 + 6n_j^2 r^2 + 6n_j^2 r - 2n_j r^2 - 4n_j r + r - 1$, $C_P = -3n_j r^2 (1 - n_j)(2 - n_j r)(n_j r - 2n_j + 1)$, $D_P = -n_j r^2 (1 - n_j)(1 - n_j r)(2 - n_j r)(n_j r - 2n_j + 1)$. The signs of $C_P$ and $D_P$ coincide and are positive if $r < r^P = \frac{2m-1}{n}$. If $D_P < 0$, $P_P$ has an uneven number of positive roots. Similar to (A1), we can show that there cannot be three positive roots. Thus, when $r$ is relatively small, $L^P_j$ has inverted-U shape irrespectively of the presence of complex roots (Figure 1a). If $D_P > 0$, $P_P$ has no positive roots and $L^P_j$ is a decreasing function of $\tau_j$.

**References**


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