Fiscal policy and the unbalanced pension system[1]

Mariam Mamedli[2]

Abstract

Ageing of the population forces the government to carry out pension reforms in order to insure future sustainability of the pension system. Due to the aggravated recently public finances, the reforming of the social security system is becoming even more urgent as the government won't be able to cover the deficit of the pension fund with the transfers from the federal budget. We suggest that the reforms of the pension system may be considered as the measures of fiscal consolidation. In order to define the optimal policy mix of measures we consider the shocks to the retirement age, life expectancy and productivity on the basis of the OLG model. The welfare analysis has shown that the higher income tax is socially optimal in financing the pensions when the deficit of the pension fund is covered out of the state budget. This illustrates the substitutive nature of the income tax and social contributions. Moreover, it was shown that the optimal combination of the pension reforms and traditional fiscal instruments can decrease the level of the public debt.

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[2]National Research University Higher School of Economics,Laboratory for Macroeconomic analysis; mmamedli@hse.ru

1 Introduction

Many countries are launching social security reforms in order to secure the sustainability of the social security system in the future, provided increasing expenditures on social payments. The main drivers of this dynamics are the demographic changes: fertility rates below the replacement level and higher life expectancy.

Although several reforms of the social security system have been implemented, federal transfers remain one of the key sources of balancing the budget of the pension system. Moreover, their share is expected to rise steadily in the future: according to OECD estimates, this part of fiscal expenditures will increase from 9.3% of the GDP in 2010 to 11.7% of the GDP in 2050.[1] In Russia, the deficit of the pension fund is covered as well by the transfer from the federal budget. The transfer amounted to 4.3% of the GDP in 2013 and 3.4% - in 2014.[2]

However, the financing of the pension fund deficit out of the federal budget has become even more complicated after the financial crisis of 2008-2009 and the European debt crisis, started in 2010. Pension reforms (introduction of the higher retirement age, higher social deductions, lower pensions) can be considered as an alternative to the traditional measures of the fiscal consolidation.

The aim of this research is to define the optimal set of measures (consolidation measures and the reforms of the PAYG pension system) chosen by the social planner. The analysis is based on the overlapping generations model (OLG) initially developed by Yaari (1965) and Blanchard (1985) and extended further by Buiter (1988), Giovannini (1988), Weil (1989) and Bovenberg (1993). In order to investigate the optimal policy mix we extend the model of Heijdra and Bettendorf (2006), who have analyzed the economic consequences of lower pensions and higher retirement age in an open economy with traded and non-traded sector. They, however, consider exogenous interest rate along with the rudimentary the pension system: it allows to analyze intergenerational redistribution yet is assumed to be balanced. We extend their model to investigate unbalanced budget of the pension fund in the closed economy with endogenous interest rate. Moreover, while Heijdra and Bettendorf (2006) consider the consequences of two shocks on the welfare of each generation, we model the social welfare function. This allows to specify the socially optimal fiscal policy as well as to analyze the change in the welfare due to the demographic shocks and the productivity shock.

We consider how the optimal fiscal policy changes with an increase in the retirement age. The results show that the optimal set of measures depends on structural characteristics of the economy (birth and death rates, productivity, type of the pension system). Although higher retirement age can decrease the deficit of the pension fund with

[1]OECD, Pensions at glance 2013

[2]Transfer has decreased due to the freeze of the accumulation part of the pension savings

optimally chosen income tax, it may also lead to a decrease in pensions. However, the adjustment of the income tax along with the higher retirement age can lead to a decrease in the public debt. When the longer life expectancy is considered, numeric results suggest that both social contributions and pensions should adjust to be socially optimal in the new equilibrium, while the optimal income tax remains unchanged. This result holds for both balanced and unbalanced pension system, although in the former case an increase in the income tax is optimal for the negative growth rates in order to insure higher government expenditures and public debt after the shock. The results for the productivity shock suggest that the slower decrease in the productivity with age can actually improve the public finances, decreasing both, the public debt and the deficit of the pension fund.

The paper is organized as follows. Section 2 presents an extended OLG model of Heijdra and Bettendorf (2006) with the pension system. In Section 3 the consequences of three shocks are considered (increase in the retirement age, increase in the life expectancy for both two types of pension systems and the productivity shock). Section 4 summarizes the results of the research with detailed layout of the results in the appendix.

2 The model

The model of Heijdra and Bettendorf (2006) was extended by introducing an unbalanced pension system in the closed economy with an endogenous interest rate. The deficit of the pension system is considered as a liability of the benevolent government, who conducts fiscal policy so to maximize the social welfare.

2.1 Households

Individual households

The representative consumer born in the period υ maximize an expected present value of instantaneous utility, which is additively separable with respect to the personal consumption and government expenditures. Individual consumption and public good are assumed to be imperfect substitutes, with the elasticity of substitution $\kappa > 0$.

$$U(\upsilon, t) = \int_t^\infty \left[(1-\kappa)\ln \bar{c}(\upsilon, t) + \kappa \ln g(\upsilon, t)\right] \mathrm{e}^{(\rho+\beta)(t-\tau)} d\tau, \tag{1}$$

where $\bar{c}$ is a personal consumption, g stands for the per capita government expenditures, $\rho > 0$ is the rate of time preference and $\beta \geq 0$ is the probability of death.

The households receive an interest rate $r(\tau)$ on the financial wealth, $\bar{a}(\upsilon, \tau)$, and have a non interest income net of lump-sum taxes or transfers, $WI(\upsilon, \tau)$. The payment $\beta a(\upsilon, \tau)$ is the actuarially fair annuity paid by the life insurance company.[1] Interest and non interest net labor income are spend on consumption and savings. Household financial

[1] See Yaari (1965), Blanchard (1985)

wealth consists of capital goods ($\bar{k}$) and government bonds ($\bar{a}^G$), both denominated in terms of consumer goods.

A dot above the variable stands for the variable's time derivative (change in time), thus, $\dot{\bar{a}}(\upsilon,\tau) = d\bar{a}(\upsilon,\tau)/d\tau$. The household budget constraint in terms of the consumer good is:

$$\dot{\bar{a}}(\upsilon,\tau) = (r(\tau)+\beta)\bar{a}(\upsilon,\tau) + WI(\upsilon,\tau) - \bar{c}(\upsilon,\tau) \tag{2}$$

$$\bar{a}(\upsilon,\tau) = \bar{k}(\upsilon,\tau) + \bar{a}^G(\upsilon,\tau) \tag{3}$$

Following Bettendorf and Heijdra (2006) we use the PAYG pension scheme, introduced by Nielsen (1994). We assume that the young individuals, ageing from zero to π, pay a lump-sum tax t_W. After an age threshold of π the households start to receive a lump-sum transfer z. Net labor income takes the following form:

$$WI(\upsilon,\tau) = \begin{cases} (1-t_L)W^N(\upsilon,\tau) - t_W & \text{for } \tau-\upsilon \le \pi, \\ (1-t_L)W^N(\upsilon,\tau) + z & \text{for } \tau-\upsilon > \pi. \end{cases}, \tag{4}$$

where $W^N(\upsilon,\tau)$ is a wage of the worker born in period υ at the time τ. Labor productivity is assumed to depend on the age of the worker. The worker of the generation υ at time τ supplies $n(\upsilon,\tau)$ efficiency units of labor:

$$n(\upsilon,\tau) = E(\tau-\upsilon)\bar{l}(\upsilon,\tau), \tag{5}$$

where $\bar{l}(\upsilon,\tau) = 1$ is the labor hours and following Blanchard (1985) $E(\tau-\upsilon)$ is the efficiency index, which falls exponentially with the worker's age:

$$E(\tau-\upsilon) = \omega_0 \mathrm{e}^{-\alpha(\tau-\upsilon)}, \tag{6}$$

where ω_0 is a positive constant and $\alpha > 0$ specifies the speed at which the efficiency falls with age.

At each period t the household chooses the paths of consumption and financial assets so to maximize the present value of the lifetime utility (1) subject to the budget constraint (2) and a transversality condition. The initial value of the financial assets $a(\upsilon,t)$ and the government consumption per each household are taken as given.

The optimal path of the household consumption is defined by Euler condition:

$$\frac{\dot{\bar{c}}(\upsilon,t)}{\bar{c}(\upsilon,t)} = r(t) - \rho \tag{7}$$

Consumption in each period is proportional to the total wealth:

$$\bar{c}(\upsilon,t) = (\rho+\beta)(\bar{a}(\upsilon,t) + \bar{a}^H(\upsilon,t)), \tag{8}$$

where $\bar{a}^H$ is a human wealth defined as the present value of the after-tax labor income:

$$\bar{a}^H(\upsilon, t) = \int_t^\infty WI(\upsilon, \tau)\mathrm{e}^{(\rho+\beta)(t-\tau)}d\tau \tag{9}$$

Demography

Along with Bettendorf and Heijdra (2006) we model the framework that allows to consider a non-zero population growth, by distinguishing the probability of death $\beta \geq 0$, and the probability of birth, $\eta > 0$.[2] The population size $L(t)$ grows with net growth rate n_L:

$$\frac{\dot{L}(t)}{L(t)} = \eta - \beta = n_L \tag{10}$$

Taking into account the initial condition $L(0) = 1$, the population size is:

$$L(t) = \mathrm{e}^{n_L t} \tag{11}$$

The size of the generation born in the current period is assumed to be proportional to the size of the population in this period:

$$L(\upsilon, \upsilon) = \eta L(\upsilon) \tag{12}$$

The size of each generation falls exponentially with the probability of death β:

$$L(\upsilon, t) = \mathrm{e}^{\beta(\upsilon-t)}L(\upsilon, \upsilon), t \geq \upsilon \tag{13}$$

The current size of the generation born at time υ can be obtained by substituting (11) and (12) into (13):

$$L(\upsilon, t) = \eta \mathrm{e}^{\eta\upsilon}\mathrm{e}^{-\beta t} \tag{14}$$

Aggregate household sector

The aggregate variables are the integral of the variable values, specific for each living generation, weighted by the size of corresponding generation. Aggregate consumption, for example, can be defined as follows:

$$C(t) = \int_{-\infty}^t L(\upsilon, t)\bar{c}(\upsilon, t)d\upsilon, \tag{15}$$

where $L(\upsilon, t)$ and $\bar{c}(\upsilon, t)$ are given by (14) and (8), respectively.

Aggregate consumption is proportional to the household's wealth, where $A(t)$ is

[2] This framework was developed by Buiter (1988)

aggregate financial wealth and $A^G(t)$ is aggregate human wealth:

$$C(t) = (\rho + \beta)\left[A(t) + A^H(t)\right] \tag{16}$$

The change in the aggregate consumption is obtained by differentiating (15) with respect to time and taking into account (14):

$$\dot{C}(t) = \int_{-\infty}^{t} L(\upsilon,t)\bar{c}(\upsilon,t)d\upsilon + \eta L(t)\bar{c}(t,t) - \beta C(t) \tag{17}$$

The growth rate of the aggregate consumption is obtained by substituting (7) into (17) and dividing by $C(t)$:

$$\frac{\dot{C}(t)}{C(t)} = [r(t) - \rho] + \frac{\eta L(t)\bar{c}(t,t) - \beta C(t)}{C(t)} \tag{18}$$

The first item on the right-hand side is the growth of individual consumption, while the second term represents the so-called generational turnover (Bettendorf, Heijdra, 2006), which depends on the demographic parameters. Aggregate consumption increases with the arrival of new agents and decreases with the death of the older generation.

The growth rate of the aggregate consumption can be simplified to:[3]

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho + \alpha + n^L - (\rho + \beta)\frac{\eta\gamma L(t) + (\alpha + \eta)A(t)}{C(t)}, \tag{19}$$

$$\gamma(t) = \frac{d(t)}{r(t) + \beta} + (r(t) + \alpha + \beta)\left(\frac{\mathrm{e}^{-\beta\pi}}{1 - \mathrm{e}^{-\eta\pi}}\right)\left(\frac{z + d(t)}{r(t) + \beta}\right)\left(\frac{\mathrm{e}^{-r(t)\pi} - \mathrm{e}^{-n_L\pi}}{n^L - r(t)}\right) \tag{20}$$

The aggregate consumption growth, therefore, exceeds the growth of individual consumption if the net population growth is positive ($n_L > 0$), the labor productivity decreases over time ($\alpha > 0$). It can be lower if newborns consume less or due to the redistribution from the young to the old through the pension system. In contract to Bettendorf and Heijdra (2006) γ depends on the surplus (deficit) of the pension fund.

Aggregate financial wealth is defined as follows:

$$A(t) = \int_{-\infty}^{t} L(\upsilon,t)\bar{a}(\upsilon,t)d\upsilon \tag{21}$$

The definition of the aggregate savings can be found by differentiating an equation (21) for the aggregate financial wealth with respect to time and taking into account that the newborn generation does not have any financial wealth, $\bar{a}(t,t) = 0$:

[3] For greater detail see Appendix 1

$$\dot{A}(t) = -\beta A(t) + \int\limits_{-\infty}^{t} L(v,t)\dot{\bar{a}}(v,t)dv \tag{22}$$

By substituting (2) in (22) we get:[4]

$$\dot{A}(t) = r(t)A(t) + WI(t) - C(t), \tag{23}$$

$$WI(t) = \frac{\eta\omega_0}{\alpha+\eta}(1-t_L)F_N(k_N(t),1))L(t) - D(t), \tag{24}$$

where $F_N(k_N(t),1))$ is the marginal product of labor and $D(t)$ is a surplus of the pension system.

The aggregate labor supply in period t measured in efficiency units is proportional to the population size in the corresponding period and is obtained from (5), (6), (11) and (14):

$$N(t) = \int\limits_{-\infty}^{t} L(v,t)\bar{n}(v,t)dv = \frac{\eta\omega_0}{\alpha+\eta}L(t) \tag{25}$$

2.2 Firms

As opposed to Bettendorf and Heijdra (2006) we consider a closed economy with one domestic production sector. The output is produced according to the Cobb-Douglas technology $Y = F(K,N) = K^{\varepsilon}N^{1-\varepsilon}$, where K and N represent capital and efficiency labor units. The production function is characterized by the the constant returns to scale, positive and diminishing marginal products of both factors and unitary substitution elasticity. Efficiency units of labor are:

$$N(t) = \int\limits_{-\infty}^{t} E(\tau - v)L(v,t)dv \tag{26}$$

Producers maximize the profit, choosing the optimal level of capital and labor:

$$\Pi(t) = Y(t) - \int\limits_{-\infty}^{t} W^N(v,t)L(v,t)dv - W^K(t)K(t), \tag{27}$$

where $W^K(t)$ is a capital rent and $W^N(v,t)$ is the wage of the worker of the vintage v at time t.

The first order conditions are:

$$W^K(t) = F_K(k_N(t),1) \tag{28}$$

[4] for grater details see Appendix 2

$$W^N(t) \equiv \frac{W^N(\upsilon,t)}{E(\tau-\upsilon)} = F_N(k_N(t),1), \tag{29}$$

where $F_K = \partial F/\partial K_N$, $F_N = \partial F/\partial N$, $W^N(t)$ is the wage per efficiency unit of labor and $k_N(t) = K(t)/N(t)$ is the capital per efficiency unit of labor.

The produced output is allocated to the private consumption, investment and government expenditures.

$$Y(t) = C(t) + I(t) + G(t) \tag{30}$$

2.3 Portfolio investments

The optimal investment decision is based on the maximization of net present value of cash flows from the owned capital stock subject to the capital accumulation identity:

$$V(t) = \int_t^\infty \left[W^K(\tau)K(\tau) - I(\tau)\right] \mathrm{e}^{-R(t,\tau)} d\tau \tag{31}$$

$$s.t.\dot{K}(\tau) = I(\tau) - \delta K(\tau) \tag{32}$$

where I represents the gross investment and $R(t,\tau) = \int_t^\tau r(s)ds$ is a discount factor.

Thus, (32), the first order condition for the problem, specifies that the rental rate W^K equals to the return on the capital $r(t)$ taking into account the amortization rate δ.

$$W^K = r(t) - \delta \tag{33}$$

2.4 Public sector

Government budget identity defines the accumulation path of public debt A^G, which depends on the current government expenditures $G(t)$, revenues from the labor tax and additional income (or expenditure) coming from the surplus (or deficit) of the pension fund.

$$\dot{A^G}(t) = r(t)A^G(t) + G(t) - t_L W^N(t)N(t) - D(t) \tag{34}$$

Taking into account the transversality condition:

$$\lim_{\tau \longrightarrow \infty} A^G(t)\mathrm{e}^{R(t,\tau)} = 0 \tag{35}$$

Public debt is:

$$A^G(t) = \int_{-\infty}^{\tau} \left[t_L W^N(\tau)N(\tau) - G(\tau) + D(\tau)\right] \mathrm{e}^{-r(t,\tau)} d\tau \tag{36}$$

2.5 Pension system

The key difference with the paper of Bettendorf and Heijdra (2006) is the assumption that the PAYG pension system can be run on non balanced-budget basis, with a surplus

$D(t) > 0$ or deficit, $D(t) < 0$.

$$t_W(1 - e^{-\eta\pi})L(t) = z\mathrm{e}^{-\eta\pi}L(t) + D(t) \tag{37}$$

The left-hand side of (37) represents the total social contributions paid by the young, while on the right-hand side there are total pensions paid to the old and the surplus (or deficit) of the pension fund if the sum of social contributions and pensions don't match.

2.6 Equilibrium of model and the social welfare function

The key equations in the per capita terms are presented in Table 1 below. The endogenous variables are k, y, c, a, a^G, r, W^N,W^K, γ, n. The exogenous variables are β, α, η, π, ρ, z, t_W, t_L.

The Eq.T1 corresponds to the accumulation of capital per capita, and it is obtained by combining (30) and (32). Eq.T2 stands for the optimal path of per capita consumption, obtained from (19) in per capita terms. Eq.T3 is the government budget constraint expressed in per capita terms, derived from the government budget constraint (34).The last dynamic equation, Eq.T4, represents accumulation of per capita assets and is obtained from the (23), taking into account (24) and (37).

$$\dot{k}(t) = ny(t) - c(t) - g(t) - (n_L + \delta)k(t) \tag{$T1.1$}$$

$$\dot{c}(t) = (r(t) - \rho + \alpha)c(t) - (\rho + \beta)(\eta\gamma(t) + (\alpha + \eta)a(t)) \tag{$T1.2$}$$

$$\dot{a}^G(t) = (r(t) - n_L)a^G(t) + g(t) - nt_L W^N(t) + d(t) \tag{$T1.3$}$$

$$\dot{a}(t) = (r(t) - n_L)a(t) - c(t) + n(1 - t_L)W^N(t) + d(t) \tag{$T1.4$}$$

$$\gamma(t) = \frac{-d}{r(t) + \beta} + \left(\frac{\mathrm{e}^{-\beta\pi}}{1 - \mathrm{e}^{-\eta\pi}}\right)\left(\frac{z - d}{r(t) + \beta}\right)(r(t) + \alpha + \beta)\left(\frac{\mathrm{e}^{-r(t)\pi} - \mathrm{e}^{-n_L\pi}}{n^L - r(t)}\right) \tag{$T1.5$}$$

$$t_W(1 - \mathrm{e}^{-\eta\pi}) = z\mathrm{e}^{-\eta\pi} - d \tag{$T1.6$}$$

$$W^K(t) = \varepsilon k_N(t)^{\varepsilon - 1} = \varepsilon\left(\frac{k(t)}{n}\right)^{\varepsilon - 1} \tag{$T1.7$}$$

$$r(t) = W^K(t) - \delta \tag{$T1.8$}$$

$$W^N(t) = (1 - \varepsilon)y(t) \tag{$T1.9$}$$

$$y(t) = k_N^{\varepsilon}(t) = \left(\frac{k(t)}{n}\right)^{\varepsilon} \tag{$T1.10$}$$

$$n = \frac{\eta\omega_0}{\alpha + \eta} \tag{$T1.11$}$$

$$a(t) = k(t) + a^G(t) \tag{$T1.12$}$$

Since the analytical solution of the model is complicated by the non-linearity of the equations and the high power of the equation which defines the steady-state level of the capital per capita we are solving numerically the system of equations T1.1-T1.3, taking into account T.4-T.12.[5] Restricting $\dot{k}(t) = 0$, and $\dot{a}^G(t) = 0$ in the steady-state we use a root-finding method (the bisection method) to define the level of capital per capita bring the growth consumption per capita to zero $\dot{c}(t) = 0$. The program runs through all possible combinations of fixed and variable parameters on set initially intervals to determine the steady-state level of k^* and the corresponding combinations of parameters which brings $\dot{c}(t) = 0$.

With an aim to distinguish the socially optimal policy mix of measures we check if the found set of possible equilibria satisfies the stability condition of the equilibrium and the condition on the limit of the public debt.[6]

We define the social welfare function as a present value of the utility of all currently living and future generations taking into account their share in the population. The first term in (38) represents the welfare of retirees, while the second - the welfare of the young.

$$SW(\tau) = \int_{t}^{\infty} \int_{-\infty}^{\tau-\pi} L(\upsilon,\tau)[(1-\kappa)\ln \bar{c}(\upsilon,\tau) + \kappa \ln \bar{g}(\upsilon,\tau)]e^{(\rho+\beta)(t-\tau)} d\upsilon d\tau + \tag{38}$$

$$+ \int_{t}^{\infty} \int_{\tau-\pi}^{\tau} L(\upsilon,\tau)[(1-\kappa)\ln \bar{c}(\upsilon,\tau) + \kappa \ln \bar{g}(\upsilon,\tau)]e^{(\rho+\beta)(t-\tau)} d\upsilon d\tau$$

In order to derive the social welfare as a function of the steady-state value of the k^* we express the individual consumption for young and elder generation as a function of their individual human wealth (a_y^H and a_o^H, respectively).[7]

$$SW(t) = \chi \mathrm{e}^{\eta\pi} \left[(1-\kappa)(\ln((\rho+\beta)a_o^H) + (r-\rho)\frac{\pi\eta+1}{\eta}) + \kappa \ln g \right] - \tag{39}$$

$$-\chi \left[(1-\mathrm{e}^{-\eta\pi})((1-\kappa)\ln((\rho+\beta)a_y^H) - \kappa \ln g) - (1-\kappa)(r-\rho)(1-\mathrm{e}^{\eta\pi} - \eta\pi\mathrm{e}^{-\eta\pi})\eta^{-1}) \right],$$

$$\chi = \frac{\mathrm{e}^{n_L t}}{n_L - \rho - \beta}.$$

3 Results of the shocks

3.1 Initial assumptions and restrictions

With the goal to specify socially optimal fiscal policy some of the parameters were fixed, allowing to focus on the fiscal instruments subject to the characteristics of the pension system and demographic conditions. For the results presented below capital depreciation rate δ was set to 3%, output elasticity of capital ε - to 0.33, rate of time

[5]All calculations were conducted using the Matlab language

[6]The stability condition insures that the determinant of the Jacobian matrix of the log-linearized dynamic system of equations T.1.1, T1.2 and T1.4 is less than zero. In this case model is locally saddle-point stable.

[7]For greater detail see Appendix 3

preference ρ - to 1.5%. The share of the government expenditures in the utility function, κ, was set to 0.5, while the value of government expenditures was fixed at 25% of the GDP, which is the common value to the OECD countries. Moreover, the speed of productivity decline with age, α, equals to 1.25%, meaning that the worker is twice less productive at the retirement age in the first two cases, where this parameter is fixed.

The share of pensions equal 30% of the median life-time wage, while the optimal size of the mandatory social contributions, ψ, as a percentage of the median wage, is chosen from the maximization of the social welfare function.[1]

$$t_W = \psi \omega_0 F_N(k(t), 1) \mathrm{e}^{-\alpha\tau} \tag{40}$$

$$\tau = \pi - \frac{1}{\alpha} \ln \left(\frac{1 + \mathrm{e}^{\alpha\pi}}{2} \right) \tag{41}$$

3.2 Increase of the retirement age

First, we consider an increase of the retirement age, π, from 55 to 70 with the step of five years. For the results below the death rate β equals to 1.25%, bringing the life expectancy to 80 years. The birth rate η varies from 1% to 3%.[2] This allows to analyze both negative and positive population growth. The results for the selected growth rates are presented in the Table 2 below. The table presents the steady-state of the economy, where $\bar{W}$ stands for the median wage, c_{share} is the share of private consumption per capita in the output and SW_y and SW_o provide the welfare of the corresponding generations.

First, the case of fixed income tax was considered in order to focus on the optimal changes in the pension system as the result of the higher retirement age. Numeric results suggest that higher retirement age leads to the lower pensions for the considered birth rates. This dynamics can be explained by the lower median wage due to the extended working period. At the same time the social contributions are decreasing with the higher retirement age. The reason for this result is twofold: first, the lower median wage corresponds to the higher working period; second, the optimal share of contributions, ψ is chosen by the program at a lower level as lower pensions are payed to retirees. Meanwhile, the budget is run with deficit for the considered population growth rates. However, the change in the deficit of the pension fund in the equilibrium is not monotonous. It generally increases at negative and low growth rates ($n_L \leq 0.5\%$) and decreases afterwards.

[1]The results are robust to the change in the share of pensions, with the change in the equilibrium level of key variables, although the comparative results are the same

[2]For the birth rate an interval $[0\%; 3\%]$ was considered. However, the results with the birth rates lower than 1% were filtered out by the stability condition and the debt limit restriction

Table 2. *Key model variables and policy instruments for* $\pi = 60$

η	1%	1,25%	1,5%	1,75%	2%	2,2%	2,4%	2,6%	2,8%	3%
ψ	29%	21%	15%	11%	9%	7%	5%	4%	3%	2%
t_W	0,173	0,139	0,107	0,084	0,071	0,057	0,042	0,034	0,026	0,018
z	0,179	0,198	0,215	0,228	0,237	0,244	0,250	0,256	0,258	0,263
$\bar{W}$	0,894	0,992	1,074	1,139	1,184	1,218	1,248	1,278	1,291	1,314
W^N	1,821	1,796	1,783	1,769	1,742	1,730	1,718	1,713	1,691	1,686
$y*n$	1,214	1,347	1,459	1,547	1,608	1,655	1,695	1,735	1,753	1,785
g	0,303	0,337	0,365	0,387	0,402	0,414	0,424	0,434	0,438	0,446
k	9,052	9,784	10,437	10,890	10,987	11,140	11,257	11,461	11,273	11,420
a^G	0,006	0,127	0,051	0,030	0,454	0,415	0,099	0,307	0,333	0,202
c	0,661	0,717	0,755	0,779	0,794	0,801	0,804	0,803	0,802	0,797
c_{share}	54,5%	53,2%	51,8%	50,4%	49,4%	48,4%	47,4%	46,3%	45,7%	44,6%
n_L	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,15%	1,35%	1,55%	1,75%
r	1,47%	1,59%	1,66%	1,74%	1,88%	1,95%	2,02%	2,05%	2,18%	2,21%
d_{share}	-1,66%	-1,52%	-1,62%	-1,64%	-1,35%	-1,42%	-1,62%	-1,54%	-1,55%	-1,61%
SW	-0,304	-5,345	-9,115	-12,820	-17,428	-20,980	-24,994	-29,691	-36,941	-45,272
SW_o	11,946	9,004	7,322	6,061	4,811	4,223	3,772	3,577	3,033	3,033
SW_y	-12,251	-14,349	-16,437	-18,881	-22,239	-25,203	-28,766	-33,267	-39,974	-48,305

Table 3. *Key model variables and policy instruments for* $\pi = 65$

η	1%	1,25%	1,5%	1,75%	2%	2,2%	2,4%	2,6%	2,8%	3%
ψ	26%	18%	13%	10%	7%	5%	4%	3%	2%	1%
t_W	0,152	0,118	0,091	0,074	0,054	0,040	0,033	0,025	0,017	0,009
z	0,175	0,196	0,211	0,223	0,233	0,240	0,246	0,251	0,255	0,258
$\bar{W}$	0,8762	0,9792	1,05315	1,11705	1,1628	1,2024	1,2316	1,2528	1,2741	1,2921
W^N	1,821	1,809	1,783	1,769	1,745	1,741	1,730	1,713	1,702	1,690
$y*n$	1,214	1,356	1,459	1,547	1,611	1,666	1,706	1,735	1,765	1,790
g	0,303	0,339	0,365	0,387	0,403	0,416	0,427	0,434	0,441	0,447
k	9,052	9,983	10,437	10,890	11,037	11,365	11,487	11,461	11,503	11,509
a^G	0,078	0,070	0,122	0,384	0,264	0,075	0,327	0,448	0,419	0,105
c	0,661	0,718	0,755	0,779	0,794	0,800	0,803	0,803	0,800	0,796
c_{share}	54,5%	52,9%	51,8%	50,4%	49,3%	48,0%	47,1%	46,3%	45,3%	44,5%
n_L	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,15%	1,35%	1,55%	1,75%
r	1,47%	1,53%	1,66%	1,74%	1,87%	1,89%	1,95%	2,05%	2,12%	2,18%
d_{share}	-1,56%	-1,59%	-1,55%	-1,36%	-1,48%	-1,62%	-1,51%	-1,49%	-1,53%	-1,64%
SW	-1,475	-5,893	-10,353	-14,174	-18,483	-21,560	-25,738	-31,103	-37,603	-46,459
SW_o	11,410	8,932	6,766	5,501	4,326	3,949	3,481	2,993	2,710	2,501
SW_y	-12,884	-14,825	-17,119	-19,675	-22,809	-25,509	-29,220	-34,096	-40,314	-48,960

Since several possible equilibria were filtered out at the stage of condition verification we have considered the case where the income tax, t_L, was not fixed and could adjust to the change in the retirement age.[3] The results for $\pi = 60$ and $\pi = 65$ are presented in Tables 4 and 5 below.[4]

In this case pension system is run with deficit as well. It varies from 8.2% to 1.6% of the GDP, depending on the growth rate. Its dynamics, however, is more stable than in the previous case: the deficit of the pension fund is generally decreasing for the growth rates considered. Most of the adjustment of the pension fund falls on the size of the pensions, that work as a automatic stabilizer (as well as in the previous case, it equals 30% of the median wage which is changing alone with the working period). For most of the cases social contributions are decreasing with the 5 year increase in the retirement age due to the decline in its share in the wage, ψ. In this case, however, the value of ψ is

[3]The income tax has been chosen on the interval $[0; 50]$ so to maximize the social welfare function.

[4]For greater details see the Appendix 4

less informative since most of the adjustment falls on the income tax. For some growth rates, optimal level of ψ is chosen at the zero, coupled with the corresponding change in the income tax rate, t_L, while for other cases t_L remains unchanged. This illustrates that income tax and social contribution rates can act as substitutes in the adjustment to the new equilibrium.

The lower pensions along with decreased deficit of the pension fund result in the lower social welfare for all the growth rates considered, while the output, consumption, government expenditures and capital are generally unchanged.

Table 4. *Key model variables and policy instruments for* $\pi = 60$ *with an optimal* t_L

η	**0,5%**	**0,75%**	**1%**	**1,25%**	**1,5%**	**1,75%**	**2%**	**2,2%**	**2,6%**	**3%**
ψ	21%	6%	0%	0%	0%	1%	1%	1%	2%	2%
t_W	0,084	0,031	0,000	0,000	0,000	0,008	0,008	0,008	0,017	0,018
z	0,120	0,156	0,182	0,202	0,218	0,231	0,240	0,247	0,256	0,263
$\bar{W}$	0,401	0,519	0,608	0,675	0,726	0,771	0,801	0,825	0,854	0,876
W^N	1,904	1,881	1,858	1,833	1,809	1,795	1,768	1,757	1,718	1,686
t_L	50%	50%	50%	48%	47%	45%	44%	43%	41%	40%
$y * n$	0,816	1,058	1,238	1,375	1,480	1,571	1,632	1,680	1,740	1,785
g	0,204	0,265	0,310	0,344	0,370	0,393	0,408	0,420	0,435	0,446
k	6,658	8,424	9,614	10,399	10,890	11,384	11,488	11,670	11,560	11,420
a^G	0,043	0,008	0,203	0,044	0,402	0,238	0,400	0,184	0,038	0,202
c	0,462	0,583	0,664	0,719	0,756	0,779	0,794	0,799	0,802	0,797
c_{share}	56,6%	55,1%	53,7%	52,3%	51,1%	49,6%	48,6%	47,6%	46,1%	44,6%
n_L	-0,75%	-0,50%	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,35%	1,75%
r	1,086%	1,187%	1,294%	1,408%	1,529%	1,599%	1,737%	1,800%	2,018%	2,211%
d_{share}	-8,2%	-8,3%	-8,1%	-7,0%	-6,0%	-4,8%	-4,1%	-3,6%	-2,3%	-1,6%
SW	21,977	10,503	3,266	-2,340	-7,115	-11,055	-15,676	-19,190	-29,208	-45,272
SW_o	29,672	20,269	15,014	11,376	8,919	7,317	5,886	5,177	3,789	3,033
SW_y	-7,695	-9,766	-11,748	-13,716	-16,034	-18,372	-21,563	-24,367	-32,997	-48,305

Table 5. *Key model variables and policy instruments for* $\pi = 65$ *with an optimal* t_L

η	**0,5%**	**0,75%**	**1%**	**1,25%**	**1,5%**	**1,75%**	**2%**	**2,2%**	**2,6%**	**3%**
ψ	16%	3%	0%	0%	0%	1%	1%	0%	1%	1%
t_W	0,063	0,015	0,000	0,000	0,000	0,008	0,008	0,000	0,008	0,009
z	0,118	0,153	0,179	0,199	0,214	0,227	0,236	0,243	0,251	0,258
$\bar{W}$	0,393	0,509	0,596	0,662	0,712	0,756	0,786	0,809	0,838	0,861
W^N	1,904	1,881	1,858	1,833	1,809	1,795	1,769	1,757	1,718	1,690
t_L	50%	50%	49%	48%	46%	44%	43%	43%	41%	40%
$y * n$	0,816	1,058	1,238	1,375	1,480	1,571	1,633	1,680	1,741	1,790
g	0,204	0,265	0,310	0,344	0,370	0,393	0,408	0,420	0,435	0,447
k	6,658	8,424	9,614	10,399	10,890	11,384	11,491	11,670	11,561	11,509
a^G	0,017	0,015	0,104	0,580	0,256	0,045	0,136	0,419	0,161	0,105
c	0,462	0,583	0,664	0,719	0,756	0,779	0,794	0,799	0,802	0,796
c_{share}	56,6%	55,1%	53,7%	52,3%	51,1%	49,6%	48,6%	47,6%	46,1%	44,5%
n_L	-0,75%	-0,50%	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,35%	1,75%
r	1,086%	1,187%	1,294%	1,408%	1,529%	1,599%	1,736%	1,800%	2,018%	2,184%
d_{share}	-8,3%	-8,3%	-7,5%	-6,4%	-5,4%	-4,3%	-3,6%	-3,5%	-2,3%	-1,6%
SW	20,982	9,335	1,883	-3,802	-8,514	-12,441	-17,031	-20,612	-30,636	-46,459
SW_o	29,185	19,708	14,276	10,761	8,193	6,607	5,194	4,572	3,191	2,501
SW_y	-8,204	-10,373	-12,393	-14,563	-16,708	-19,049	-22,225	-25,184	-33,827	-48,960

3.3 Increase in the life expectancy

We analyzed an increase in the life expectancy from 70 to 80 years, considering the corresponding change in β from $\beta = 1.43\%$ to $\beta = 1.25\%$. Income tax and social contribution rates were chosen freely so to maximize social welfare function. The limit for the income tax was fixed at 50%, while social contributions were considered up to 35% with the 1% step in both cases. However, the results do not change significantly when the higher bound of 55% for the income tax is considered. For the negative growth rates the optimal income tax is chosen at the upper bound.

We have considered the fixed retirement age so that equilibria under the same birth rates were comparable. In order to check the robustness of the results the change in β was considered for different retirement ages, namely, for π that equals 55, 60, 65 and 70. The results are presented for two types of pension systems: the case of the balanced pension fund, where an additional restriction on the pension fund balance is introduced ($d = 0$) and case of unbalanced pension fund, where the its deficit is covered by the transfer from the federal budget. Since the results for different retirement ages are similar, we will provide here the results for $\pi = 60$ as it allows to consider a wider range of growth rates not filtered out by the restrictions than for $\pi = 55$ and leaves 10 years at the retirement when the life expectancy shock occurs.

Table 6. *Key model variables and policy instruments for $\beta = 1.43\%$ with an optimal t_L*

η	**0,8%**	**1%**	**1,25%**	**1,5%**	**1,75%**	**2%**	**2,3%**	**2,6%**	**3%**
ψ	5%	0%	0%	0%	1%	1%	2%	2%	2%
t_W	0,027	0,000	0,000	0,000	0,008	0,008	0,017	0,017	0,017
z	0,161	0,183	0,203	0,218	0,230	0,240	0,248	0,255	0,262
$\bar{W}$	0,537	0,611	0,676	0,726	0,766	0,800	0,827	0,852	0,874
t_L	50%	50%	48%	47%	45%	44%	42%	41%	40%
$y * n$	1,094	1,244	1,378	1,479	1,561	1,630	1,685	1,735	1,781
y	2,804	2,800	2,756	2,712	2,676	2,648	2,601	2,569	2,522
g	0,274	0,311	0,345	0,370	0,390	0,407	0,421	0,434	0,445
k	8,601	9,751	10,471	10,882	11,178	11,431	11,401	11,455	11,329
a^G	0,093	0,187	0,040	0,352	0,195	0,333	0,049	0,029	0,139
c	0,617	0,682	0,738	0,775	0,799	0,814	0,823	0,824	0,818
c_{share}	56,36%	54,85%	53,56%	52,41%	51,22%	49,95%	48,81%	47,46%	45,91%
n_L	-0,63%	-0,43%	-0,18%	0,07%	0,32%	0,57%	0,87%	1,17%	1,57%
r	1,24%	1,25%	1,39%	1,53%	1,66%	1,75%	1,93%	2,05%	2,24%
d_t	-0,089	-0,101	-0,096	-0,089	-0,075	-0,067	-0,050	-0,040	-0,029
d_{share}	-8,18%	-8,08%	-6,95%	-5,99%	-4,83%	-4,09%	-2,97%	-2,32%	-1,61%
SW	7,451	3,316	-1,897	-6,219	-9,910	-13,304	-18,050	-23,224	-33,123

Table 7. *Key model variables and policy instruments for $\beta = 1.25\%$ with an optimal t_L*

η	**0,8%**	**1%**	**1,25%**	**1,5%**	**1,75%**	**2%**	**2,3%**	**2,6%**	**3%**
ψ	5%	0%	0%	0%	1%	1%	2%	2%	2%
t_W	0,027	0,000	0,000	0,000	0,008	0,008	0,017	0,017	0,018
z	0,162	0,182	0,202	0,218	0,231	0,240	0,249	0,256	0,263
$\bar{W}$	0,540	0,608	0,675	0,726	0,771	0,801	0,831	0,854	0,876
t_L	50%	50%	48%	47%	45%	44%	42%	41%	40%
$y * n$	1,101	1,238	1,375	1,480	1,571	1,632	1,692	1,740	1,785
y	2,822	2,786	2,750	2,713	2,692	2,653	2,612	2,577	2,529
g	0,275	0,310	0,344	0,370	0,393	0,408	0,423	0,435	0,446
k	8,766	9,614	10,399	10,890	11,384	11,488	11,550	11,560	11,420
a^G	0,106	0,203	0,044	0,402	0,238	0,400	0,062	0,038	0,202
c	0,602	0,664	0,719	0,756	0,779	0,794	0,802	0,802	0,797
c_{share}	54,70%	53,65%	52,31%	51,08%	49,63%	48,61%	47,36%	46,11%	44,62%
n_L	-0,45%	-0,25%	0,00%	0,25%	0,50%	0,75%	1,05%	1,35%	1,75%
r	1,19%	1,29%	1,41%	1,53%	1,60%	1,74%	1,89%	2,02%	2,21%
d_t	-0,090	-0,100	-0,096	-0,089	-0,076	-0,067	-0,050	-0,040	-0,029
d_{share}	-8,18%	-8,08%	-6,95%	-5,99%	-4,83%	-4,09%	-2,97%	-2,32%	-1,61%
SW	9,109	3,266	-2,340	-7,115	-11,055	-15,676	-21,687	-29,208	-45,272

Under the unbalanced pension system an increase in the life expectancy leads to the lower private consumption, higher output and, as the result, higher government expenditures. Social welfare increases for the low birth rate (in this case 1%) and starts to decrease for the higher birth rates due to the lower welfare of young as the result of the lower savings and interest rate. The welfare of the elder generation is increasing to the longer life expectancy. Public debt is increasing after the shock due to the higher government expenditures and lower interest rate, while optimal income tax rate remains virtually unchanged. At the same time pension system is run with deficit, which is increasing for most of the cases due to the low social contributions and increasing pensions due to the higher median wage.

Table 8. *Key model variables and policy instruments for* $\beta = 1.25\%$ *and* $d = 0$

η	**0,8%**	**1%**	**1,25%**	**1,5%**	**1,75%**	**2%**	**2,3%**	**2,6%**	**3%**
ψ	49%	36%	27%	21%	16%	13%	10%	8%	6%
t_W	0,255	0,215	0,176	0,146	0,121	0,101	0,082	0,067	0,051
z	0,157	0,176	0,197	0,213	0,225	0,235	0,245	0,252	0,259
$\bar{W}$	0,523	0,588	0,656	0,710	0,749	0,784	0,817	0,840	0,864
t_L	0,380	0,380	0,380	0,380	0,380	0,380	0,380	0,380	0,380
$y * n$	1,066	1,198	1,337	1,447	1,527	1,597	1,665	1,712	1,760
y	2,731	2,695	2,675	2,653	2,618	2,595	2,570	2,536	2,493
g	0,266	0,299	0,334	0,362	0,382	0,399	0,416	0,428	0,440
k	7,949	8,697	9,568	10,183	10,463	10,751	10,995	11,011	10,934
a^G	0,185	0,217	0,269	0,324	0,373	0,443	0,556	0,684	0,955
c	0,597	0,659	0,716	0,754	0,779	0,794	0,803	0,805	0,800
c_{share}	55,98%	55,03%	53,54%	52,13%	51,02%	49,75%	48,25%	47,03%	45,48%
n_L	-0,45%	-0,25%	0,00%	0,25%	0,50%	0,75%	1,05%	1,35%	1,75%
r	1,47%	1,59%	1,66%	1,74%	1,87%	1,95%	2,05%	2,18%	2,36%
SW	3,018	-2,125	-6,366	-10,171	-14,304	-18,275	-23,791	-31,447	-47,783

Under the balanced system an increase in the life time expectancy leads to the corresponding increase in social contributions in contract to the previous case, where the financing of the pensions is carried out virtually through income tax revenues. However, if two balanced pension systems under different life expectancy are considered, the optimal share of social contributions in the median wage is constant as well as the optimal tax income for the corresponding growth rates. In this case, pensions, income tax rate and contributions rate work as automatic stabilizers, where the optimal rates are unchanged, yet the values paid adjust to the new higher level of the median wage (due to the higher level of capital).

With the transition to the balanced pension system, as expected, the income tax is lower for the corresponding growth rates as in this case pensions are financed by social contributions with an increasing optimal level. At the same time, public debt in the equilibrium with the balanced pension system is higher, due to the lower tax revenues (both the rate and the aggregate wage is lower due to the fall in capital).

3.4 Increased labor productivity

Higher productivity is modeled as the lower speed with which the product of labor is falling with age. The results are presented in Tables 5 and 6 for $\alpha = 1.25\%$ and $\alpha = 1.1\%$.[5] An increase in labor productivity leads to the higher capital and, therefore, per capita output as well as government expenditures and private consumption. An increase in the productivity results in the higher wage, increasing the level of pensions, whose level is set at 30% of the median wage. It puts the higher pressure on the pension system and, thus, the government budget. However, income tax rate, which is chosen optimally in this numeric example, remains virtually unchanged, varying within $40 - 50\%$ for different growth rates. For some growth rates tax payments do not cover the increased liabilities of the pension system. In these cases social contributions are increasing through the optimal level of ψ. However, despite the fact that the deficit of the pension fund is increasing, public debt is lower in the new equilibrium due to the higher income tax payments (driven by the higher wage).

Table 9. *Key model variables and policy instruments for* $\alpha = 1.25\%$ *with an optimal* t_L

η	0,5%	0,75%	1%	1,25%	1,5%	1,75%	2%	2,2%	2,5%	3%
ψ	21%	6%	0%	0%	0%	1%	1%	1%	1%	2%
t_W	0,084	0,031	0,000	0,000	0,000	0,008	0,008	0,008	0,008	0,018
z	0,120	0,156	0,182	0,202	0,218	0,231	0,240	0,247	0,255	0,263
$\bar{W}$	0,401	0,519	0,608	0,675	0,726	0,771	0,801	0,825	0,849	0,876
t_L	50%	50%	50%	48%	47%	45%	44%	43%	42%	40%
$y * n$	0,816	1,058	1,238	1,375	1,480	1,571	1,632	1,680	1,730	1,785
g	0,204	0,265	0,310	0,344	0,370	0,393	0,408	0,420	0,432	0,446
k	6,658	8,424	9,614	10,399	10,890	11,384	11,488	11,670	11,646	11,420
a^G	0,043	0,008	0,203	0,044	0,402	0,238	0,400	0,184	0,237	0,202
c	0,462	0,583	0,664	0,719	0,756	0,779	0,794	0,799	0,802	0,797
c_{share}	56,65%	55,10%	53,65%	52,31%	51,08%	49,63%	48,61%	47,57%	46,39%	44,62%
n_L	-0,75%	-0,50%	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,25%	1,75%
r	1,09%	1,19%	1,29%	1,41%	1,53%	1,60%	1,74%	1,80%	1,95%	2,21%
d_{share}	-8,24%	-8,32%	-8,08%	-6,95%	-5,99%	-4,83%	-4,09%	-3,57%	-2,90%	-1,61%
SW	21,977	10,503	3,266	-2,340	-7,115	-11,055	-15,676	-19,190	-26,187	-45,272

Table 10. *Key model variables and policy instruments for* $\alpha = 1.1\%$ *with an optimal* t_L

η	0,5%	0,75%	1%	1,25%	1,5%	1,75%	2%	2,2%	2,5%	3%
ψ	23%	8%	0%	0%	0%	1%	1%	0%	1%	2%
t_W	0,103	0,046	0,000	0,000	0,000	0,008	0,009	0,000	0,009	0,019
z	0,134	0,171	0,201	0,222	0,237	0,249	0,258	0,264	0,271	0,280
$\bar{W}$	0,446	0,571	0,671	0,740	0,791	0,830	0,860	0,880	0,905	0,935
t_L	50%	50%	50%	49%	47%	45%	44%	44%	42%	40%
$y * n$	0,882	1,130	1,327	1,463	1,565	1,642	1,700	1,741	1,790	1,848
g	0,220	0,282	0,332	0,366	0,391	0,411	0,425	0,435	0,447	0,462
k	7,021	8,770	10,300	11,062	11,518	11,748	11,807	11,879	11,888	11,797
a^G	0,050	0,080	0,007	0,521	0,203	0,029	0,191	0,523	0,019	0,013
c	0,503	0,628	0,712	0,765	0,799	0,821	0,832	0,837	0,837	0,826
c_{share}	57,08%	55,59%	53,65%	52,31%	51,08%	49,96%	48,96%	48,05%	46,77%	44,69%
n_L	-0,75%	-0,50%	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,25%	1,75%
r	1,19%	1,29%	1,29%	1,41%	1,53%	1,66%	1,80%	1,89%	2,02%	2,22%
d_{share}	-8,22%	-8,21%	-8,32%	-7,17%	-6,17%	-4,98%	-4,22%	-4,05%	-2,99%	-1,66%
SW	18,587	8,168	2,990	-2,368	-6,829	-11,074	-15,467	-19,052	-25,522	-43,193

[5] More detailed tables for α decreasing from 1.25% to 1.1% are given in the Appendix 4

Although both, private consumption and government expenditures are higher in the new equilibrium, the dynamics of the social welfare is different for the growth rates considered. It is lower in the new equilibrium for the negative and small rates of population growth and is increasing with the higher productivity for higher growth rates. Social welfare is increasing for η higher than 1.5% for a decrease in α from 1.25% to 1.1%. Different sign in the change of the social welfare can be explained by the relation of the welfare of young and elder generations: for small growth rates an increase in the welfare of young is lower that the fall the welfare of of the older generation.

4 Conclusion

We extend the OLG model with the infinitely living households developed by Bettendorf and Heijdra (2006) by introducing an unbalanced pension system, where the deficit of the pension fund is covered by the transfer from the government budget. This assumption makes income tax and characteristics of the pension system interact as imperfect substitutes to insure the stability of the public debt in the equilibrium.

The developed framework can be used to analyze further the optimal set of the reforms of the pension system and fiscal policy measures, increasing the number of fiscal instruments, that can be chosen optimally.

The results of this research suggest that the financing of the pension fund deficit through the income tax is optimal, when the deficit is covered by the transfer from the state budget. Moreover, the reforms of the pension system such as higher pension age can lower the value of the public debt, and, therefore, can be used together with the traditional measures of the fiscal consolidation.

The results of the research allow to extend the research of the fiscal measures of consolidation and define the welfare optimal measures of fiscal consolidation and pension reforms. These results can be useful in the analysis of consequences of demographic shocks for the public finances and in the development of the optimal consolidation measures.

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Appendix

1. Derivation of the aggregate Euler equation

The equation (18) can simplified as follows.

$$\frac{\dot{C}(t)}{C(t)} = [r(t) - \rho] + \frac{\eta L(t)\bar{c}(t,t) - \beta C(t)}{C(t)} \tag{A1}$$

As new generations are born without any financial assets ($\bar{a}(t,t) = 0$), thus from (8) $\bar{c}(t,t) = (\rho + \beta)\bar{a}^H(t,t)$ and taking into account (16) we get:

$$\frac{\eta L(t)\bar{c}(t,t) - \beta C(t)}{C(t)} = (\rho + \beta)\frac{\eta L(t)\bar{a}^H(t,t) - \beta(A(t) + A^H(t))}{C(t)}$$

Aggregate human wealth is defined as follows:

$$A^H(t) = \left[\bar{a}^H(\upsilon,t)\right]_{t-\upsilon>\pi} + \left[\bar{a}^H(\upsilon,t)\right]_{0<t-\upsilon\leq\pi}$$

where:

$$\left[\bar{a}^H(\upsilon,t)\right]_{t-\upsilon>\pi} = \int_t^\infty (1-t_L)W^N(\upsilon,t)\mathrm{e}^{R(t,\tau)+\beta(t-\tau)}d\tau + \int_t^\infty z\mathrm{e}^{R(t,\tau)+\beta(t-\tau)}d\tau$$

and $R(t,\tau) = \int_t^\tau r(s)ds$

$$\left[\bar{a}^H(\upsilon,t)\right]_{0<t-\upsilon\leq\pi} = \int_t^\infty (1-t_L)W^N(\upsilon,t)\mathrm{e}^{R(t,\tau)+\beta(t-\tau)}d\tau - \int_t^{\upsilon+\pi} t_W\mathrm{e}^{R(t,\tau)+\beta(t-\tau)}d\tau + \int_{\upsilon+\pi}^\infty z\mathrm{e}^{R(t,\tau)+\beta(t-\tau)}d\tau$$

For the simplicity of the analysis let us assume the constant interest rate r (so that equations could be applicable to the steady state).

$$\left[\bar{a}^H(\upsilon,t)\right]_{0<t-\upsilon\leq\pi} = \int_t^\infty (1-t_L)W^N(\upsilon,t)\mathrm{e}^{(r+\beta)(t-\tau)}d\tau - \frac{t_W}{r+\beta}\left(1-\mathrm{e}^{-(r+\beta)(\upsilon+\pi-t)}\right) + \frac{z}{r(t)+\beta}\mathrm{e}^{-(r+\beta)(\upsilon+\pi-t)}$$

We know that age dependent wage can be written as follows:

$$W^N(\upsilon,t) = E(\tau-\upsilon)F_N(k(t),1) = \omega_0 \mathrm{e}^{-\alpha(\tau-\upsilon)}F_N(k(t),1)$$

$$\int_t^\infty (1-t_L)W^N(\upsilon,t)\mathrm{e}^{(r+\beta)(t-\tau)}d\tau = \mathrm{e}^{\alpha(\upsilon-t)}\Omega_0(t)$$

where $\Omega_0(t)$ is defined as follows:

$$\Omega_0(t) = \omega_0 \int_t^\infty (1-t_L)F_N(k(t),1)\mathrm{e}^{(r+\alpha+\beta)(t-\tau)}d\tau$$

Substituting this definition into the expressions for human wealth of workers and retirees, noted above, we get:

$$\int_{-\infty}^{t-\pi} L(\upsilon,t)\left[\bar{a}^H(\upsilon,t)\right]_{t-\upsilon>\pi} d\upsilon =$$

$$= \int_{-\infty}^{t-\pi} \eta \mathrm{e}^{\eta\upsilon}\mathrm{e}^{-\beta t}\left[\mathrm{e}^{\alpha(\upsilon-t)}\Omega_0(t)\upsilon + \frac{z}{r+\beta}\right] d\upsilon =$$

$$= L(t)\left[\frac{\eta}{\alpha+\eta}\Omega_0(t)\mathrm{e}^{-(\alpha+\eta)\pi} - \frac{z}{r+\beta}\mathrm{e}^{-\eta\pi}\right].$$

$$\int_{t-\pi}^{t} L(\upsilon,t)\left[\bar{a}^H(\upsilon,t)\right]_{0<t-\upsilon\leq\pi} d\upsilon =$$

$$= \int_{t-\pi}^{t} \eta \mathrm{e}^{\eta\upsilon}\mathrm{e}^{-\beta t}\left[\mathrm{e}^{\alpha(\upsilon-t)}\Omega_0(t) - \frac{t_W}{r+\beta}\left(1-\mathrm{e}^{-(r+\beta)(\upsilon+\pi-t)}\right) + \frac{z}{r+\beta}\mathrm{e}^{-(r+\beta)(\upsilon+\pi-t)}\right] d\upsilon =$$

$$= L(t)\left[\frac{\eta}{\alpha+\eta}\Omega_0(t)\left(1-\mathrm{e}^{-(\alpha+\eta)\pi}\right) - \frac{t_W}{r+\beta}(1-\mathrm{e}^{-\eta\beta}) + \frac{\eta(t_W+z)}{r+\beta}e^{-\beta\pi}\left(\frac{\mathrm{e}^{-r\pi}-\mathrm{e}^{-n_L\pi}}{n^L-r}\right)\right]$$

Thus aggregate human wealth is:

$$A^H(t) = L(t)\left[\frac{\lambda\Omega_0(t)}{\alpha+\eta} + \eta\mathrm{e}^{-\beta\pi}\frac{t_W+z}{r+\beta}\left(\frac{\mathrm{e}^{-r\pi}-\mathrm{e}^{-n_L\pi}}{n^L-r}\right)\right]$$

where it was used that:

$$t_W(1-\mathrm{e}^{-\eta\pi}) = z\mathrm{e}^{-\eta\pi} + d^{pens}(t) \Rightarrow t_W + z = \frac{z+d^{pens}(t)}{1-\mathrm{e}^{-\eta\pi}}$$

From the expression for working-age households, taking into account that $t_W = d + (t_W + z)\mathrm{e}^{-\eta\pi}$:

$$\bar{a}^H(t,t) = \Omega_0(t) + \left(\frac{t_W + z}{r(t)+\beta}\right)\left(\mathrm{e}^{-(r(t)+\beta)\pi} - \mathrm{e}^{-\eta\pi}\right) - \frac{t_W}{r(t)+\beta} =$$

$$= \Omega_0(t) + \mathrm{e}^{-\beta\pi}\left(\frac{t_W + z}{r(t)+\beta}\right)\left(\mathrm{e}^{-r(t)\pi} - \mathrm{e}^{-n_L\pi}\right) - \frac{d(t)}{r(t)+\beta}$$

After substituting this expression in the equation for A^H and eliminating $\Omega_0(t)$ we get:

$$\eta L(t)\bar{a}^H(t,t) = (\alpha+\eta)A^H(t) - \eta\gamma L(t)$$

where

$$\gamma = \frac{d(t)}{r+\beta} + (r+\alpha+\beta)\left(\frac{\mathrm{e}^{-\beta\pi}}{1-\mathrm{e}^{-\eta\pi}}\right)\left(\frac{z+d(t)}{r+\beta}\right)\left(\frac{\mathrm{e}^{-r\pi} - \mathrm{e}^{-n_L\pi}}{n^L - r}\right)$$

Taking into account the expression for $\bar{a}^H(t,t)$ and taking into account (16) we get:

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho + \alpha + n^L - (\rho+\beta)\frac{\eta\gamma L(t) + (\alpha+\eta)A(t)}{C(t)}$$

2. Derivation of aggregate non-interest income

Aggregate non-interest income is defined as follows:

$$WI(t) = \int_{-\infty}^{t} L(v,t)WI(v,t)dv,$$

where $WI(v,t)$ is defined as follows:

$$WI(v,\tau) = \begin{cases} (1-t_L)W^N(v,\tau) - t_W & \text{for } \tau - v \le \pi, \\ (1-t_L)W^N(v,\tau) + z & \text{for } \tau - v > \pi. \end{cases}$$

Thus $WI(t)$ is split into parts, non-interest income of the retirees and non-interest income of the young:

$$WI(t) = \int_{-\infty}^{t-\pi} L(v,t)\left[(1-t_L)WI(v,t) + z\right]dv + \int_{t-\pi}^{t} L(v,t)\left[(1-t_L)WI(v,t) - t_W\right]dv$$

After applying the expression for $W^N(v,t) = E(t-v)F_N(k_N(t,1)) = \omega_0\mathrm{e}^{-\alpha(t-v)}F_N(k_N(t,1))$, which comes from taking into account the definition of the efficiency index $E(\tau - v)$ and fact that the wage of particular worker born at period v is equal to the marginal product of labor, adjusted for his or her productivity.

$$WI(t) = \int_{-\infty}^{t-\pi} L(\upsilon,t)\left[(1-t_L)\omega_0 \mathrm{e}^{-\alpha(t-\upsilon)} F_N(k_N(t,1)) + z\right] d\upsilon +$$
$$+ \int_{t-\pi}^{t} L(\upsilon,t)\left[(1-t_L)\omega_0 \mathrm{e}^{-\alpha(t-\upsilon)} F_N(k_N(t,1)) - t_W\right] d\upsilon$$

Rearranging we get:

$$WI(t) = (1-t_L)F_N(k_N(t,1))\left[\int_{-\infty}^{t-\pi} L(\upsilon,t)\omega_0 \mathrm{e}^{-\alpha(t-\upsilon)} d\upsilon + \int_{t-\pi}^{t} L(\upsilon,t)\omega_0 \mathrm{e}^{-\alpha(t-\upsilon)} d\upsilon\right] +$$
$$+ z\int_{-\infty}^{t-\pi} L(\upsilon,t)d\upsilon - t_W \int_{t-\pi}^{t} L(\upsilon,t)d\upsilon =$$
$$= (1-t_L)F_N(k_N(t,1)) \int_{-\infty}^{t} L(\upsilon,t)\omega_0 \mathrm{e}^{-\alpha(t-\upsilon)} d\upsilon +$$
$$+ z\int_{-\infty}^{t-\pi} L(\upsilon,t)d\upsilon - t_W \int_{t-\pi}^{t} L(\upsilon,t)d\upsilon$$

Noting from (5) that $\bar{n}(\upsilon,t) = E(t-\upsilon) = \omega_0 \mathrm{e}^{-\alpha(t-\upsilon)}$ and applying the notion of $N(t)$ from (24) we get:

$$WI(t) = (1-t_L)F_N(k_N(t,1))\frac{\eta\omega_0}{\alpha+\eta}L(t) + z\int_{-\infty}^{t-\pi} L(\upsilon,t)d\upsilon - t_W \int_{t-\pi}^{t} L(\upsilon,t)d\upsilon$$

Applying (14) we get:

$$WI(t) = (1-t_L)F_N(k_N(t,1))\frac{\eta\omega_0}{\alpha+\eta}L(t) + z\int_{-\infty}^{t-\pi} \eta \mathrm{e}^{\eta\upsilon-\beta t} d\upsilon - t_W \int_{t-\pi}^{t} \eta \mathrm{e}^{\eta\upsilon-\beta t} d\upsilon =$$
$$= (1-t_L)F_N(k(t,1))\frac{\eta\omega_0}{\alpha+\eta}L(t) - (1-\mathrm{e}^{-\eta\pi})t_W \mathrm{e}^{n_L t} + z\mathrm{e}^{-\eta\pi}\mathrm{e}^{n_L t}$$

Taking into account the fact that the pension system is run on a non balanced manner, thus using (34) the equation of the aggregate income can be simplified as follows:

$$WI(t) = (1-t_L)\frac{\eta\omega_0}{\alpha+\eta}F_N(k_N(t,1))L(t) - D(t)$$

3. Social welfare

$$SW = \int_{t}^{\infty} \int_{-\infty}^{\tau-\pi} L(\upsilon,\tau)[(1-\kappa)ln\bar{c}(\upsilon,\tau) + \kappa ln\bar{g}(\upsilon,\tau)]e^{(\rho+\beta)(t-\tau)}d\upsilon d\tau +$$

$$+ \int_{t}^{\infty} \int_{\tau-\pi}^{\tau} L(\upsilon,\tau)[(1-\kappa)ln\bar{c}(\upsilon,\tau) + \kappa ln\bar{g}(\upsilon,\tau)]e^{(\rho+\beta)(t-\tau)}d\upsilon d\tau$$

Taking into account the Eurler equation:

$$\bar{c}(\upsilon,\tau) = \bar{c}(\upsilon,t)\mathrm{e}^{(r-\rho)(\tau-t)}$$

$$\bar{c}(\upsilon,\upsilon) = \bar{c}(\upsilon,t)\mathrm{e}^{(r-\rho)(\upsilon-t)}$$

$$(\rho+\beta)[\bar{a}(\upsilon,\upsilon) + \bar{a}^H(\upsilon,\upsilon)] = (\rho+\beta)[\bar{a}(\upsilon,t) + \bar{a}^H(\upsilon,t)]\mathrm{e}^{-(r(t)-\rho)(\upsilon-t)}$$

Applying that $\bar{a}(\upsilon,\upsilon) = 0$ and simplifying we get:

$$\bar{c}(\upsilon,\tau) = (\rho+\beta)(\bar{a}(\upsilon,\tau) + \bar{a}^H(\upsilon,\tau)) = (\rho+\beta)\bar{a}^H(\upsilon,\upsilon)\mathrm{e}^{-(r-\rho)(\upsilon-t)}$$

$$\bar{a}^H_{old}(\upsilon,t) = \frac{1}{r+\alpha+\beta}\left(\omega(1-\varepsilon)(1-t_L)\left(\frac{k}{n}\right)^{\varepsilon}\mathrm{e}^{\alpha(\upsilon-t)}\right) + \frac{z}{r+\beta}$$

$$\bar{a}^H_{young}(\upsilon,t) = \frac{1}{r+\alpha+\beta}\left(\omega(1-\varepsilon)(1-t_L)\left(\frac{k}{n}\right)^{\varepsilon}\mathrm{e}^{\alpha(\upsilon-t)}\right) - \frac{tw}{r+\beta} + \frac{tw+z}{r+\beta}\mathrm{e}^{-(r+\beta)(\upsilon+\pi-t)}$$

$$\begin{aligned} SW(t) = \frac{\mathrm{e}^{n_L t}}{n_L-\rho-\beta}\big[((1-\kappa)\ln((\rho+\beta)a^H_{old}) + \\ +(1-\kappa)(r-\rho)\frac{\pi\eta+1}{\eta} + \kappa\ln g)\mathrm{e}^{\eta\pi} - \\ -(1-\kappa)\ln((\rho+\beta)a^H_{young})(1-\mathrm{e}^{-\eta\pi}) - \\ -(1-\kappa)(r-\rho)(1-\mathrm{e}^{\eta\pi}-\eta\pi\mathrm{e}^{-\eta\pi})\eta^{-1} - \kappa\ln g(1-\mathrm{e}^{-\eta\pi})\big] \end{aligned}$$

4. Numeric results

Increase in the retirement age

Table 1. Key model variables and policy instruments for $\pi = 55$

η	1%	1,25%	1,5%	1,75%	2%	2,2%	2,4%	2,6%	2,8%	3%
ψ	34%	24%	18%	14%	10%	9%	7%	6%	4%	4%
t_W	0,205	0,162	0,131	0,108	0,081	0,074	0,059	0,051	0,035	0,035
z	0,181	0,202	0,218	0,231	0,242	0,247	0,254	0,257	0,263	0,264
$\bar{W}$	0,906	1,012	1,089	1,155	1,210	1,235	1,270	1,287	1,317	1,322
W^N	1,809	1,795	1,771	1,757	1,745	1,718	1,714	1,691	1,691	1,662
$y * n$	1,206	1,346	1,449	1,537	1,611	1,644	1,690	1,712	0,467	1,760
g	0,301	0,337	0,362	0,384	0,403	0,411	0,423	0,428	0,438	0,440
k	8,875	9,759	10,228	10,676	11,037	10,917	11,166	11,011	11,273	10,934
a^G	0,137	0,073	0,144	0,309	0,009	0,538	0,419	0,732	0,053	1,144
c	0,660	0,717	0,754	0,779	0,794	0,801	0,804	0,805	0,802	0,800
c_{share}	54,8%	53,3%	52,1%	50,7%	49,3%	48,8%	47,6%	47,0%	47,0%	45,5%
n_L	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,15%	1,35%	1,55%	1,75%
r	1,53%	1,60%	1,72%	1,80%	1,87%	2,02%	2,05%	2,18%	2,18%	2,36%
d_{share}	-1,47%	-1,58%	-1,52%	-1,41%	-1,66%	-1,32%	-1,44%	-1,31%	-6,18%	-1,27%
SW	0,133	-4,138	-8,421	-12,114	-15,738	-20,200	-23,849	-29,695	-35,148	-45,814
a_o^H	33,457	33,383	32,492	32,092	31,760	30,376	30,326	29,149	29,327	27,816
a_y^H	22,563	23,266	23,056	23,058	23,266	22,179	22,372	21,516	21,947	20,659
SW_o	11,903	9,526	7,511	6,325	5,507	4,547	4,306	3,677	3,747	3,128
SW_y	-11,770	-13,664	-15,933	-18,438	-21,245	-24,748	-28,154	-33,372	-38,895	-48,942

Table 2. Key model variables and policy instruments for $\pi = 60$

η	1%	1,25%	1,5%	1,75%	2%	2,2%	2,4%	2,6%	2,8%	3%
ψ	29%	21%	15%	11%	9%	7%	5%	4%	3%	2%
t_W	0,173	0,139	0,107	0,084	0,071	0,057	0,042	0,034	0,026	0,018
z	0,179	0,198	0,215	0,228	0,237	0,244	0,250	0,256	0,258	0,263
$\bar{W}$	0,894	0,992	1,074	1,139	1,184	1,218	1,248	1,278	1,291	1,314
W^N	1,821	1,796	1,783	1,769	1,742	1,730	1,718	1,713	1,691	1,686
$y * n$	1,214	1,347	1,459	1,547	1,608	1,655	1,695	1,735	1,753	1,785
g	0,303	0,337	0,365	0,387	0,402	0,414	0,424	0,434	0,438	0,446
k	9,052	9,784	10,437	10,890	10,987	11,140	11,257	11,461	11,273	11,420
a^G	0,006	0,127	0,051	0,030	0,454	0,415	0,099	0,307	0,333	0,202
c	0,661	0,717	0,755	0,779	0,794	0,801	0,804	0,803	0,802	0,797
c_{share}	54,5%	53,2%	51,8%	50,4%	49,4%	48,4%	47,4%	46,3%	45,7%	44,6%
n_L	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,15%	1,35%	1,55%	1,75%
r	1,47%	1,59%	1,66%	1,74%	1,88%	1,95%	2,02%	2,05%	2,18%	2,21%
d_{share}	-1,66%	-1,52%	-1,62%	-1,64%	-1,35%	-1,42%	-1,62%	-1,54%	-1,55%	-1,61%
SW	-0,304	-5,345	-9,115	-12,820	-17,428	-20,980	-24,994	-29,691	-36,941	-45,272
a_{old}^H	34,092	33,335	33,104	32,674	31,436	30,930	30,452	30,353	29,171	29,070
a_{young}^H	23,696	23,622	23,962	23,986	23,104	22,918	22,799	22,785	21,956	21,985
W_{young}	11,946	9,004	7,322	6,061	4,811	4,223	3,772	3,577	3,033	3,033
W_{young}	-12,251	-14,349	-16,437	-18,881	-22,239	-25,203	-28,766	-33,267	-39,974	-48,305

Table 3. Key model variables and policy instruments for $\pi = 65$

η	1%	1,25%	1,5%	1,75%	2%	2,2%	2,4%	2,6%	2,8%	3%
ψ	26%	18%	13%	10%	7%	5%	4%	3%	2%	1%
t_W	0,152	0,118	0,091	0,074	0,054	0,040	0,033	0,025	0,017	0,009
z	0,175	0,196	0,211	0,223	0,233	0,240	0,246	0,251	0,255	0,258
$\bar{W}$	0,8762	0,9792	1,05315	1,11705	1,1628	1,2024	1,2316	1,2528	1,2741	1,2921
W^N	1,821	1,809	1,783	1,769	1,745	1,741	1,730	1,713	1,702	1,690
$y * n$	1,214	1,356	1,459	1,547	1,611	1,666	1,706	1,735	1,765	1,790
g	0,303	0,339	0,365	0,387	0,403	0,416	0,427	0,434	0,441	0,447
k	9,052	9,983	10,437	10,890	11,037	11,365	11,487	11,461	11,503	11,509
a^G	0,078	0,070	0,122	0,384	0,264	0,075	0,327	0,448	0,419	0,105
c	0,661	0,718	0,755	0,779	0,794	0,800	0,803	0,803	0,800	0,796
c_{share}	54,5%	52,9%	51,8%	50,4%	49,3%	48,0%	47,1%	46,3%	45,3%	44,5%
n_L	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,15%	1,35%	1,55%	1,75%
r	1,47%	1,53%	1,66%	1,74%	1,87%	1,89%	1,95%	2,05%	2,12%	2,18%
d_{share}	-1,56%	-1,59%	-1,55%	-1,36%	-1,48%	-1,62%	-1,51%	-1,49%	-1,53%	-1,64%
SW	-1,475	-5,893	-10,353	-14,174	-18,483	-21,560	-25,738	-31,103	-37,603	-46,459
a_o^H	33,964	33,980	32,960	32,526	31,454	31,495	31,013	30,202	29,702	29,179
a_y^H	23,989	24,557	24,150	23,983	23,462	23,713	23,381	22,824	22,530	22,238
SW_o	11,410	8,932	6,766	5,501	4,326	3,949	3,481	2,993	2,710	2,501
SW_y	-12,884	-14,825	-17,119	-19,675	-22,809	-25,509	-29,220	-34,096	-40,314	-48,960

Table 4. Key model variables and policy instruments for $\pi = 70$

η	1%	1,25%	1,5%	1,75%	2%	2,2%	2,4%	2,6%	2,8%	3%
ψ	23%	16%	11%	6%	4%	3%	2%	1%	1%	1%
t_W	0,133	0,103	0,076	0,046	0,032	0,024	0,016	0,008	0,008	0,008
z	0,173	0,192	0,208	0,230	0,236	0,242	0,246	0,250	0,251	0,251
$\bar{W}$	0,866	0,961	1,040	1,149	1,181	1,209	1,231	1,250	1,255	1,255
W^N	1,833	1,809	1,795	1,757	1,743	1,730	1,715	1,702	1,691	1,673
$y * n$	1,222	1,356	1,469	1,622	1,667	1,706	1,737	1,765	1,772	1,771
g	0,306	0,339	0,367	0,405	0,417	0,427	0,434	0,441	0,443	0,443
k	9,244	9,983	10,646	11,262	11,401	11,487	11,492	11,503	11,394	11,157
a^G	0,073	0,148	0,093	0,475	0,204	0,381	0,410	0,240	0,717	1,130
c	0,662	0,718	0,755	0,794	0,800	0,803	0,803	0,800	0,799	0,799
c_{share}	54,2%	52,9%	51,4%	49,0%	48,0%	47,1%	46,2%	45,3%	45,1%	45,1%
n_L	-0,25%	0,00%	0,25%	0,75%	0,95%	1,15%	1,35%	1,55%	1,65%	1,75%
r	1,41%	1,53%	1,60%	1,80%	1,88%	1,95%	2,04%	2,12%	2,18%	2,29%
d_{share}	-1,57%	-1,50%	-1,58%	-1,36%	-1,55%	-1,49%	-1,50%	-1,59%	-1,45%	-1,32%
SW	-1,838	-7,039	-10,921	-19,114	-22,640	-26,900	-32,164	-38,763	-43,417	-49,396
a_o^H	34,670	33,849	33,583	32,050	31,466	30,870	30,155	29,561	28,965	28,032
a_y^H	24,949	24,761	24,958	24,078	23,860	23,447	22,970	22,609	22,093	21,324
SW_o	11,511	8,409	6,629	4,126	3,493	2,983	2,531	2,204	1,952	1,566
SW_y	-13,349	-15,448	-17,550	-23,240	-26,133	-29,883	-34,695	-40,967	-45,369	-50,962

Table 5. Key model variables and policy instruments for $\pi = 55$ with an optimal t_L

η	0,5%	0,75%	1%	1,25%	1,5%	1,75%	2%	2,2%	2,6%	3%
ψ	26%	10%	2%	0%	1%	1%	1%	1%	0%	0%
t_W	0,106	0,053	0,012	0,000	0,007	0,008	0,008	0,008	0,000	0,000
z	0,123	0,159	0,186	0,207	0,224	0,234	0,245	0,251	0,262	0,269
$\bar{W}$	0,409	0,530	0,620	0,689	0,746	0,782	0,818	0,836	0,872	0,895
W^N	1,904	1,881	1,858	1,833	1,821	1,783	1,768	1,745	1,718	1,688
t_L	50%	50%	50%	49%	47%	46%	45%	44%	43%	42%
$y * n$	0,816	1,058	1,238	1,375	1,490	1,560	1,632	1,669	1,741	1,787
g	0,204	0,265	0,310	0,344	0,372	0,390	0,408	0,417	0,435	0,447
k	6,658	8,424	9,614	10,399	11,108	11,161	11,488	11,437	11,561	11,459
a^G	0,022	0,049	0,069	0,107	0,035	0,318	0,549	0,371	0,183	0,453
c	0,462	0,583	0,664	0,719	0,756	0,780	0,794	0,800	0,802	0,796
c_{share}	56,6%	55,1%	53,7%	52,3%	50,8%	50,0%	48,6%	47,9%	46,1%	44,5%
n_L	-0,75%	-0,50%	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,35%	1,75%
r	1,086%	1,187%	1,294%	1,408%	1,470%	1,660%	1,737%	1,865%	2,018%	2,199%
d_{share}	-8,3%	-8,3%	-8,2%	-7,6%	-6,3%	-5,4%	-4,7%	-4,1%	-3,6%	-2,9%
SW	23,002	11,687	4,622	-0,826	-5,021	-10,044	-14,091	-18,203	-27,361	-43,012
a^{H_old}	31,801	32,037	31,798	31,704	32,535	31,206	31,171	30,441	29,680	28,623
a^H_{young}	24,713	25,614	25,920	25,731	25,936	24,560	24,326	23,620	23,003	22,003
SW_o	30,162	20,842	15,640	12,157	9,954	7,769	6,671	5,675	4,708	4,104
SW_y	-7,160	-9,155	-11,018	-12,983	-14,976	-17,814	-20,761	-23,878	-32,069	-47,116

Table 6. Key model variables and policy instruments for $\pi = 60$ with an optimal t_L

η	0,5%	0,75%	1%	1,25%	1,5%	1,75%	2%	2,2%	2,6%	3%
ψ	21%	6%	0%	0%	0%	1%	1%	1%	2%	2%
t_W	0,084	0,031	0,000	0,000	0,000	0,008	0,008	0,008	0,017	0,018
z	0,120	0,156	0,182	0,202	0,218	0,231	0,240	0,247	0,256	0,263
$\bar{W}$	0,401	0,519	0,608	0,675	0,726	0,771	0,801	0,825	0,854	0,876
W^N	1,904	1,881	1,858	1,833	1,809	1,795	1,768	1,757	1,718	1,686
t_L	50%	50%	50%	48%	47%	45%	44%	43%	41%	40%
$y * n$	0,816	1,058	1,238	1,375	1,480	1,571	1,632	1,680	1,740	1,785
g	0,204	0,265	0,310	0,344	0,370	0,393	0,408	0,420	0,435	0,446
k	6,658	8,424	9,614	10,399	10,890	11,384	11,488	11,670	11,560	11,420
a^G	0,043	0,008	0,203	0,044	0,402	0,238	0,400	0,184	0,038	0,202
c	0,462	0,583	0,664	0,719	0,756	0,779	0,794	0,799	0,802	0,797
c_{share}	56,6%	55,1%	53,7%	52,3%	51,1%	49,6%	48,6%	47,6%	46,1%	44,6%
n_L	-0,75%	-0,50%	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,35%	1,75%
r	1,086%	1,187%	1,294%	1,408%	1,529%	1,599%	1,737%	1,800%	2,018%	2,211%
d_{share}	-8,2%	-8,3%	-8,1%	-7,0%	-6,0%	-4,8%	-4,1%	-3,6%	-2,3%	-1,6%
SW	21,977	10,503	3,266	-2,340	-7,115	-11,055	-15,676	-19,190	-29,208	-45,272
a^{H_o}	31,695	31,905	31,649	32,015	31,629	32,204	31,422	31,402	30,275	29,070
a^H_y	25,103	26,011	26,039	25,944	25,269	25,334	24,492	24,364	23,089	21,985
SW_o	29,672	20,269	15,014	11,376	8,919	7,317	5,886	5,177	3,789	3,033
SW_y	-7,695	-9,766	-11,748	-13,716	-16,034	-18,372	-21,563	-24,367	-32,997	-48,305

Table 7. Key model variables and policy instruments for $\pi = 65$ with an optimal t_L

η	0,5%	0,75%	1%	1,25%	1,5%	1,75%	2%	2,2%	2,6%	3%
ψ	16%	3%	0%	0%	0%	1%	1%	0%	1%	1%
t_W	0,063	0,015	0,000	0,000	0,000	0,008	0,008	0,000	0,008	0,009
z	0,118	0,153	0,179	0,199	0,214	0,227	0,236	0,243	0,251	0,258
$\bar{W}$	0,393	0,509	0,596	0,662	0,712	0,756	0,786	0,809	0,838	0,861
W^N	1,904	1,881	1,858	1,833	1,809	1,795	1,769	1,757	1,718	1,690
t_L	50%	50%	49%	48%	46%	44%	43%	43%	41%	40%
$y*n$	0,816	1,058	1,238	1,375	1,480	1,571	1,633	1,680	1,741	1,790
g	0,204	0,265	0,310	0,344	0,370	0,393	0,408	0,420	0,435	0,447
k	6,658	8,424	9,614	10,399	10,890	11,384	11,491	11,670	11,561	11,509
a^G	0,017	0,015	0,104	0,580	0,256	0,045	0,136	0,419	0,161	0,105
c	0,462	0,583	0,664	0,719	0,756	0,779	0,794	0,799	0,802	0,796
c_{share}	56,6%	55,1%	53,7%	52,3%	51,1%	49,6%	48,6%	47,6%	46,1%	44,5%
n_L	-0,75%	-0,50%	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,35%	1,75%
r	1,086%	1,187%	1,294%	1,408%	1,529%	1,599%	1,736%	1,800%	2,018%	2,184%
d_{share}	-8,3%	-8,3%	-7,5%	-6,4%	-5,4%	-4,3%	-3,6%	-3,5%	-2,3%	-1,6%
SW	20,982	9,335	1,883	-3,802	-8,514	-12,441	-17,031	-20,612	-30,636	-46,459
a^{H_o}	31,595	31,781	32,000	31,867	31,925	32,484	31,695	31,244	30,126	29,179
a_y^H	25,555	26,299	26,316	25,725	25,501	25,550	24,709	24,385	23,130	22,238
SW_o	29,185	19,708	14,276	10,761	8,193	6,607	5,194	4,572	3,191	2,501
SW_y	-8,204	-10,373	-12,393	-14,563	-16,708	-19,049	-22,225	-25,184	-33,827	-48,960

Table 8. Key model variables and policy instruments for $\pi = 70$ with an optimal t_L

η	0,5%	0,75%	1%	1,25%	1,5%	1,75%	2%	2,2%	2,6%	3%
ψ	0%	0%	0%	0%	0%	0%	0%	2%	0%	2%
t_W	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,016	0,000	0,017
z	0,117	0,150	0,175	0,195	0,211	0,223	0,231	0,237	0,248	0,253
$\bar{W}$	0,390	0,500	0,585	0,649	0,704	0,742	0,772	0,788	0,828	0,843
W^N	1,927	1,881	1,858	1,833	1,821	1,795	1,770	1,745	1,730	1,686
t_L	53%	53%	49%	47%	45%	44%	43%	41%	41%	39%
$y*n$	0,826	1,058	1,238	1,375	1,490	1,571	1,634	1,669	1,752	1,785
g	0,206	0,265	0,310	0,344	0,372	0,393	0,408	0,417	0,438	0,446
k	6,895	8,424	9,614	10,399	11,108	11,384	11,513	11,437	11,797	11,423
a^G	0,165	1,224	0,506	0,417	0,051	0,245	0,288	0,069	0,110	0,364
c	0,464	0,583	0,664	0,719	0,756	0,779	0,794	0,800	0,801	0,797
c_{share}	56,2%	55,1%	53,7%	52,3%	50,8%	49,6%	48,6%	47,9%	45,7%	44,6%
n_L	-0,75%	-0,50%	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,35%	1,75%
r	0,992%	1,187%	1,294%	1,408%	1,470%	1,599%	1,730%	1,865%	1,951%	2,210%
d_{share}	-10,0%	-8,4%	-7,0%	-5,9%	-5,0%	-4,2%	-3,5%	-2,3%	-2,3%	-0,9%
SW	22,645	8,294	0,554	-5,103	-9,245	-13,703	-18,209	-22,433	-31,152	-48,214
a^{H_o}	31,152	30,133	31,869	32,197	32,982	32,336	31,617	31,180	30,685	29,149
a_y^H	27,020	25,098	26,134	26,007	26,378	25,588	24,814	23,996	23,754	22,043
SW_o	31,262	19,609	13,699	10,066	7,885	6,051	4,682	3,611	2,928	1,800
SW_y	-8,617	-11,315	-13,145	-15,169	-17,130	-19,755	-22,892	-26,044	-34,080	-50,014

Increase in the life expectancy

Table 9. Key model variables and policy instruments for $\beta = 1.43\%$ with an optimal t_L

η	0,8%	1%	1,25%	1,5%	1,75%	2%	2,3%	2,6%	3%
ψ	5%	0%	0%	0%	1%	1%	2%	2%	2%
t_W	0,027	0,000	0,000	0,000	0,008	0,008	0,017	0,017	0,017
z	0,161	0,183	0,203	0,218	0,230	0,240	0,248	0,255	0,262
$\bar{W}$	0,537	0,611	0,676	0,726	0,766	0,800	0,827	0,852	0,874
W^N	1,869	1,866	1,838	1,808	1,784	1,766	1,734	1,713	1,682
t_L	50%	50%	48%	47%	45%	44%	42%	41%	40%
$y * n$	1,094	1,244	1,378	1,479	1,561	1,630	1,685	1,735	1,781
y	2,804	2,800	2,756	2,712	2,676	2,648	2,601	2,569	2,522
g	0,274	0,311	0,345	0,370	0,390	0,407	0,421	0,434	0,445
k	8,601	9,751	10,471	10,882	11,178	11,431	11,401	11,455	11,329
a^G	0,093	0,187	0,040	0,352	0,195	0,333	0,049	0,029	0,139
c	0,617	0,682	0,738	0,775	0,799	0,814	0,823	0,824	0,818
c_{share}	56,36%	54,85%	53,56%	52,41%	51,22%	49,95%	48,81%	47,46%	45,91%
n	0,390	0,444	0,500	0,545	0,583	0,615	0,648	0,675	0,706
n_L	-0,63%	-0,43%	-0,18%	0,07%	0,32%	0,57%	0,87%	1,17%	1,57%
r	1,24%	1,25%	1,39%	1,53%	1,66%	1,75%	1,93%	2,05%	2,24%
dt	-0,089	-0,101	-0,096	-0,089	-0,075	-0,067	-0,050	-0,040	-0,029
d_{share}	-8,18%	-8,08%	-6,95%	-5,99%	-4,83%	-4,09%	-2,97%	-2,32%	-1,61%
SW	7,451	3,316	-1,897	-6,219	-9,910	-13,304	-18,050	-23,224	-33,123
a^{H_o}	29,884	30,566	30,709	30,121	30,095	29,857	29,233	28,723	27,665
a_y^H	24,261	25,101	24,833	24,008	23,604	23,218	22,399	21,859	20,885
SW_o	16,612	13,679	10,106	7,686	5,944	4,820	3,535	2,795	2,010
SW_y	-9,162	-10,363	-12,003	-13,904	-15,854	-18,123	-21,584	-26,019	-35,134

Table 10. Key model variables and policy instruments for $\beta = 1.25\%$ with an optimal t_L

η	0,8%	1%	1,25%	1,5%	1,75%	2%	2,3%	2,6%	3%
ψ	5%	0%	0%	0%	1%	1%	2%	2%	2%
t_W	0,027	0,000	0,000	0,000	0,008	0,008	0,017	0,017	0,018
z	0,162	0,182	0,202	0,218	0,231	0,240	0,249	0,256	0,263
$\bar{W}$	0,540	0,608	0,675	0,726	0,771	0,801	0,831	0,854	0,876
W^N	1,881	1,858	1,833	1,809	1,795	1,768	1,742	1,718	1,686
t_L	50%	50%	48%	47%	45%	44%	42%	41%	40%
$y * n$	1,101	1,238	1,375	1,480	1,571	1,632	1,692	1,740	1,785
y	2,822	2,786	2,750	2,713	2,692	2,653	2,612	2,577	2,529
g	0,275	0,310	0,344	0,370	0,393	0,408	0,423	0,435	0,446
k	8,766	9,614	10,399	10,890	11,384	11,488	11,550	11,560	11,420
a^G	0,106	0,203	0,044	0,402	0,238	0,400	0,062	0,038	0,202
c	0,602	0,664	0,719	0,756	0,779	0,794	0,802	0,802	0,797
c_{share}	54,70%	53,65%	52,31%	51,08%	49,63%	48,61%	47,36%	46,11%	44,62%
n	0,390	0,444	0,500	0,545	0,583	0,615	0,648	0,675	0,706
n_L	-0,45%	-0,25%	0,00%	0,25%	0,50%	0,75%	1,05%	1,35%	1,75%
r	1,19%	1,29%	1,41%	1,53%	1,60%	1,74%	1,89%	2,02%	2,21%
dt	-0,090	-0,100	-0,096	-0,089	-0,076	-0,067	-0,050	-0,040	-0,029
d_{share}	-8,18%	-8,08%	-6,95%	-5,99%	-4,83%	-4,09%	-2,97%	-2,32%	-1,61%
SW	9,109	3,266	-2,340	-7,115	-11,055	-15,676	-21,687	-29,208	-45,272
a^{H_o}	32,163	31,649	32,015	31,629	32,204	31,422	30,986	30,275	29,070
a_y^H	26,200	26,039	25,944	25,269	25,334	24,492	23,800	23,089	21,985
SW_o	19,260	15,014	11,376	8,919	7,317	5,886	4,603	3,789	3,033
SW_y	-10,151	-11,748	-13,716	-16,034	-18,372	-21,563	-26,290	-32,997	-48,305

Table 11. Key model variables and policy instruments for $\beta = 1.43\%$ and $d = 0$

η	0,8%	1%	1,25%	1,5%	1,75%	2%	2,3%	2,6%	3%
ψ	49%	36%	27%	21%	16%	13%	10%	8%	6%
t_W	0,254	0,214	0,176	0,146	0,121	0,102	0,082	0,067	0,051
z	0,156	0,176	0,197	0,213	0,225	0,236	0,245	0,252	0,258
$\bar{W}$	0,521	0,588	0,655	0,709	0,750	0,786	0,817	0,839	0,861
W^N	1,8129	1,7962	1,7807	1,7656	1,7460	1,7341	1,7126	1,6885	1,6561
t_L	0,38	0,38	0,38	0,38	0,38	0,38	0,38	0,38	0,38
$y * n$	1,061	1,197	1,336	1,445	1,528	1,601	1,664	1,710	1,754
y	2,719	2,694	2,671	2,648	2,619	2,601	2,569	2,533	2,484
g	0,265	0,299	0,334	0,361	0,382	0,400	0,416	0,428	0,438
k	7,847	8,692	9,529	10,133	10,479	10,830	10,984	10,972	10,821
a^G	0,166	0,198	0,241	0,287	0,331	0,394	0,470	0,556	0,704
c	0,610	0,675	0,733	0,772	0,798	0,814	0,823	0,825	0,820
c_{share}	57,46%	56,34%	54,87%	53,46%	52,22%	50,84%	49,45%	48,24%	46,79%
n	0,390	0,444	0,500	0,545	0,583	0,615	0,648	0,675	0,706
n_L	-0,63%	-0,43%	-0,18%	0,07%	0,32%	0,57%	0,87%	1,17%	1,57%
r	1,51%	1,59%	1,67%	1,75%	1,86%	1,93%	2,05%	2,20%	2,40%
SW	2,132	-1,969	-5,761	-8,998	-12,228	-15,195	-19,471	-24,826	-35,007
a^{H_o}	32,171	31,913	31,720	31,394	30,694	30,369	29,494	28,422	26,954
a^H_y	20,610	21,089	21,570	21,796	21,634	21,660	21,250	20,626	19,689
SW_o	11,936	9,207	7,069	5,571	4,347	3,601	2,771	2,077	1,339
SW_y	-9,804	-11,176	-12,830	-14,569	-16,575	-18,796	-22,242	-26,904	-36,346

Table 12. Key model variables and policy instruments for $\beta = 1.25\%$ and $d = 0$

η	0,8%	1%	1,25%	1,5%	1,75%	2%	2,3%	2,6%	3%
ψ	49%	36%	27%	21%	16%	13%	10%	8%	6%
t_W	0,255	0,215	0,176	0,146	0,121	0,101	0,082	0,067	0,051
z	0,157	0,176	0,197	0,213	0,225	0,235	0,245	0,252	0,259
$\bar{W}$	0,523	0,588	0,656	0,710	0,749	0,784	0,817	0,840	0,864
W^N	1,821	1,797	1,783	1,769	1,745	1,730	1,713	1,691	1,662
t_L	0,380	0,380	0,380	0,380	0,380	0,380	0,380	0,380	0,380
$y * n$	1,066	1,198	1,337	1,447	1,527	1,597	1,665	1,712	1,760
y	2,731	2,695	2,675	2,653	2,618	2,595	2,570	2,536	2,493
g	0,266	0,299	0,334	0,362	0,382	0,399	0,416	0,428	0,440
k	7,949	8,697	9,568	10,183	10,463	10,751	10,995	11,011	10,934
a^G	0,185	0,217	0,269	0,324	0,373	0,443	0,556	0,684	0,955
c	0,597	0,659	0,716	0,754	0,779	0,794	0,803	0,805	0,800
c_{share}	55,98%	55,03%	53,54%	52,13%	51,02%	49,75%	48,25%	47,03%	45,48%
n	0,390	0,444	0,500	0,545	0,583	0,615	0,648	0,675	0,706
n_L	-0,45%	-0,25%	0,00%	0,25%	0,50%	0,75%	1,05%	1,35%	1,75%
r	1,47%	1,59%	1,66%	1,74%	1,87%	1,95%	2,05%	2,18%	2,36%
SW	3,018	-2,125	-6,366	-10,171	-14,304	-18,275	-23,791	-31,447	-47,783
a^{H_o}	34,212	33,441	33,349	33,015	32,007	31,442	30,793	29,719	28,351
a^H_y	22,035	22,183	22,760	22,998	22,616	22,471	22,233	21,608	20,745
SW_o	13,931	10,477	8,238	6,617	5,159	4,276	3,525	2,827	2,178
SW_y	-10,913	-12,601	-14,604	-16,788	-19,463	-22,551	-27,315	-34,274	-49,961

Increase in the productivity

Table 13. Key model variables and policy instruments for $\alpha = 1.25\%$ with an optimal t_L

η	0,40%	0,50%	0,75%	1,00%	1,25%	1,50%	1,75%	2,00%	2,20%	2,50%	3,00%
ψ	32%	21%	6%	0%	0%	0%	1%	1%	1%	1%	2%
t_W	0,109	0,084	0,031	0,000	0,000	0,000	0,008	0,008	0,008	0,008	0,018
z	0,102	0,120	0,156	0,182	0,202	0,218	0,231	0,240	0,247	0,255	0,263
$\bar{W}$	0,340	0,401	0,519	0,608	0,675	0,726	0,771	0,801	0,825	0,849	0,876
W^N	1,904	1,904	1,881	1,858	1,833	1,809	1,795	1,768	1,757	1,730	1,686
t_L	50%	50%	50%	50%	48%	47%	45%	44%	43%	42%	40%
$y*n$	0,692	0,816	1,058	1,238	1,375	1,480	1,571	1,632	1,680	1,730	1,785
g	0,173	0,204	0,265	0,310	0,344	0,370	0,393	0,408	0,420	0,432	0,446
k	5,650	6,658	8,424	9,614	10,399	10,890	11,384	11,488	11,670	11,646	11,420
a^G	0,037	0,043	0,008	0,203	0,044	0,402	0,238	0,400	0,184	0,237	0,202
c	0,398	0,462	0,583	0,664	0,719	0,756	0,779	0,794	0,799	0,802	0,797
c_{share}	57,46%	56,65%	55,10%	53,65%	52,31%	51,08%	49,63%	48,61%	47,57%	46,39%	44,62%
n_L	-0,85%	-0,75%	-0,50%	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,25%	1,75%
r	1,09%	1,09%	1,19%	1,29%	1,41%	1,53%	1,60%	1,74%	1,80%	1,95%	2,21%
d_{share}	-8,23%	-8,24%	-8,32%	-8,08%	-6,95%	-5,99%	-4,83%	-4,09%	-3,57%	-2,90%	-1,61%
SW	28,207	21,977	10,503	3,266	-2,340	-7,115	-11,055	-15,676	-19,190	-26,187	-45,272
a^{H_o}	30,922	31,695	31,905	31,649	32,015	31,629	32,204	31,422	31,402	30,497	29,070
a^H_y	24,122	25,103	26,011	26,039	25,944	25,269	25,334	24,492	24,364	23,480	21,985
SW_o	34,975	29,672	20,269	15,014	11,376	8,919	7,317	5,886	5,177	4,189	3,033
SW_y	-6,768	-7,695	-9,766	-11,748	-13,716	-16,034	-18,372	-21,563	-24,367	-30,376	-48,305

Table 14. Key model variables and policy instruments for $\alpha = 1.2\%$ with an optimal t_L

η	0,40%	0,50%	0,75%	1,00%	1,25%	1,50%	1,75%	2,00%	2,20%	2,50%	3,00%
ψ	32%	21%	7%	0%	0%	0%	1%	1%	1%	1%	2%
t_W	0,113	0,087	0,038	0,000	0,000	0,000	0,008	0,008	0,008	0,009	0,018
z	0,106	0,125	0,161	0,188	0,209	0,224	0,236	0,245	0,252	0,261	0,268
$\bar{W}$	0,354	0,416	0,538	0,628	0,695	0,747	0,786	0,816	0,839	0,869	0,894
W^N	1,904	1,904	1,881	1,858	1,833	1,809	1,783	1,757	1,745	1,730	1,684
t_L	50%	50%	50%	50%	49%	47%	45%	44%	43%	42%	40%
$y*n$	0,714	0,840	1,085	1,266	1,403	1,507	1,587	1,647	1,694	1,753	1,805
g	0,179	0,210	0,271	0,317	0,351	0,377	0,397	0,412	0,423	0,438	0,451
k	5,826	6,853	8,640	9,832	10,611	11,092	11,350	11,438	11,605	11,803	11,518
a^G	0,008	0,008	0,071	0,143	0,642	0,340	0,163	0,316	0,108	0,169	0,139
c	0,410	0,476	0,598	0,679	0,734	0,770	0,793	0,806	0,812	0,813	0,806
c_{share}	57,46%	56,64%	55,10%	53,65%	52,31%	51,08%	49,96%	48,96%	47,94%	46,39%	44,68%
n_L	-0,85%	-0,75%	-0,50%	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,25%	1,75%
r	1,09%	1,09%	1,19%	1,29%	1,41%	1,53%	1,66%	1,80%	1,87%	1,95%	2,22%
d_{share}	-8,31%	-8,32%	-8,22%	-8,16%	-7,02%	-6,04%	-4,88%	-4,13%	-3,61%	-2,93%	-1,63%
SW	27,798	21,631	10,261	3,177	-2,381	-7,021	-11,429	-15,985	-19,468	-25,742	-44,686
a^{H_o}	31,477	32,272	32,483	32,210	32,087	32,150	31,970	31,179	31,136	30,939	29,355
a^H_y	24,396	25,421	26,209	26,417	25,831	25,609	25,055	24,213	24,071	23,758	22,140
SW_o	34,441	29,172	19,843	14,647	11,183	8,636	6,737	5,356	4,673	3,982	2,776
SW_y	-6,643	-7,541	-9,583	-11,470	-13,564	-15,657	-18,166	-21,341	-24,141	-29,723	-47,461

Table 15. Key model variables and policy instruments for $\alpha = 1.1\%$ with an optimal t_L

η	0,40%	0,50%	0,75%	1,00%	1,25%	1,50%	1,75%	2,00%	2,20%	2,50%	3,00%
ψ	34%	23%	8%	0%	0%	0%	1%	1%	0%	1%	2%
t_W	0,129	0,103	0,046	0,000	0,000	0,000	0,008	0,009	0,000	0,009	0,019
z	0,114	0,134	0,171	0,201	0,222	0,237	0,249	0,258	0,264	0,271	0,280
$\bar{W}$	0,381	0,446	0,571	0,671	0,740	0,791	0,830	0,860	0,880	0,905	0,935
W^N	1,881	1,881	1,858	1,857	1,833	1,809	1,783	1,757	1,741	1,718	1,684
t_L	50%	50%	50%	50%	49%	47%	45%	44%	44%	42%	40%
$y * n$	0,753	0,882	1,130	1,327	1,463	1,565	1,642	1,700	1,741	1,790	1,848
g	0,188	0,220	0,282	0,332	0,366	0,391	0,411	0,425	0,435	0,447	0,462
k	5,993	7,021	8,770	10,300	11,062	11,518	11,748	11,807	11,879	11,888	11,797
a^G	0,026	0,050	0,080	0,007	0,521	0,203	0,029	0,191	0,523	0,019	0,013
c	0,436	0,503	0,628	0,712	0,765	0,799	0,821	0,832	0,837	0,837	0,826
c_{share}	57,88%	57,08%	55,59%	53,65%	52,31%	51,08%	49,96%	48,96%	48,05%	46,77%	44,69%
n_L	-0,85%	-0,75%	-0,50%	-0,25%	0,00%	0,25%	0,50%	0,75%	0,95%	1,25%	1,75%
r	1,19%	1,19%	1,29%	1,29%	1,41%	1,53%	1,66%	1,80%	1,89%	2,02%	2,22%
d_{share}	-8,26%	-8,22%	-8,21%	-8,32%	-7,17%	-6,17%	-4,98%	-4,22%	-4,05%	-2,99%	-1,66%
SW	24,037	18,587	8,168	2,990	-2,368	-6,829	-11,074	-15,467	-19,052	-25,522	-43,193
a^{H_o}	31,290	32,089	32,228	33,399	33,230	33,250	33,019	32,163	31,441	31,120	30,169
a^H_y	23,611	24,638	25,550	27,207	26,577	26,320	25,716	24,827	24,302	23,745	22,630
SW_o	30,536	25,958	17,504	13,889	10,523	8,055	6,218	4,883	4,229	3,285	2,345
SW_y	-6,499	-7,371	-9,336	-10,899	-12,891	-14,884	-17,291	-20,349	-23,281	-28,807	-45,538