Government of Russian Federation

Federal State Autonomous Educational Institution of High Professional Education

«National Research University Higher School of Economics»

National Research University
High School of Economics
Faculty of Psychology

Syllabus for the course
«Digital signal processing»
(Цифровая обработка сигналов)

030300.68 «Cognitive sciences and technologies: from neuron to cognition», Master of Science

Author:
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Approved by: Department of Data Analysis and Artificial Intelligence

Recommended by:

Moscow, 2014
1. Teachers

Author, lecturer: Alexei Ossadtchi,
Faculty of Computer Science, Department of Data Analysis and Artificial Intelligence, professor;
Faculty of Social Sciences, Department of Psychology, Center for cognition and decision making, senior scientist.

Tutor: Nikita Novikov,
Faculty of Social Sciences, Department of Psychology, Center for cognition and decision making, research scientist.

Teaching assistant: Zafer Iscan (to be confirmed)

2. Scope of Use

The present program establishes minimum demands of students’ knowledge and skills, and determines content of the course.

The present syllabus is aimed at department teaching the course, their teaching assistants, and students of the Master of Science program 030300.68 «Cognitive sciences and technologies: from neuron to cognition».

This syllabus meets the standards required by:
- Educational standards of National Research University Higher School of Economics;
- Educational program «Psychology» of Federal Master’s Degree Program 030300.68, 2014;
- University curriculum of the Master’s program in psychology (030300.68) for 2014.

3. Summary

In modern science system's based approach is frequently exercised as it allows to formalize the problems encountered in the real world by representing them in the well studied framework that provides for efficient analysis and solution. In order to get a full grip of his powerful machinery researchers need to understand the fundamentals principles of the theory. The existing classes are either too specialized and mathematically detailed or too much of a cook-book nature. The goal of this class is to provide master students of non-mathematical background with a unifying view on the theory behind the system-oriented approach. The main goal is to develop the intuition behind the complex concepts. We will attempt to do it by showing the similarity of discrete and continuous treatments of the system's theory and interpret the results using the natural concepts of linear algebra.

The second part of the class is primarily dedicated to the fundamentals of estimation theory. We start from the basic concepts and estimation of model parameters in the white and colored noise cases. We will then introduce the Maximum Likelihood and Bayesian approaches to the problem of parameter estimation. We will illustrate the Bayesian methodology via examples of solving the inverse problem in neuroimaging and spectroscopy. We will use the Bayesian concept to develop Kalman filters - advanced model-based estimators taking into account the dynamical properties of the signals.
4. **Learning Objectives**

Learning objectives of the “Digital signal processing” class are to provide students with the unifying view on the theory behind system oriented approach in the modern science and to demonstrate the equivalence of “continuous” and “discrete” approaches. The following grand topics will be covered and supported by an extensive weekly homework assignments and practical sessions.

1. Signals
2. Transforms and decompositions
3. Statistical description of signals in original and transformed spaces
4. Signal models
5. Systems
6. Fundamentals of estimation theory

5. **Learning outcomes**

After completing the study of the “Digital signal processing” the student should:

- Know the definition of signal, basic model signal types, understand the difference between the continuous and discrete signals, be able to formally write an expression of an arbitrary signal, be able to show the link between the continuous and discrete treatments, be able to explain the role linear algebra plays in signal processing.

- Understand the notion of a transform, be able to interpret a transform in the linear algebra framework, be familiar with basic transforms used (Fourier transform, Wavelet transform), be able to formulate one’s own transform and write expressions for the coefficients and synthesis equation, be able to connect statistical properties of signals on both sides of the transform operation.

- Know basic signal models (AR, ARMA, Markov model), list the parameters, understand how parameters affect the nature of the resulting process and its statistical properties.

- Understand the notion of system, understand why LTI systems is an important class of systems, know what convolution operation is, be able to calculate the output of an arbitrary LTI system, be able to relate statistical properties of the output signal to those of the input.

- Understand system parameters estimation problem, know basic criteria used to judge the quality of an estimator, understand the notion of the lower bound on the variance of the estimate, know basic estimation approaches (ML, Bayesian, Least squares) and understand the interrelation between those.

After completing the study of the discipline «Digital signal processing» the student should have the following competences:

<table>
<thead>
<tr>
<th>Competence</th>
<th>Code</th>
<th>Code (UC)</th>
<th>Descriptors (indicators of achievement of the result)</th>
<th>Educativa forms and methods aimed at generation and development of the competence</th>
</tr>
</thead>
<tbody>
<tr>
<td>The ability to reflect developed methods of activity.</td>
<td>SC-1</td>
<td>SC-M1</td>
<td>The student is able to reflect developed mathematical methods to psychological fields and problems.</td>
<td>Lectures and tutorials, group discussions, presentations, paper reviews.</td>
</tr>
<tr>
<td>Competence</td>
<td>Code</td>
<td>Code (UC)</td>
<td>Descriptors (indicators of achievement of the result)</td>
<td>Educative forms and methods aimed at generation and development of the competence</td>
</tr>
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</tr>
<tr>
<td>The ability to propose a model to invent and test methods and tools of professional activity</td>
<td>SC-2</td>
<td>SC-M2</td>
<td>The student is able to improve and develop research methods of optimization, approximation and computational problem solving.</td>
<td>Classes, homeworks.</td>
</tr>
<tr>
<td>Capability of development of new research methods, change of scientific and industrial profile of self-activities</td>
<td>SC-3</td>
<td>SC-M3</td>
<td>The student obtain necessary knowledge in signals and systems theory and basics of estimation theory</td>
<td>Homeworks, paper reviews, additional topics</td>
</tr>
<tr>
<td>The ability to describe problems and situations of professional activity in terms of humanitarian, economic and social sciences to solve problems which occur across sciences, in allied professional fields.</td>
<td>PC-5</td>
<td>IC-M5.3_5. 4_5.6_2. 4.1</td>
<td>The student is able to describe psychological problems in terms of computational mathematics.</td>
<td>Lectures and tutorials, group discussions, presentations, paper reviews. Understanding of signals and systems fundamentals. Ability to formalize real-life problems. Understanding of the estimation theory fundamentals. Maximum likelihood vs Maximum <em>aposteriori</em> probability estimation. Understanding of the uses in real-life examples.</td>
</tr>
<tr>
<td>The ability to detect, transmit common goals in the professional and social activities</td>
<td>PC-8</td>
<td>SPC-M3</td>
<td>The student is able to identify systems theory aspects in psychological and neurobiological research tasks. Suggest a method to tackle the problem and rank several available techniques in the order of applicability in the current situation</td>
<td>Discussion of paper reviews; cross discipline lectures</td>
</tr>
</tbody>
</table>
6. Place of the discipline in the Master’s program structure

The “Digital signal processing” course is an attempt to give master students with non-mathematical background a unifying view on the system-oriented approach used in the modern science. The class is a minor-type course offered by the Data analysis and artificial intelligence department of HSE to the first year of the Master’s program «Cognitive sciences and technologies». It is recommended for students of the Master’s program who are planning to exploit quantitative methods in their research and want to build to an intuition to tackle the advanced topics in signals and data analysis.

Prerequisites

The course is based on the basic knowledge of mathematical analysis and linear algebra concepts obtained by the students during their first semester at the program. The course is oriented to students with humanitarian background and will maximize the use of plots and pictures rather than equations and formal proofs to introduce the main concepts.

The following knowledge and competence are needed to study the discipline:
- A good command of the English language.
- A basic knowledge in mathematics.
1. Mathematical analysis basics
2. Fundamentals of linear algebra

Main competences developed after completing the study this discipline can be used to learn the following disciplines:
- Advanced topics in estimation theory.
- Adaptive signal processing
- Advanced multivariate data analysis and statistical inference

Comparison with the other courses at HSE

This class is unique in both the scope and the level of delivery that assumes only very basic mathematical background. The main stress is on building the intuition and teaching students to formalize the real-life problems and tackle them from the systems theory point of view. The scope of the class is very broad and ranges from basic signal types to the advanced topics of estimation theory. The unifying idea is the notion of the system. Signals, their properties and how they change when go through the system. Is time domain analysis the most convenient way for making sense of what an LTI systems does with the signal? This question leads to the introduction of transforms and the links to the state-space representation and linear algebra basics are shown illustrating that a transform is a simple change of basis operation. Which transforms are helpful? What do we learn when viewing signals in the transformed domain? Can we use transforms to analyze not only signals but systems? Great, we know how an LTI system works but what if I want to find its parameters based on the observed input/output data. This leads to the estimation theory part of the class where we will consider the fundamentals of this beautiful theory and exercise its uses in everyday practice of a scientist from estimation of a sample mean to solving the inverse problems in neuroimaging or spectroscopy.

Importantly, the class reflects the author’s point of view on the described topics that was formed in the mind of the author of this class who first obtained an engineering degree and then had a chance to go over the extensive and quite a broad training in signal processing. The author is also a practicing signal processing engineer and a scientist whose research is focused on development of practical algorithms for analysis of multivariate data in the field of neuroimaging.
7. Schedule

One pair consists of 2 academic hours for lecture or 2 academic hours of practical session (seminar) Please, see the Course description section for assignment of lectures.

<table>
<thead>
<tr>
<th>№</th>
<th>Topic</th>
<th>Total hours</th>
<th>Contact hours</th>
<th>Self-study</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Signals</td>
<td>20</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Transforms and decompositions</td>
<td>26</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>3.</td>
<td>Statistical description of signals in the original and transformed spaces</td>
<td>26</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>4.</td>
<td>Signal models</td>
<td>20</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5.</td>
<td>Systems</td>
<td>22</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>6.</td>
<td>Estimation theory</td>
<td>22</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>7.</td>
<td>Practical examples and Summary</td>
<td>16</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Total:</td>
<td>152</td>
<td>28</td>
<td>28</td>
</tr>
</tbody>
</table>

8. Requirements and Grading

<table>
<thead>
<tr>
<th>Type of grading</th>
<th>Type of work</th>
<th>Characteristics</th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homework(HW)</td>
<td></td>
<td>Solving homework tasks and examples, weekly</td>
<td>20</td>
</tr>
<tr>
<td>Midterm exam(MTE)</td>
<td></td>
<td>Written exam, solving problems similar to those in the HWs</td>
<td>2</td>
</tr>
<tr>
<td>Final exam(FE)</td>
<td></td>
<td>Written exam, 120 min, solving problems similar to those in the HWs</td>
<td>1</td>
</tr>
<tr>
<td>Final Grade formula</td>
<td></td>
<td>0.5<em>HW+0.15</em>MTE1+0.15<em>MTE2 +0.2</em>FE</td>
<td></td>
</tr>
</tbody>
</table>

9. Assessment

The assessment consists of classwork and homework, assigned after each lecture. Students have to demonstrate their knowledge in each lecture topic concerning both theoretical facts, and practical tasks’ solving. All tasks are connected through the discipline and have increasing complexity.

Intermediate assessment is in the form of two midterm exams. Students have to demonstrate knowledge of theory facts, but the most of tasks would evaluate their ability to solve practical examples, present straight operation, and recognition skills to solve them.

Final assessment is the final exam. Students have to demonstrate knowledge of theory facts, but the most of tasks would evaluate their ability to solve practical examples, present straight operation, and recognition skills to solve them. While comprehensive will primarily focus on the second part of the class (Estimation theory)

The grade formula:

Each exam (2 midterms and the final) will consist of 5 problems, giving two marks each

Final course mark is obtained from the following formula:

\[ O_{final} = 0.5*O_{hw} + 0.15*O_{mte1} + 0.15*O_{mte2} + 0.2*O_{FE} \]
The grades are rounded in favour of examiner/lecturer with respect to regularity of class and home works. All grades, having a fractional part greater than 0.5, are rounded up.

Table of Grade Accordance

<table>
<thead>
<tr>
<th>Ten-point Grading Scale</th>
<th>Five-point Grading Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - very bad</td>
<td>Unsatisfactory - 2</td>
</tr>
<tr>
<td>2 – bad</td>
<td>FAIL</td>
</tr>
<tr>
<td>3 – no pass</td>
<td>Satisfactory – 3</td>
</tr>
<tr>
<td>4 – pass</td>
<td>Good – 4</td>
</tr>
<tr>
<td>5 – highly pass</td>
<td>Good – 4</td>
</tr>
<tr>
<td>6 – good</td>
<td>Satisfactory – 3</td>
</tr>
<tr>
<td>7 – very good</td>
<td>Good – 4</td>
</tr>
<tr>
<td>8 – almost excellent</td>
<td>Excellent – 5</td>
</tr>
<tr>
<td>9 – excellent</td>
<td></td>
</tr>
<tr>
<td>10 – perfect</td>
<td></td>
</tr>
</tbody>
</table>

10. Course Description

The following list describes main topics in the order as they will be presented. Lecture numbers are given in square brackets

1 Signals [1,2]
   1.1 Deterministic Signals (discrete, continuous)
   1.2 Random signals, random processes
      ➤ Mean, std, autocorrelation function, distribution function
      ➤ Properties of random signals (stationarity, ergodicity)
      ➤ Scalar and vector random processes
   1.3 Signals as vectors
   1.4 Hilbert space, infinite dim Euclidian space, notion of distance
   1.5 Demonstrated equivalence of discrete and continuous forms of treatment

2 Transforms and decompositions [3,4,5]
   2.1 General idea of a transform
   2.2 A transform as a change of basis
   2.3 Calculating transform coefficients or why do we need an orthogonal basis?
   2.4 Fourier transform
      ➤ Periodic signals
      ➤ Aperiodic signals
      ➤ Discrete Fourier transform
   2.5 State-space representation and discrete transforms as a matrix-vector multiplication
   2.6 Time-frequency representation of signals
   2.7 Short-time Fourier transform
   2.8 Wavelet transform

3 Statistical description of signals in the original and transformed spaces [6,7,8]
   3.1 Second order statistical description of random signals revisited
   3.2 Periodogram, Power spectral density(PSD)
   3.3 Wiener-Hinchin theorem
   3.4 Methods to compute PSD

4 Signal models [9]
4.1 Autoregressive model
4.2 Autoregressive moving average model
4.3 Parameter estimation: problem statement

5 Systems [10,11,12,13]
5.1 Input-output relation, a system as an operator
5.2 Calculation of an output of a system for an arbitrary input, pulse response
5.3 Linear time invariant (LTI) systems
5.4 Transient processes
5.5 Passing random process through an LTI system
5.6 Time domain
5.7 Frequency domain, Transfer function or why a linear system cannot “bring in new frequencies”
5.8 Z- transform as a method for analysis of discrete-time LTIs
5.9 Digital Temporal filters as most frequently used LTIs
5.10 FIR, IIR filters,
5.11 Stability, implementation details
5.12 Digital filters as shaping filters for ARMA processes
5.13 Synthesis of signals with desired second order statistical properties

6 Estimation theory [14-20]
6.1 Properties of an estimate, criteria overview
6.2 Notion of an optimal estimator
6.3 Cramer-Rao lower bound
6.4 The concept of a general minimum variance unbiased (MVU) estimator
6.5 Sufficient statistics, finding an MVU estimator
6.6 Best linear unbiased estimator (BLUE)
6.7 Maximum Likelihood (ML) estimation
6.8 Gaussian mixture model, Expectation-Maximization algorithm, clustering
6.9 The Bayesian philosophy
6.10 Maximum aposteriori probability (MAP) estimators
6.11 Linear vector observation model
6.12 Estimation of parameters in white Gaussian noise (WGN) case
6.13 Estimation of parameters in colored Gaussian noise (CGN) case
6.14 Wiener filter (time and frequency domain)
6.15 Process model
6.16 Kalman filter (smoother, predictor)
6.17 Wiener filter vs Kalman filter

7 Examples and Summary [21, 22]

11. Term Educational Technology
The following educational technologies are used in the study process:
- discussion and analysis of the results of the home task in the group;
- individual education methods, which depend on the progress of each student;
- analysis of skills to formulate common problem in terms of mathematics and solve it;

12. Recommendations for course lecturer
Course lecturer is advised to use interactive learning methods, which allow participation of the majority of students, such as slide presentations, combined with writing materials on board, and usage of interdisciplinary papers to present connections between mathematics and psychology.

The course is intended to be adaptive, but it is normal to differentiate tasks in a group if necessary, and direct fast learners to solve more complicated tasks.

13. **Recommendations for students**

The course is interactive. Lectures are combined with classes. Students are invited to ask questions and actively participate in group discussions. There will be special office hours for students, which would like to get more precise understanding of each topic. Teaching assistant will also help you. All tutors are ready to answer your questions online by official e-mails that you can find in the “contacts” section.

14. **Final exam questions**

1. Deterministic Signals (discrete, continuous)
2. Random signals, random processes
3. Mean, std, autocorrelation function, distribution function
4. Properties of random signals (stationarity, ergodicity)
5. Scalar and vector random processes
6. Signals as vectors, Hilbert space, infinite dim Euclidean space, notion of distance
7. Demonstrate equivalence of discrete and continuous forms of treatment
8. General idea of a transform
9. A transform as a change of basis
10. Calculating transform coefficients or why do we need an orthogonal basis?
11. Fourier transform, periodic signals
12. Fourier transform, Aperiodic signals
13. Discrete Fourier transform
14. State-space representation and discrete transforms as a matrix-vector multiplication
15. Time-frequency representation of signals, Short-time Fourier transform
16. Time-frequency representation of signals, Wavelet transform
17. Second order statistical description of random signals (Periodogram, Power spectral density (PSD))
18. Wiener-Hinchin theorem, methods to compute PSD
19. Autoregressive model
20. Autoregressive moving average model
22. Input-output relation, a system as an operator
23. Calculation of an output of a system for an arbitrary input, pulse response, Linear time invariant (LTI) systems
24. Transient processes
25. Passing random process through an LTI system (time domain analysis)
26. Passing random process through an LTI system (frequency domain, transfer function)
27. Z-transform as a method for analysis of discrete-time LTIs
28. Digital Temporal filters as most frequently used LTIs
29. FIR, IIR filters, stability, implementation details
30. Digital filters as shaping filters for ARMA processes
31. Synthesis of signals with desired second order statistical properties
32. Parameter estimation, problem statement, examples
33. Notion of an optimal estimator
34. Cramer-Rao lower bound
35. The concept of a general minimum variance unbiased (MVU) estimator
36. Sufficient statistics, finding an MVU estimator
37. Best linear unbiased estimator (BLUE), scalar case
38. Best linear unbiased estimator (BLUE), vector case
39. Maximum Likelihood (ML) estimation
40. Gaussian mixture model, Expectation-Maximization algorithm, clustering
41. Maximum a-posteriori probability (MAP) estimators
42. Linear vector observation model: estimation of parameters in white Gaussian noise (WGN) case
43. Linear vector observation model: estimation of parameters in colored Gaussian noise (CGN) case
44. Wiener filter (time and frequency domain)
45. Kalman filter (smoother, predictor)
46. Wiener filter vs. Kalman filter

15. Reading and Materials

This course is intended to provide students with the unfying view on system's theory and help them to develop the corresponding intuition. Unfortunately, there is no single book and we will draw from a range of books listed below.

16. Required Reading

Selected chapters from

17. Recommended Reading


18. List of papers for review
Will be provided as the class progresses
19. Course telemaintenance

All materials of the discipline are posted in informational educational site at NRU HSE portal www.hse.ru. Students are provided with links on relevant papers, tests, electronic books, articles, etc.

15. Equipment

The course requires a laptop, projector, and acoustic systems.

Lecture materials, course structure and the syllabus are prepared by Alexei Ossadtchi, Ph.D.