Abstract from the course program "Mathematics for Economists" by M. Levin, K. Bukin, B. Demeshev, G. Kantorovich

1. Multidimensional calculus, basics of optimization

1. Euclidean spaces: basic notions and definitions
    - vector
    - distance
    - open and closed sets
    - neighborhood of a point, limiting points, boundary points
    - bounded sets, compact sets
2. Functions and their generalizations
    - vector-functions
    - limit of a function
    - continuity
3. Multidimensional calculus and some ideas drawn from the linear algebra
    - Linear structure
        - linear space
        - linear dependence/independence
        - basis, dimension
        - linear mapping
    - Norm. Scalar (dot) product
        - norm
        - scalar product
        - orthogonality
        - angle between vectors
    - Total differential
        - partial derivative
        - relation between partial and total derivatives
        - implicit function theorem
4. Optimization in many variables. Unconstrained optimization at first
    - concept of extrema
    - conditions of extrema

2. Linear Algebra

1. Basic notions, definitions and propositions

- operations on matrices
- matrix multiplication
- inverse matrix, its properties
- rank of a matrix
- linear spaces and subspaces, their properties
- Gauss method of solving linear systems
- systems of linear equations
- eigenvalues and eigenvectors (definition, relation to the matrix rank, case of a symmetric matrix)
- quadratic forms: sign-definiteness of for
- kernel and image of a linear operator
- Eucleadean spaces
- orthogonalization by Gramm-Schmidt's method
- quadratic form sign-definiteness criterion (by eigenvalues)
- Sylvester's criterion
- reduction of a matrix to a diagonal form

3. Convex analysis and Kuhn-Tucker theorem

1. Outset of a non-linear programming problem

- Convexity
  - convexity of a set
  - convex and concave functions, their properties
- separability theorem, separating hyperplane
- saddle point
- necessary and sufficient conditions of a quadratic form sign-definiteness (n=2)
- strict convexity of a function

2. Unconstrained optimization in many variables

- Taylor's expansion in a single variable case

- Jacobi's matrix
- Jacobi's matrix for a composite function
- theorem of existence of inverse operator
- sufficient conditions for extrema

3. Constrained optimization
    - Lagrange's classic problem
    - optimization of a quadratic form on a unit sphere
    - directional derivative
4. Constrained optimization with inequality constraints
    - problem setting, function requirements
    - necessary and sufficient conditions for extrema and corollaries from that theorem
    - problem modification for the nonnegative variables
    - differential characteristics of Kuhn-Tucker conditions
    - the meaning of Lagrange multiplier

4. Theory of probability and statistics

1. random variable, sample space
2. cumulative distribution function and its density
3. uniform distribution
4. normal distribution, reduction of the Gaussian variable to variable
5. standard expectation $E[X]$, $E[f(X)]$
6. initial and central moments
7. joint distributions of the random variables
8. conditional distributions
9. iterated expectations formula
10. limiting densities
11. covariation and correlation
12. standard normal vector and its properties
13. marginal and conditional normal distributions
14. quadratic forms in a standard normal vector
15. X^2 distribution and its properties
16. Student's distribution and its properties

17. Fisher’s distribution and its properties
18. point estimation of parameters
19. unbiasness and efficiency of estimators
20. elements of large-sample distribution theory
21. convergence in probability and convergence in distribution
22. asymptotic distribution
23. interval estimation
24. hypothesis testing
25. errors of the first and second type
26. critical region of the test, decision rule