

# SPECIAL FEATURES OF ACOUSTIC-GRAVITY WAVES PROPAGATION IN THE SOLAR CHROMOSPHERE



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УНИВЕРСИТЕТ

## OUTLINE

**1. Introduction**

**2. Basic equations for acoustic-gravity waves in the nonisothermal atmosphere**

**3. Wave perturbations near the resonance level**

**4. Calculations results for resonance perturbation in the Solar chromosphere**

**5. Conclusion**

**XIVth Hvar Astrophysical Colloquium**

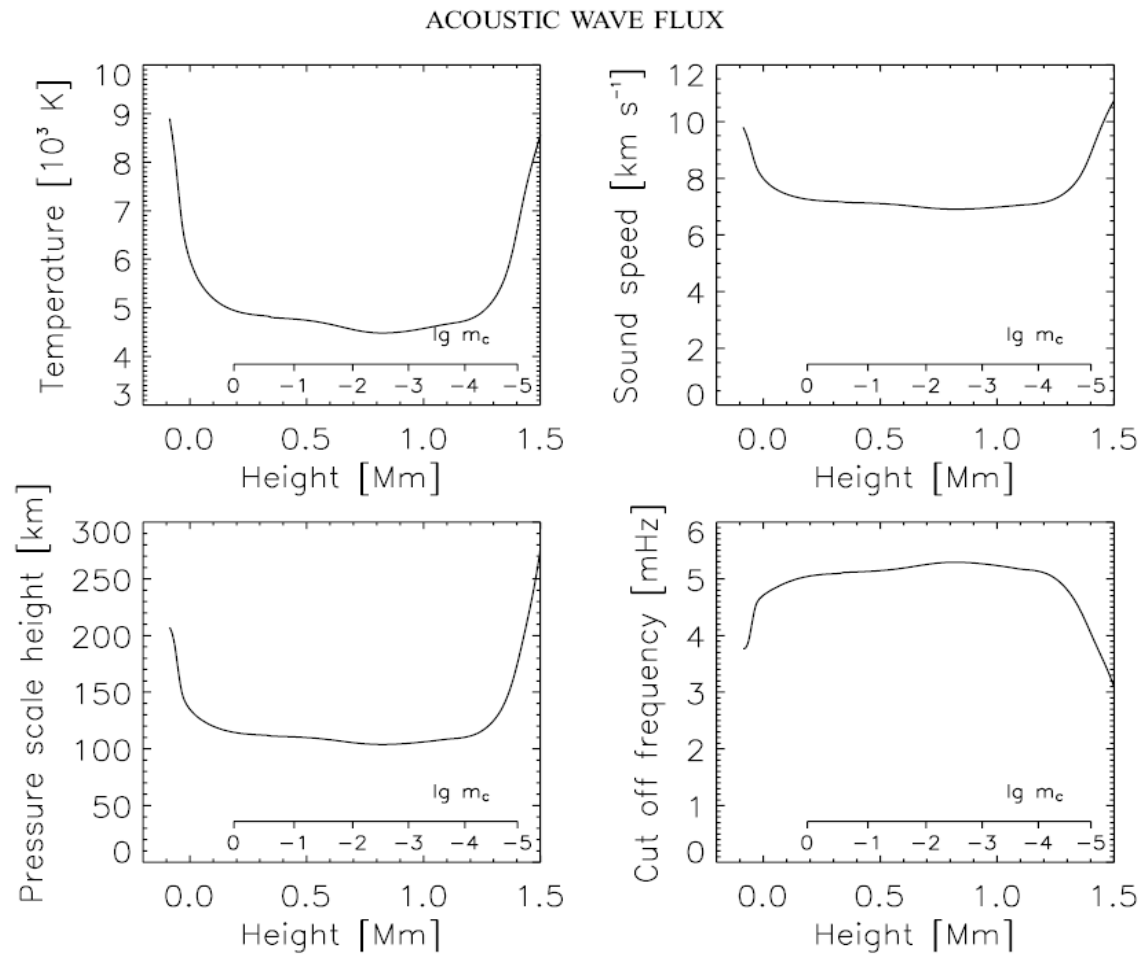
**SOLAR AND SOLAR-TERRESTRIAL PHYSICS:  
NOW AND IN THE FUTURE**

**26 - 30 September 2016, Hvar, Croatia**

## 1. Introduction

In this report some new features of acoustic-gravity wave propagation from the sources at the photospheric heights through a nonisothermic Solar chromosphere are examined. For simplicity we produce calculation properties of wave perturbation within the framework of the plane-layered model of the atmospheric.

Here you can see how the temperature, pressure and sound speed depend from height over the cromosphere bottom (A. Fossum and M. Carlsson, The Astrophysical Journal, 646:579–592, 2006)



One of the main unanswered questions in solar physics is why the Solar corona is hotter than the stellar surface. Eighty years ago Schwarzschild and Biermann independently proposed that comparatively high frequency acoustic waves play an important role in the heating of the solar chromosphere and corona (Biermann, L., Z. Astrophys., 25, 161, 1948). Recent studies by Fossum and Carlsson (2005, 2006) and Carlsson et al. (2007) conclude that high frequency waves are not sufficient to heat the solar chromosphere. Others reseachers (e.g. Cuntz et al. 2007; Wedemeyer-Böhmer et al. 2007; Kalkofen 2007) put in question these results and argue for high frequency waves to play an important role. There are assertions, that stably stratified atmospheres can support and propagate not only acoustic waves but also internal gravity waves. The significance of acoustic-gravity waves for the energy balance of the solar chromosphere reconsiders until now.

As a result, the propagation of acoustic-gravity disturbances to the upper chromosphere, where these disturbances can be transformed into the plasma disturbances and take part in heating of the Solar corona, is very important.

## 2. Basic equations for acoustic-gravity waves in the nonisothermal atmosphere

The linearized system of equations of gas dynamics for the pressure disturbances  $p$ , the horizontal velocity  $u$  and the vertical velocity  $w$  is well known. Let  $z$  be the vertical coordinate,  $x$  the horizontal coordinate.

A set of linearized gas dynamics equations

$$\rho_0 \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x},$$

$$\rho_0 \frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} - \rho g,$$

$$\frac{\partial \rho}{\partial t} + w \frac{d\rho_0}{dz} + \rho_0 \left( \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) = 0,$$

$$\frac{\partial p}{\partial t} + w \frac{dp_0}{dz} = c^2 \left( \frac{\partial \rho}{\partial t} + w \frac{d\rho_0}{dz} \right)$$

Where  $\rho_0$  is the density of the equilibrium atmosphere,  $c$  is the adiabatic sound speed.

In this equations the regular density  $\rho_0(z) = \frac{p_0(z)}{gH(z)}$

The regular pressure

$$p_0(z) = p_{00} \exp \left[ -\int_0^z \frac{dz'}{H(z')} \right]$$

The regular density described by a relation

$$\rho_0(z) = \frac{p_0(z)}{gH(z)},$$

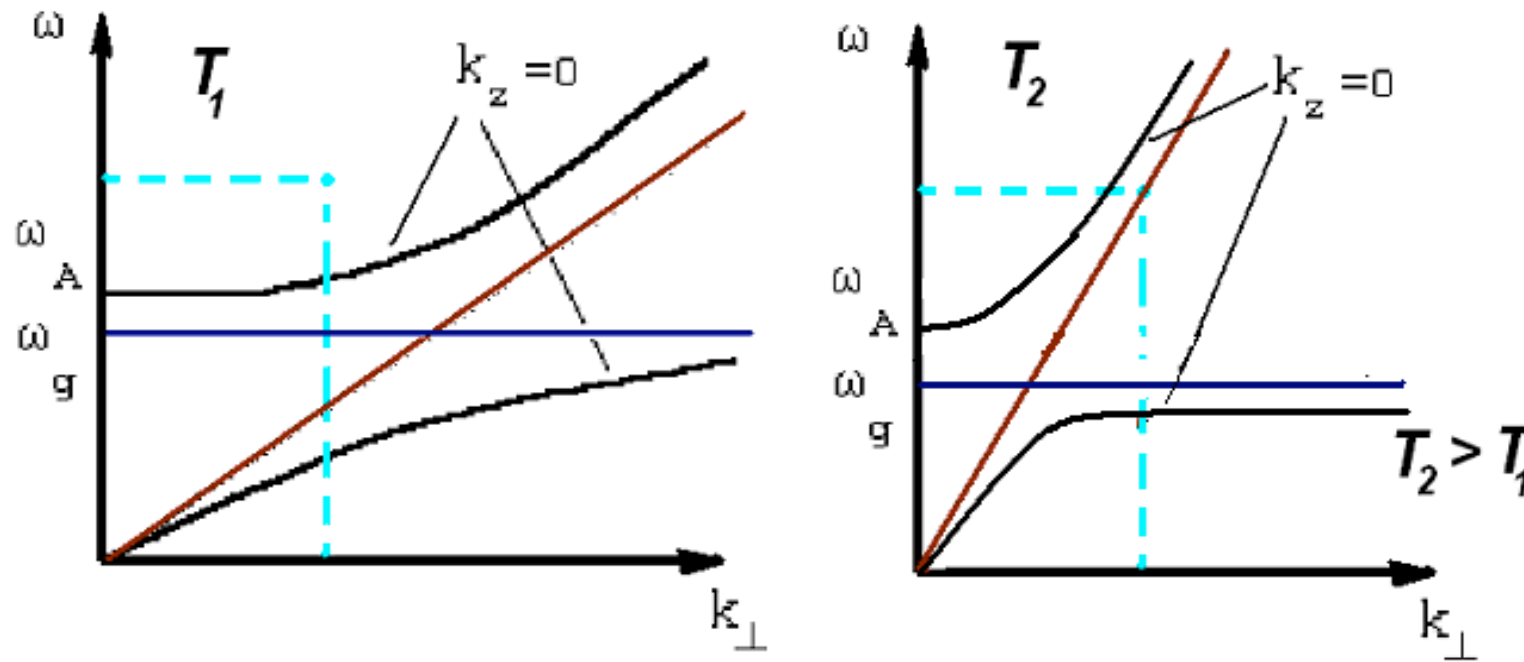
the sound speed  $c$  depends on heights in the nonisothermal atmosphere.

In the case of an arbitrary height dependence of the temperature, the following equation for the vertical velocity component can be obtained from this system:

$$\begin{aligned}
 & c^2 \left( \frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x^2} - c^2 \frac{\partial^2}{\partial y^2} \right) \frac{\partial^4 w}{\partial t^2 \partial z^2} + \gamma g \left[ c^2 \left( \frac{dH}{dz} + 1 \right) \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) - \frac{\partial^2}{\partial t^2} \right] \frac{\partial^3 w}{\partial t^2 \partial z} \\
 & - \left[ \frac{\partial^2}{\partial t^2} \left( \frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x^2} - c^2 \frac{\partial^2}{\partial y^2} \right)^2 + c^2 \gamma g^2 \frac{dH}{dz} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^2 + \right. \\
 & \left. + \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) g^2 (1 - \gamma) \left( \frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x^2} - c^2 \frac{\partial^2}{\partial y^2} \right) \right] w = 0
 \end{aligned}$$

The exact solution of this equation is possible only in some special cases.

The preliminary study of this problem can be produced in the framework of geometrical-optics approximation. In next figures we can see transformation of the dispersion dependence between a wave frequency and a horizontal wave number when the atmospheric temperature increases with altitude.



The up propagating infrasonic wave reflects from higher-temperature domain, but some energy leaks upper then resonance level  $z = z_*$ , where  $\omega - c(z_*)k_{\perp} = 0$ . Actually, the geometrical-optics approximation is not correct near the resonance level. Then we need to examine a wave propagation problem as a separated task.



### 3. Wave perturbations near the resonance level

We analyze a more complicated model of acoustic-gravity waves in the cromosphere with a realistic altitude temperature profile. The linearized system of equations for the wave perturbation can be reduced to the following form:

$$\left[ -\omega^2 + \omega_g^2(z) \right] W - i \frac{\omega}{\rho_E} \left[ \frac{\partial}{\partial z} + \Gamma(z) \right] P = 0,$$

$$V = (k_{\perp} / \omega) P$$

$$\left[ c_s^2(z) k_{\perp}^2 - \omega^2 \right] P - i \omega \rho_E c_s^2(z) \left[ \frac{\partial}{\partial z} - \Gamma(z) \right] W = 0.$$

Here,  $V = (\rho / \rho_{00})^{1/2} v$ ,  $W = (\rho / \rho_{00})^{1/2} w$ , and  $P = (\rho_{00} / \rho_0)^{1/2} p$  are new variables, where  $\rho_0$  and  $\rho_{00}$  are the basic state densities in the current layer and at the bottom, respectively,  $\omega_g$  is the Brunt Vaisala frequency,  $\Gamma(z)$  is the Ekkard parameter. Field variables are proportional to  $\exp(-i\omega t + ik_{\perp} x)$  for a monochromatic signal with frequency  $\omega$  in plane atmospheric layers. In this model, the horizontal wave number  $k_{\perp}$  is altitude independent, and its main value is defined by the scale of the sources.

Let us consider in more detail the processes near the resonance level  $z = z_*$ , which are described by the system of equations

$$[-\omega^2 + \omega_g^2(z_*)]W - i \frac{\omega}{\rho_{00}} \left[ \frac{\partial}{\partial z} + \Gamma(z_*) \right] P = 0$$

$$[-\omega^2 + c^2(z)k_{\perp}^2]P - i\omega\rho_{00}c^2(z_*) \left[ \frac{\partial}{\partial z} - \Gamma(z_*) \right] W = 0$$

Analytical study of this equations show that for upward propagation at the level  $z = z_*$  the conditions  $W = 0$  and  $\frac{dW}{dz} = 0$  are fulfilled. The absence of disturbances of both the vertical velocity and its derivative leads to the conclusion that above the level  $z = z_*$  the solutions both for  $W$  and for  $P$  are identically equal to zero, as well as the averaged vertical energy flux.

In order to counter balance the pressure jump at the level in question, the finite mass should be concentrated at the level  $z = z_*$ , which is taken into account in the general solution by means of a delta function.

Hence, if for the wave perturbation in the nonisothermal atmosphere at some level  $z = z_*$  a condition  $\omega = c(z_*)k_{\perp}$  is satisfied, then the averaged vertical energy flux is equal to zero, and **above the first of such levels, wave perturbation are absent along the vertical propagation path.**

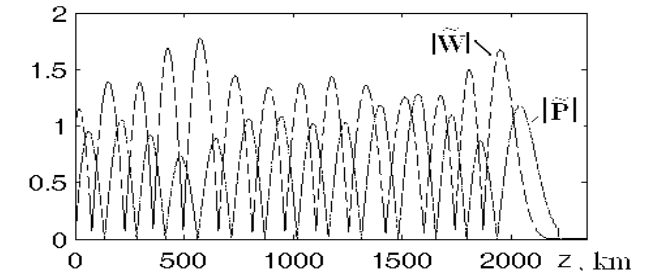
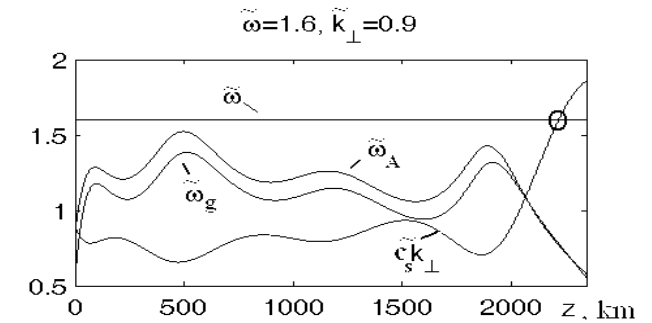
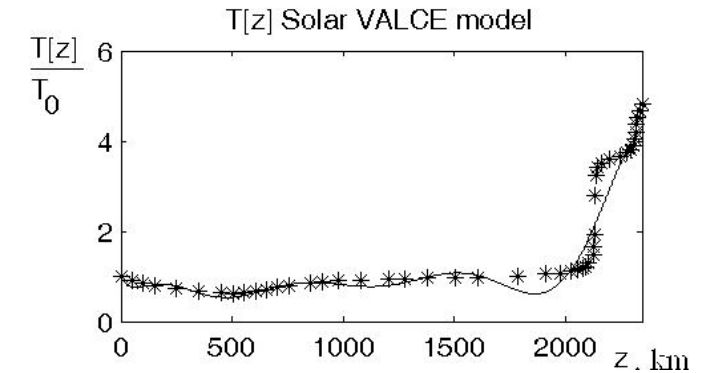
Under the real conditions, the resonance in the form of a delta function in the pressure disturbance, and in the horizontal velocity ( $V = P/c(z = z_*)\rho_0$ ), is limited by a dissipation and nonlinearity.

## 4. Calculations results for resonance perturbation in the Solar chromosphere

We determine perturbations of the pressure and vertical velocity by means of full-wave numerical calculations. We assume that on the photosphere level there is a monochromatic source of vertical velocity and that at the altitudes higher than transition region the atmosphere is isothermal.

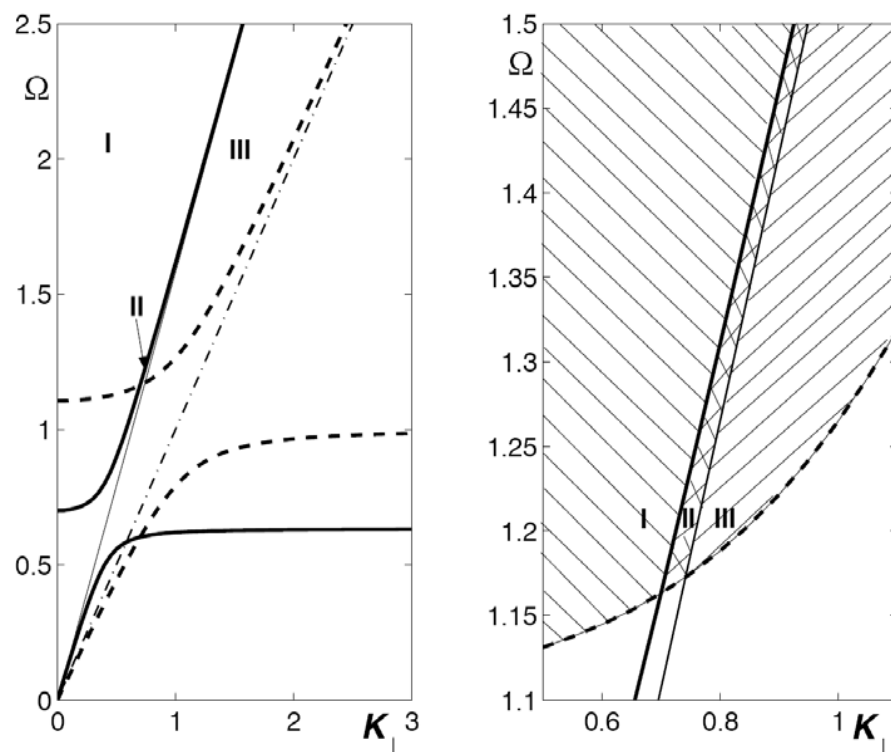
The wave fields are conveniently calculated in dimensionless variables, as which we selected  $\tilde{\omega} = \omega / \omega_{g0}$ ,  $\tilde{k}_{\perp} = k_{\perp} c_{s0} / \omega_{g0}$ ,  $\tilde{c}_s = c_s(z) / c_{s0}$ ,  $\tilde{W} = W / W_0$ ,  $\tilde{P} = P / \rho_{00} c_{s0}^2$ . Here, subscript "0" indicates the value of the variable on the bottom chromosphere surface. We selected the altitude temperature profile following to the experimental models of the Solar chromosphere (Vernazza, J. E., Avrett, E. H., & Loeser, R. Ap.J.S, 45, 635, 1981). We determine the spline temperature profile using the model values marked by stars in this figure. The temperature spline profiles was approximated by a polynomial of the tenth order (a solid curve).

The results of numerical calculation are shown here for  $\tilde{\omega} = 1.6$  and  $\tilde{k}_{\perp} = 0.9$ . On the upper panels there are the height dependence  $\tilde{c}(z)\tilde{k}_{\perp}$ , the straight line is the frequency  $\tilde{\omega}$  and eigen frequencies of the medium  $\omega_g(z)$ ,  $\omega_A(z)$ . On the bottom panels, the solid curves are the altitude dependence of pressure amplitude  $|\tilde{P}(z)|$  and the dashed curves are the altitude dependence of the vertical velocity amplitude  $|\tilde{W}(z)|$ .





## *Three regimes of acoustic-gravity waves propagation in the atmosphere with a temperature jump*



Dispersion curves with  $k_z = 0$  ( $T_2 / T_1 = 2.5$ ,  $K_{\perp} = k_{\perp} c_1 / \omega_{g1}$ ,  
 $\Omega = \omega / \omega_{\sigma 1}$ )

### Regime (I):

Waves partially reflect from the jump and partially they pass into the upper medium

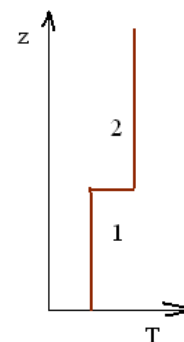
### Regime (II):

Complete internally reflection with exponential damping in the upper medium

### Regime (III):

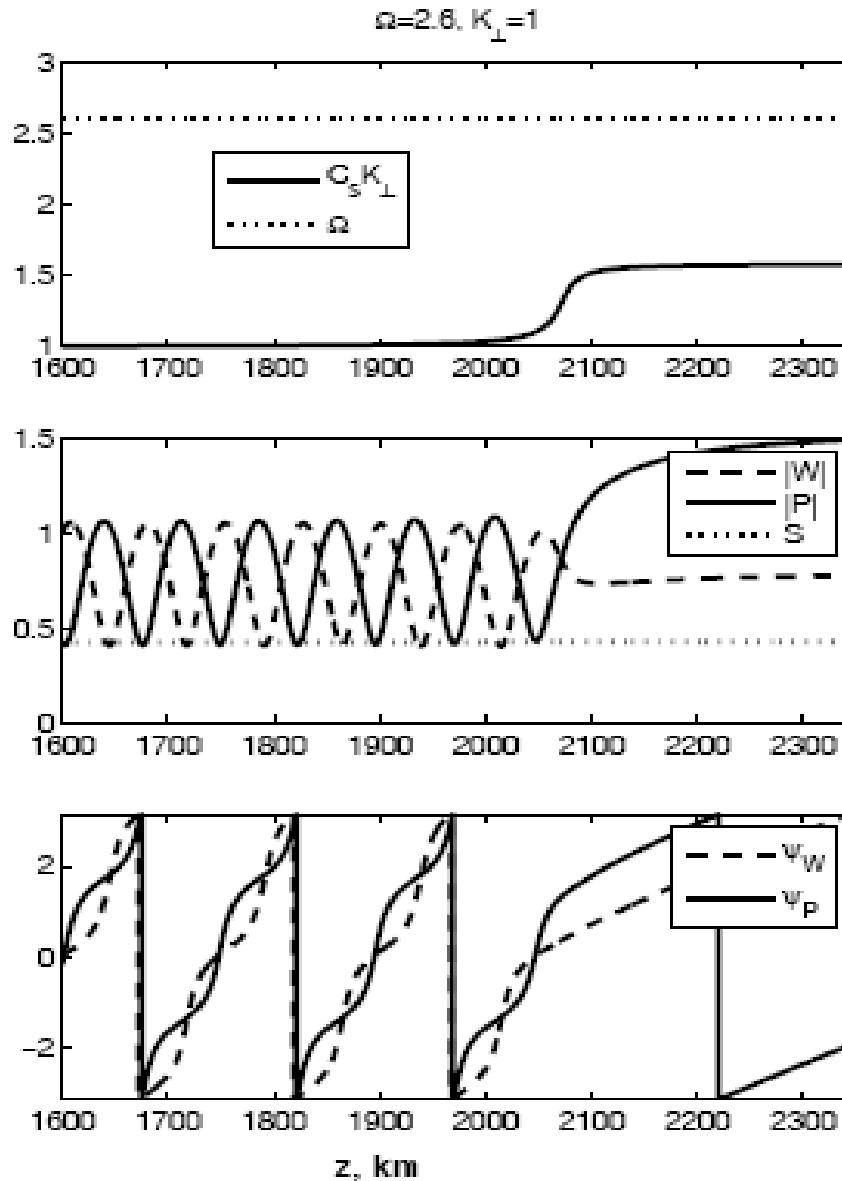
In the presence of the resonance equality the temperature jump performs the solid wall role. Higher than with there is no wave field.

The regime is analytical determined in the case of the smooth altitude temperature and it is confirmed by the numerical calculation



## Numerical analysis, using of wave equations in Riccati form

### Regime (I):

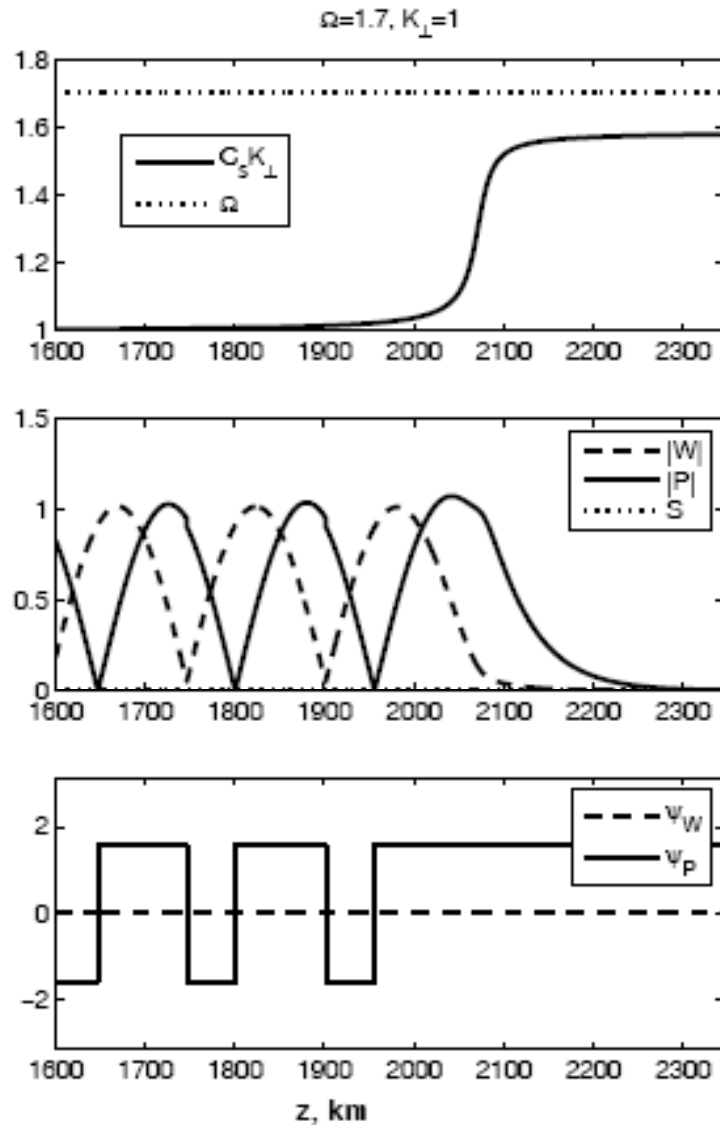


Waves partially reflect from the jump and partially they pass into the upper medium

Conditions of realization and dependence of amplitudes  $|W|$ , and the phases  $\Psi_w, \Psi_p$  of vertical velocity and pressure from the coordinate  $z$ . Average energy flux  $S$  is independent from the coordinate  $z$ .

## *Numerical analysis, using of wave equations in Riccati form*

### Regime (II):

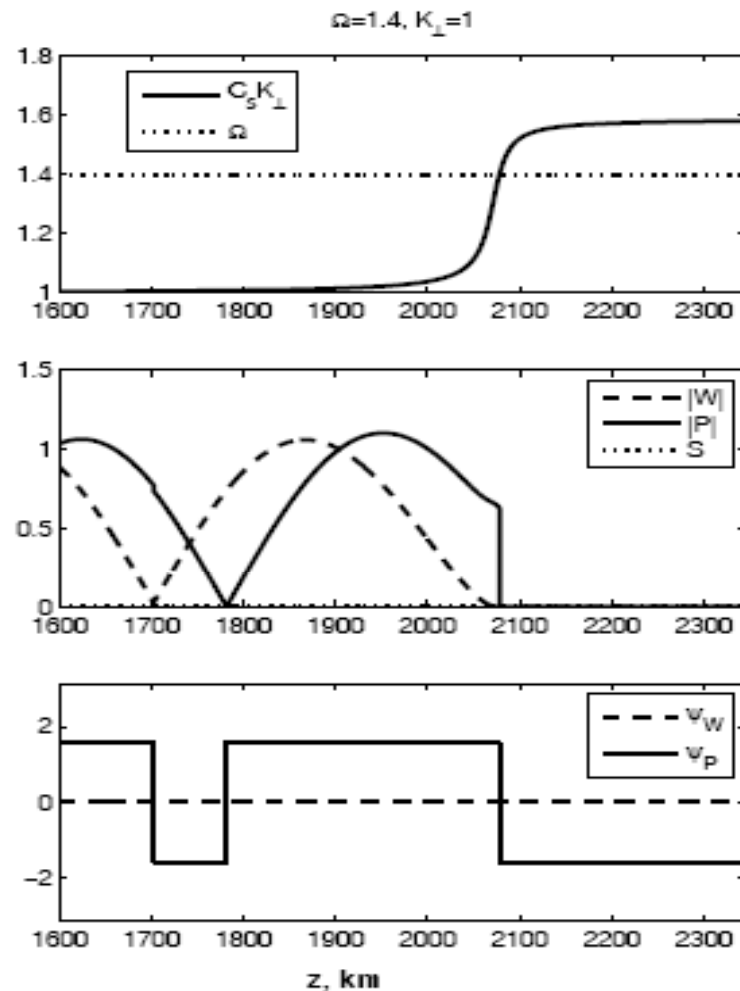


Complete internally reflection with exponential damping wave amplitude in the upper medium

Conditions of realization and dependence of amplitudes  $|W|$ , and the phases  $\Psi_w, \Psi_p$  of vertical velocity and pressure from the coordinate  $z$ . , Energy flux  $S$  is equal to zero.

## *Numerical analysis, using of wave equations in Riccaty form*

### Regime (III):



In the presence of the resonance equality exist effect of solid wall. Higher than with there are no wave field.

Conditions of realization and dependence of amplitudes  $|W|$ , and the phases  $\Psi_w, \Psi_p$  of vertical velocity and pressure from the coordinate  $z$ . Energy flux  $S$  is equal to zero.

## ***Conclusion***

So, the wave component of pressure has singularities near the altitude  $z_*$ , where the horizontal phase velocity is equal to the local sound speed. The vertical velocity and its derivative in perturbation turns to zero at this altitude. The wave perturbations are absent above this altitude. propagate. The dissipation and nonlinearity limit the pressure wave singularities.

Three regimes of acoustic-gravity waves propagation in the Solar atmosphere are possible.

**Partial passage through the temperature profile-** is realized with the fixed value of horizontal wave number for the sufficiently high frequencies (for horizontal scales about 1000 km, the wave periods can be smaller than 50 sec).

**Internal reflectance** is realized at the lower frequencies. In this case the amplitude of wave field falls exponentially, but it is comparatively slow because of the high value of  $H$ . Then it is possible the transformation of the wave perturbations into the plasma perturbation, which can heating the corona.

**The third new regime** is realized, if inside the chromosphere resonance condition is satisfied. Then temperature discontinuity plays the role of solid wall and higher than there are no wave fields.