

Some general results on preferential attachment and clustering coefficient

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Abstract

Many social, biological, and information systems can be represented by networks, whose vertices are items and links are relations between these items. That is why the evolution of complex networks attracted a lot of attention in recent years. In particular, numerous random graph models have been proposed to reflect and predict important quantitative and topological aspects of growing real-world networks. Turns out that many real-world networks of diverse nature have some typical properties: small diameter, power-law degree distribution, high clustering, and others. In this talk, I will focus on general approaches to the analysis of such properties and I will review several recent works in this field. The most well-known approach to the modeling of complex networks is preferential attachment. The main idea of this approach is that at each time step a new vertex is added to a graph and is joined to some vertices already existing in the graph chosen with probabilities proportional to their degrees. Preferential attachment allows to obtain a graph with a power-law degree distribution, and many different models are based on this idea: LCD, Buckley-Osthus, Holme-Kim, RAN, and others. I will present a general framework for analyzing preferential attachment models. In particular, I will discuss the PA-class of models which is defined in terms of constraints that are sufficient for the study of the degree distribution. Also, an additional constraint allows to analyze the behavior of the local and global clustering coefficients and the assortativity property in this class. Finally, I will present a general result on the behavior of the global clustering coefficient in scale-free graphs. Namely, I will show that for any sequence of graphs with a power-law degree distributions with a parameter $\gamma \in (2, 3)$ the global clustering coefficient tends to zero. This result is quite surprising, since there is a common belief that for many real-world networks both the average local and the global clustering coefficients tend to a non-zero limit as the networks become large.