New type of semiclassical asymptotics eigenstates near the boundaries of spectral clusters for Schrödinger-type operators

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We consider the eigenvalue problem for a two-dimensional perturbed oscillator in $L^2(\mathbb{R}^2)$

$$\left(-\frac{\hbar^2}{2}\left(\frac{\partial^2}{\partial q_1^2}+\frac{\partial^2}{\partial q_2^2}\right)+\frac{q_1^2+q_2^2}{2}+\varepsilon V\right)\psi=\lambda\psi, \qquad \|\psi\|_{L^2(\mathbb{R}^2)}=1, \tag{1}$$

where $V(q_1, q_2)$ is an arbitrary polynomial of degree 4 and $\hbar > 0$, $\varepsilon > 0$ are small parameters with $\varepsilon \ll \hbar$. In [1], an example of problem (1) was used to propose a method for finding a series of asymptotic eigenvalues near the boundaries of spectral clusters which are formed around the eigenvalues of the unperturbed equation (at $\varepsilon = 0$). This method is based on a new integral representation.

Applying the operator averaging and coherent transformation to problem (1) on the lth irreducible representation of the algebra of symmetries of the unperturbed operator, we obtain the eigenvalue problem in the space $\mathcal{P}_\ell$ of polynomials of degree less than or equal to ℓ. Here the number ℓ is of the order of $\hbar^{-1}$. The desired polynomial satisfies a second-order differential equation. We first study the multiple-point spectral problem in the class of antiholomorphic functions with zero characteristic exponents at finite singular points. Further, we obtain the asymptotics of the desired polynomial by using the operation of projection on the space $\mathcal{P}_\ell$.

In the subsequent papers [2, 3], this method was used to obtain asymptotics of a series of eigenvalues of the hydrogen atom in a magnetic field near the lower boundaries of spectral clusters and series of asymptotic eigenvalues of the Hartree-type operator near the boundaries of spectral clusters.

References

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[3] A.V. Pereskokov, *Teoret. Mat. Fiz.*, **178**:1, 88–106 (2014)