Spherical varieties

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Abstract

The aim of this course is to give introduction to the spherical varieties i.e. such compactifications of homogeneous spaces of reductive group for which a Borel subgroup B acts with a dense open orbit. The particular case of these varieties are Grassmanians, flag varieties, symmetric spaces and toric varieties. The remarkable property of spherical homogeneous spaces is that their compactifications can be described in combinatorial way (Luna-Vust theory of scherical embedding). In our course we shall study the Luna-Vust theory, Local structure of spherical varieties, G-equivariant invariants of homogeneous spaces, such as rank lattice, little Weyl group and spherical roots. We also shall discuss the equivariant geometry of cotangent bundle and its moment map and give the relations with the little Weyl group.

Program

- Scherical varieties. General properties and equivalent definitions.
- Local structure of spherical varieties.
- Akhiezer theorem on modality and complexity. Vinberg theorem on complexity. Horospherical contraction.
- Finiteness of number of B-orbits in spherical variety. Springer-Richardson monoid. Action of the Weyl group on the set of B-orbits. Bott–Samelson resolutions of Schubert varieties.
- Luna-Vust theory of spherical embeddings. Invariant valuations, colours.
- Demazure construction of wonderful compactifications. Bialynicki-Birula cell decomposition.
- Equivariant geometry of cotangent bundle. Structure of the moment map. Variety of generic horosheres and the little Weyl group.
- Moment polytope and Brion's polytope.

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