

Actuality and Necessity in Anselm's Argument: a Two-dimensional Approach

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Motivation

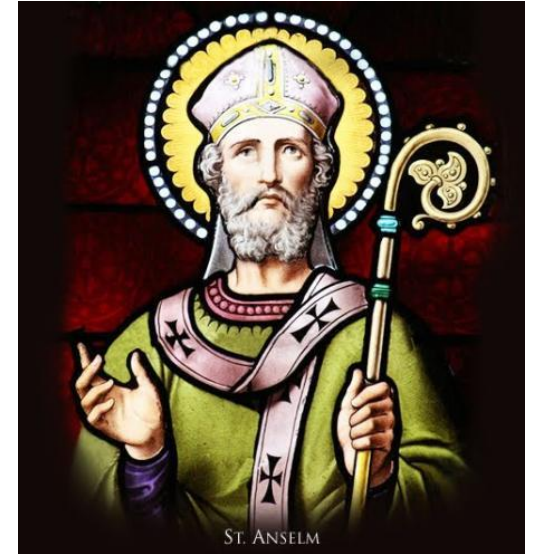
- The ontological argument (OA) has been reintroduced into the area of analytical metaphysics in 20th century due to the development of modal logic and possible worlds semantics
- However, there are few attempts to approach this argument with two-dimensional possible worlds framework

Plan

- Anselm's argument and "locutio mentis"
- MOA and Lewis' criticism
- Intensional challenge to modal realism
- Two-dimensional approach
- S5AF
- 2D-MOA

Argumentum Anselmi

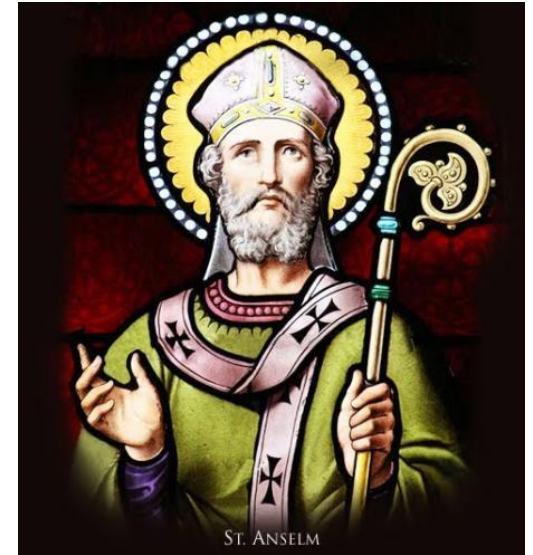
- Anselm starts with distinguishing two modes of existence:
 - in mind (*in intellectu*)
 - in reality (*in re*)
- Assuming that the latter is “greater” than the former, Anselm consequently points out that the specific object “*id quo maius cogitari nequit*” (Q) must have existence not only of the first, but also of the second kind



The core idea of Anselm's argument

- Anselm's main idea was not to deduce the "existence" from the "concept"
- He rather aimed to **show a way from one mode of existence to another**
- God's "existence in intellect" doesn't presuppose that we have any concept of God (whether adequate or not)
- "Therefore, O Lord, **not only art Thou that than which no greater can be conceived, but Thou art something greater than can be conceived**"

[Proslogion XV]



«locutio mentis»

- In the act of his “inner speech”, the ontological arguer doesn’t form any concept but directly construes an **intentional object**
- His aim is to “rediscover” that very object through some a priori analysis – this time as a **real** one
- How is it possible to fix the reference of a term without forming any concept?

Performative inference?

- Elena Dragalina-Chernaya [Dragalina-Chernaya 2011] provides an interpretation of the “deductions of existence” in possible worlds semantics for the illocutionary logic, i.e. logic of the performance of speech acts
- Given this semantic framework she interprets Anselm’s argument as a “performative inference”
- These inferences preserve certainty (i.e. speaker’s confidence of the assertion) but not the truth
- Is it enough?

Modal ontological argument (MOA)

- [Hartshorne 1961, Malcolm 1960, Plantinga 1974]
- The “referential” vocabulary is not enough to deal with the OA and is to be replaced with a “modal” vocabulary
- The *esse in intellectu* / *esse in re* opposition is very close to the opposition of *pure possibility* (existence in some possible world) and its *actualization* (existence in the actual world)
- In short, the argument is as follows: *it is possible that God exists; God is not a contingent being, i.e., either it is necessary that God exists, or it is necessary that he does not; hence, it is necessary that God exists*

Lewis' criticism of MOA

Premise 1. Whatever exists in the understanding can be conceived to exist in reality.

Premise 2. Whatever exists in the understanding would be greater if it existed in reality than if it did not.

Premise 3. Something exists in the understanding, than which nothing greater can be conceived.

Conclusion. Something exists in reality, than which nothing greater can be conceived.

1. $\forall x(Ux \supset \exists w(Ww \& xEw))$ 2. $\forall x \forall w \forall v (Ux \& Ww \& Wv \& xEw \& \sim xEv. \supset xwGxv)$

3A. $\exists x (Ux \& \sim \exists w \exists y (Ww \& ywGx@))$

3B. $\exists x \exists v (Ux \& Wv \& \sim \exists w \exists y (Ww \& ywGxv))$

3C. $\exists x (Ux \& \sim \exists v \exists w \exists y (Wv \& Ww \& ywGxv))$

3D. $\exists x (Ux \& \sim \exists w \exists y (Ww \& ywGxw))$

4. $W@$

C. $\exists x (xE @ \& \sim \exists w \exists y (Ww \& ywGx@))$

Lewis' criticism of MOA

7. VALIDITY OF THE ARGUMENT

We now have four precise, nonmodal translations of our original argument, one for each alternative translation of Premise 3. It is a routine matter to determine, by ordinary nonmodal logic, which are valid and which are not. It turns out that the arguments from 3A and 3C.

$$\frac{1, 2, 3A, 4}{\therefore C}$$

$$\frac{1, 2, 3C, 4}{\therefore C}$$

are valid, whereas the arguments from 3B and 3D

$$\frac{1, 2, 3B, 4}{\therefore C}$$

$$\frac{1, 2, 3D, 4}{\therefore C}$$

Lewis' criticism of MOA

- Lewis's analysis exposed the fact that Plantinga and others were committed to a kind of “**metaphysical egocentrism**”: they presume that the actual world is the only “real” while all others are “merely possible”
- Lewis's own approach, widely known as **modal realism**, considers the concept of “**actual**” as an **indexical**: “actual” refers at any world w to the world w
- “The world an ontological arguer calls actual is special only in that the ontological arguer resides there – and it is no great distinction for a world to harbor an ontological arguer”

[Lewis 1970, p. 184]

Challenge for modal realism

- Taking seriously the indexical account of “actuality” we are in a position to break the usually accepted connection between modality and intensionality
- This generates an “intensional challenge” for modal realism:
 - In so far as “actuality” is up to the speaker and his speech acts, **doesn't this make intensions relative to the speaker's location within that world?**
 - If it does, shouldn't we adopt a more flexible interpretation of intensions?

Two-dimensional approach



- 2D framework can be used to isolate an aspect of meaning that meets our philosophical intuitions about the epistemic necessity and to show its priority to the metaphysical one
- In this framework, the truth values of statements, relative to a given possible world, are determined in two different ways: they depend both on what the facts are and on what the sentences mean

1-intensions and 2-intensions

- *Primary intension* is a function $f: W_A \rightarrow E$ from actual worlds to extensions;
- *Secondary intension* is a function $f: W_C \rightarrow E$ from counterfactual worlds to extensions;
- *Two-dimensional intension* is a function $f: W_A \rightarrow (W_C \rightarrow E)$ that portrays the connection between the two previously mentioned intensions

Modal rationalism

- (MR) Every **epistemically possible** scenario – a complete description of what the world might be like together with the speaker's location within that world – describes a genuine **metaphysically possible (centered) world**
- **Centered world** (according to Lewis): $\langle w, a, t \rangle$
- From a semantic point of view 1-intensions are based on our ability to consider any arbitrary world both as a counterfactual and as the actual (centered)

Actuality operator

- From a syntactic point of view, diagonalization can be expressed with the operator “A” meaning “it is actually the case that”
- Let us consider models $\langle W, @, V \rangle$, in which W is nonempty set of possible worlds, $@ \in W$ is the actual world, and V assigns subsets of W to propositional variables
- Truth conditions for “ \Box ” and “A” are:
- $M \models_w \Box \varphi$ iff for all $u \in W$, $M \models_u \varphi$
- $M \models_w A\varphi$ iff $M \models_{@} \varphi$

S5AF

- The simplest logic with the actuality operator can be obtained from S5 by adding axioms which grant the distributivity of A relative to truth-functional connectives, as well as two additional axioms, linking it with the alethic modal operators: $\Box p \rightarrow Ap$ and $Ap \rightarrow \Box Ap$
- An additional operator F (“fixedly”) can be defined with clause:
- $M \vDash_w F\phi$ iff for all models $M' = \langle W, @', V \rangle$ we have $M' \vDash_w \phi$

Necessity and apriority

- F-operator captures an alternative sense of necessity, which is ineffable in the standard modal systems with “ \square ”
- It is the so-called “deep necessity”, or “apriority”, as opposed to “superficial” necessity (conveyed by “ \square ”)
- As shown in [Humberstone 2004], the following schemas are valid in S5AF:
 - $\varphi \rightarrow (\square(\varphi \leftrightarrow (\varphi \leftrightarrow A\varphi)) \ \& \ FA(\varphi \leftrightarrow A\varphi))$
 - $\neg\varphi \rightarrow (\square(\neg\varphi \leftrightarrow (\neg\varphi \leftrightarrow A\neg\varphi)) \ \& \ FA(\neg\varphi \leftrightarrow A\neg\varphi))$

Fabricating hybrids

- For a *contingent a posteriori* true φ , we have *necessarily* true $A\varphi$, and for a *contingent a posteriori* false φ , we have *necessarily* false $A\varphi$ (note that in both cases, $A\varphi$ is a priori equivalent to φ)
- Similarly, for a *contingent a posteriori* true φ , we have *a priori* true $\varphi \leftrightarrow A\varphi$; for a *contingent a posteriori* false φ , we have *a priori* false $\varphi \leftrightarrow \neg A\varphi$ (note that these formulas is necessarily equivalent to φ in the first and the second case respectively)

2D-matrix for (a contingent) p

Worlds of evaluation	w_1	w_2	w_3
Worlds of reference			
w_1	1	0	1
w_2	0	0	1
w_3	1	1	0

2D-matrix for Ap

Worlds of evaluation	w_1	w_2	w_3
Worlds of reference			
w_1	1	1	1
w_2	0	0	0
w_3	0	0	0

2D-matrix for $p \leftrightarrow Ap$

Worlds of evaluation	w_1	w_2	w_3
Worlds of reference			
w_1	1	0	1
w_2	1	1	0
w_3	0	0	1

p

1	0	1
0	0	1
1	1	0



Ap

1	1	1
0	0	0
0	0	0

$p \leftrightarrow Ap$

1	0	1
1	1	0
0	0	1

A- and C-operators

- Thus, A-operator can be regarded as the operator forming *ontologically determined* (necessarily true or necessarily false) *a priori* counterpart for an arbitrary sentence.
- FA ($\varphi \leftrightarrow A\varphi$) & ($\Box A\varphi \vee \Box \neg A\varphi$)
- We can as well introduce the dual C-operator, which forms *epistemically determined* (*a priori* true or *a priori* false) necessary counterpart for an arbitrary sentence:

- Def 1 (“apriorization”) $C\varphi =_{\text{def}} \begin{cases} \varphi \leftrightarrow A\varphi, & \text{if } \varphi \\ \varphi \leftrightarrow \neg A\varphi, & \text{if } \neg\varphi \end{cases}$

“Maius”

- The correct interpretation of “Greater” must be *a priori* and we should associate it with a kind of logical validity
 - It is primarily relation between sentences and secondly between objects
 - It seems natural to choose S5AF as a logic governing the kind of apriority that is required for our analysis
- Df2 (“maius”). φ is greater than $\psi \leftrightarrow_{df} \varphi \vDash_{S5AF} \psi$ and $\psi \not\vDash_{S5AF} \varphi$
 - Since $C\varphi \vDash_{S5AF} \varphi$ and $\varphi \not\vDash_{S5AF} C\varphi$, this definition allows us to claim that $C\varphi$ (epistemically determined counterpart of φ) is greater than (mere) φ (with intentional objects of both being in the corresponding relation)

G-sentence

- That, in turn, naturally leads us to the definition:
- Df3 (“God’s existence”) $g \leftrightarrow_{ap} (g \leftrightarrow Cg)$
- Thereby God is defined as an object whose existence is a priori greater than itself (that than which no greater can be conceived)

2D-MOA

1. $g \leftrightarrow_{ap} (g \leftrightarrow Cg)$ Df3 (“God’s existence”)
2. $\neg g$ premise
3. $\neg g \leftrightarrow Cg$ 1, 2, *a priori* equivalence
4. $Cg \leftrightarrow_{ap} (g \leftrightarrow \neg Ag)$ 2, Def 1 (“apriorization”)
5. $\neg g \leftrightarrow (g \leftrightarrow \neg Ag)$ 3, 4, *a priori* equivalence
6. $g \leftrightarrow (g \leftrightarrow Ag)$ 5, PL
7. $(g \leftrightarrow g) \leftrightarrow Ag$ 6, PL
8. Ag 8, PL
9. g 9, S5AF
10. $\neg \neg g$ 2, 9, by contradiction
11. g 10, PL

Conclusion

- 2D-MOA is less vulnerable to Lewis' criticism
- We can see that the two-dimensional interpretation of Anselm's argument in terms of S5AF demonstrates its core feature:
 - we may know *a priori* that sentence “God exists” is true without knowing *a priori* what exactly this sentence means in our world
- Howbeit, it is better than nothing, and so, according to Anselm, it is greater than nothing

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