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Fuad Aleskerov, Vyacheslav Yakuba

POLARIZATION AND OPTIMAL ALLOCATION OF MIGRANTS

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Fuad Aleskerov
National Research University Higher School of Economics (HSE), V.A.Trapeznikov Institute of Control Sciences of Russian Academy of Sciences (ICS RAS), Moscow; E-mail: alesk@hse.ru, Tel.: +7(495)621-1342.

Vyacheslav Yakuba
National Research University Higher School of Economics (HSE), V.A.Trapeznikov Institute of Control Sciences of Russian Academy of Sciences (ICS RAS), Moscow; E-mail: yakuba@ipu.ru

Алескеров, Ф. Т., Якуба, В. И.

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Предложен ряд моделей размещения мигрантов по населенным пунктам, позволяющих не увеличить существенно поляризацию в обществе. Модели проиллюстрированы на примерах.

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1. Introduction

Arrival of new immigrants which represent different cultures and nationalities increase not only the population, but also the diversity in the host society. If the goal of migration policy implies only the control of open space per inhabitant, the rate of net migration growth is enough for successful migration policy.

However, main problems to be considered are heterogeneity of the population, heterogeneity of migrants, demand on the labor market and, most important, possible polarization among local population and migrants.

To achieve an optimal resettlement of immigrants we propose to take into account the rate of society polarization. The reasons for polarization might be infinite. For example, two groups might be funs of two different football teams.

A polarization of society may emerge due to different social, cultural, ethnic and other measurable characteristics of the population.

For the formal statement of the problem we consider the settlements with initial distribution of population and initial distribution of polarization in these settlements. Then we consider migrants with the distribution of polarization among them. Main assumption is that after allocation of migrants in the settlements the final polarization can be changed, thus, this effect should be taken into account.

The text has the following structure. In Section 2 we present a survey of the publications which can explain the causes of polarization in the settlements. In Section 3 we give a definition of one-dimensional polarization index. Then in Section 4 we illustrate via examples the problem of allocation of migrants over settlements taking into account the rate of polarization in the settlements. In Section 5 we formulate several optimization models for allocation of migrants using goal functions based on the polarization index. In Section 6 we discuss dynamic consequences of the proposed models. The Appendix contains a multi-dimensional version of polarization index.

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2. Review of the publications

To the best of our knowledge, there are no publications in which the problem of efficient allocation of migrants to decrease a polarization in the settlements is discussed. On the other hand, we found few articles which might give some hints on the appearance of polarization.

In [4] it is argued that since the United Arab Emirates becomes a priority destination for migrants for last few decades, due to the booming economy and cutting-edge technologies, more than 75% of the country population consists of international employees. The results of the qualitative study of personal interviews with 50 local and international middle-to-top managers of culturally diverse organizations are presented. The findings show that the lack of cultural diversity management may lead to decrease of the efficiency of the workforce, diminishing of cross-culture understanding. There might be also biases in job allocation. The study reveals the room for more efficient HR management in culturally diverse organizations, which takes into account advantages brought by cultural diversity of workers.

In [18] the logit model is applied to the focus group data to estimate the effect of group composition on the likelihood to polarize after some deliberation process on the themes related to international migration. The study itself deals with the observations that some individuals tend to shift the individual preferences to more extreme positions, and some of them tend to moderate their views after some deliberations. The study analyses the factors behind the polarization of views and moderation of views, focusing on data from a citizen deliberation experiment on immigration in Finland. The participants who have changed their minds more than the group average, either in a more extreme or a more moderate direction were analyzed. The statistically significant variables for changes in directions of views among the groups are age, gender, left-right ideology. The education and knowledge variable are barely significant. And the variables like the trust, empathy, and the composition of the group (like-minded versus mixed groups) do not drive polarization and moderation.

In [23] several groups of factors are analyzed to understand the differences among groups of migrant colonists and indigenous people in the Ecuadorian Amazons. These factors might lead to the polarization in the communities. The results of the survey of the rural and non-rural population (migrant colonist vs. indigenous population) in the canton Pastaza of the Ecuadorian Amazon area are presented. The multinomial probit model was used to per-

form the analyses of the determinants in 5 work categories to differ migrant colonists with indigenous people: farm/nonfarm self/wage employment and environmental income (income from natural non-cultivated resources, like forest). It is inferred from the model that colonists have highest share of waged non-farm employment and settled people closer to urban areas with developed road network. Most indigenous people obtain income from farm self-employment and from environment. The colonists also take advantage from producing crop for sale and cattle ranching. This difference can be attributed for agricultural technological advances over traditional indigenous methods of production.

In [14] the results of the study of the welfare policies and budget allocation for migrant groups in South Korea show that social policies are mainly oriented on the married women, while other groups of migrants attract less attention.

In [10] the game-theoretic approach is used to model the problem of the illegal immigration to advanced country through the borders with less well-off neighboring transit countries. The target country may spend resources to neighbors to support their control of borders. But the countries may direct the aid to border security activities, other than immigration control. The Nash equilibrium characterizes the optimal allocation of resources among transit countries and optimal use of the aid by them. Different goals are taken into account, like competition for a share of aid, maximization of joint welfare, and follow to the leader.

In [17], basing on clustering analysis and multinomial logit regression, the study reveals the difference between the financial behavior of immigrants, and the native-born investors. The results show that for low-income investors the mode of financial behavior is driven by the culture of the country of origin. For the high-income investors the financial behavior is determined by their wealth and association to the economic class.

In [20] the empirical model was used to reveal the impact of immigration on crime level upon unification of East and West Germany. The crime effect of ethnic German migrants who were allocated by authorities in different regions of Germany is studied. Since self-selection was severely limited, the allocation was not driven by local crime or by local labor market conditions. The analysis shows the following results. The inflow of ethnic German immigrants strongly increased crime rates. The crime impact depends heavily on the labor market condition in a region. Poorly educated migrants with low language skills are particularly vulnerable in economically disadvantaged

regions. Successful integration of immigrants into the labor market immediately after arrival seems crucial for crime prevention. The effects are substantially stronger in regions with high preexisting crime levels or large shares of foreigners.

In [9] it is pointed out that in Qatar, like in UAE the foreign population greatly outnumbers the local citizens. It can be noticed that in Bahrain the migrants constitute the majority, while in Kuwait and Saudi Arabia they account for very significant minority of the total population. Due to highly differentiated society, while performing interviews in Qatar it is important to pay attention to the subnational level and to the communities with different status and experience. Basing on the survey data, the determinants of the generalized trust among citizens and immigrants are examined. Several specific aspects were added to the research agenda: the confidence in the rule of law and government institutions, the interaction with members of other national and subnational communities, and the attendance to religious service. More than 2 thousands interviews were performed. Using logit model several hypothesis were tested. The results are as follows. Qatari citizens show the lowest levels of generalized trust. The white-collar immigrants from countries in the Middle East show more trust than Qataris. White-collar immigrants from Asia and labor migrant from outside of the Middle East show much higher levels of generalized trust. Membership in civic or voluntary organizations is positively related to generalized trust for Qataris, but not for white-collar immigrants. Qatari citizens who attend worship services show more trust. Two hypothesis: does contact at work with persons of other nationalities increases generalized trust, and, does confidence in the police increases generalized trust, are justified by data for white-collar immigrants from Asia, but not for Qatari citizens.

In [8] the survey-based research, performed among Turkish and North African women in France, studies the differences in using of the local French social services. The results show that young Turkish women use the social services less frequently than young North African women. While the Turkish community is considered to be the least integrated immigrant community in France, the North Africans in France strongly endorse integration. The separation is positively associated with the perception of a threat to heritage culture and with the difficulties in understanding the language of the social workers.

In [15] the ethnographic study of North Korean settles in South Korea is performed. Due to severe famine in North Korea in the end of 1990s the num-

ber of immigrants to the South Korea rapidly increased. By the 2014 there are more than 27 thousands of the North settlers in the South. The qualitative ethnographic study presents the cultural gaps, issues of conflict and power analysis of the young North Korean settlers in the South Korea. North Korean propaganda constructed a well-known artificial image of South Korean capitalist society. In addition, justified by a traditional Korean idea of self-sufficiency, it generated a number of issues hardened the adjustment of North Korean migrants into the South Korean society. The factors include: cultural differences (smile means a trick for cheating), habits of everyday violent behavior in North Korea, different functions of money (absence of fines or monetary penalties, free cure for injures from fights, no monetary obligations by court in North Korea), cautious attitude to givers, even to government-funded education programs of aid to North Koreans, inside out opinions that South Korean teachers and social workers, who work with North Koreans, have a job because of North Korean settlers, since the South Korean government funded such aid programs, so «North Korea identity» becomes a good that is of value on the market.

In [24] the impact of the recent refugee crisis in Europe is discussed. The inflow of refugees to European countries, combined with the terrorist attacks in France, Belgium, and Germany may lead to major changes in European Union politics. Borders begin restricting access, military presence appears in transport facilities, growth in xenophobia and islamophobia become visible. The right-wing anti-immigration parties gain wins, the British electorate decides to leave the European Union, the risks of further military intervention in the Middle East rises.

Different types of adjustment of migrants into the Swiss communities are studied in [21]. As the economy of Switzerland specializes on the production on value-added goods and services, the majority of migrant workers are the highly skilled professionals from Germany, France, Italy, UK, and USA. There is also a big share of migrants from Asia. The cross-culture adjustment is important for the effectiveness of the Swiss-based multinational companies. In the current framework adjustment means reducing the uncertainty in the new environment. The common approach to such adjustment involves several kinds of adjustments. The General adjustment refers to comfort of living in the culture environment, including food or healthcare. The Work adjustment deals with the work requirements and tasks. The Interaction adjustment means the ease of communication including language and cross-cultural issues. The survey data from foreign employees of Swiss-based global companies living

in French area of Switzerland is used to test four hypotheses. The hypotheses are about the use of information and training, the level of all kinds of adjustments of the employees and the employee's families, about the local language proficiency, and about the importance of the Interaction adjustment. The results obtained show, in particular, low Interaction adjustment comparing to other kinds of adjustment. This may be explained by existence English-speaking expat community in Leman region and the migrant community in French-speaking Geneva Lake region (foreign workers live in the region on the long-term basis should have proficiency in French to interact with local population). As for the families, it is inferred that extensive spousal employment support positively correlated with the spousal general adjustment.

One of the possible allocation mechanisms might be a construction of camps for refugees. In [11] it is discussed that for the durable solutions for refugees to succeed, the refugee resettlement aspects in new host countries should include favorable attitudes by members of host societies, protection of the well-being of refugees, and effective integration of refugees into new countries. The review of the research literature in that article discusses studies in two areas – the public attitudes toward refugees and the factors of successful refugee resettlement. By the end of 2015, around 3 million of the total 16 million refugees under the mandate of UNHCR (UN Refugee Agency) were living in planned or managed camps, over 500 thousands were living in self-settled camps, and close to 200 thousands were living in reception or transit camps. Camps may be built by host nations to provide better control and as a way of reducing tension and competition between refugees and local communities. Ironically, refugee camps may improve attitudes toward refugees while providing a damaging living environment for them. Many refugees spend years in camps. It is shown that only limited number of countries, like Canada, adopt more enhanced methods to cope with the refugee resettlement crisis.

In [12] the review of publications of two parts of international migration research is performed. From the one hand the acculturation of migrants from psychological and sociological points of view, and from the other hand, the international assignee adjustment from the management and business administration point of view are taken into consideration. The findings are that the situation is more complicated than the standard matrix of acculturation (low/high home/foreign culture). The majority of papers deal with USA-related migration. The study of other pairs of countries is quite limited. Since global companies play a major role in international migration, the corporate culture and management interaction is important since it sets so-called informal land-

mark for all migrants. The dynamics is important since many individuals who work abroad become multicultural, and the work and the sociocultural contexts should be studied together. It is proposed that the extended families of migrants should be considered as well.

In [5] it is pointed out that since the term 'expatriates' is used as a reference to the internationally assigned employees of the multinational corporations, the expatriate analysis deals with a tiny minority of privileged workers. Many other migrants are less privileged and they are of different races, ethnicities, economic and social statuses, less educated, work on lower level jobs. The literature of the second type of international workers is quite limited due to a number of factors.

In [7] it is shown that the cultural differences play an important role in migration process. The employees more readily migrate to regions with a similar cultural background. Social connections and networks are important for the adjustment process in the foreign country. As the study shows, the intercultural training provided for expatriates is a crucial part of preparation if there are large differences in cultures. But organizations generally do not use any tools to assess candidates for international assignment which increases the risk of poor performance.

In [6] a migration, inspired by international assignments of employees by global companies, is studied by empirical methods. The attention focuses on the support of expatriates by a company before, during, and after their work outside the country of origin. It is shown that many challenges induced by international assignments are pertinent well before and after being abroad for relatively long term (for 3 to 5 years). It is inferred that social integration should be performed by the employer, at least initially. The difficulties upon repatriation are also underestimated by the organizations. In general, it is essential to provide the worker with interpersonal and cross-cultural skills required during international assignment.

In [22] a comparative analysis of management and practices of internationally assigned employers of more than hundred large multinational countries is performed. The changes in expatriate practices over time are examined on four areas, such that staffing, selection, training, and implementation.

In [13] the factors of adjustment of expatriate workers to new surroundings in global organizations are studied. The qualitative research based on in-depth interviews with the senior executives of the global companies in petrochemical, hardware manufacturing, and technology and innovations sectors was performed. The companies selected are the UK, The Netherlands, India,

and US-bases with regional offices all over the world. The results show that four factors can be revealed, the Work Adjustment, like task satisfaction, emotional satisfaction, reinforces or stimulus, and the leadership, Career Development, Family Adjustment, like cross-culture training, education and employment for the family, and General Adjustment, like health, housing, and social support systems. The interviewees highlighted that the task and emotional satisfaction are among the most important factors of adjustments, what global companies often tend to ignore.

In [1] the analysis of the contribution of migrants to economic growth in USA economy is performed. The measure of foreign-born expertise in USA in several technological areas as a share of patents in the area brought by the immigrants is constructed. The measure is defined as

$$Exp_k = \sum_c \frac{Pat(k,c)}{Pat(c)} \times MigInv(c),$$ where Exp_k – foreign born expertise in area k, $Pat(k,c)$ – number of patents in area k in country c, $Pat(c)$ – number of patents in country c, $MigInv(c)$ – number of migrants from country c.

The intuition of the measure is as follows. The frontier innovative technologies from a particular country are transferred by the migrant inventors by their implicit or methodized knowledge. The growth models might be applied to justify that, innovation of present and prior generations leads to economic growth.

The data obtained from the United States Patent and Trademark Office contains the geographic location of inventors, their technology area (patent class) and patent citations.

The name and location of inventors on patent documents are used to match them to Federal Censuses. So the profiles of inventors are generated. The profiles include labor income received (including commissions). The descriptive analysis on immigrant inventors is performed. The inventors are observed longitudinally, their career patents and citations are measured to determine their productivity. The regressions of wages and careers productivity on the foreign expertize measure, controlled for education, age, and state and occupation fixed effects. It is shown that migrant inventors were productive in those technology areas in which they were active exhibited higher levels of growth over the long run. The immigrant inventors had a positive macroeconomic impact on the US inventiveness.

However the labor market wage-gap is observed. This can be associated with assimilation issues, labor market barriers in the US. The immigrant wage-gap cannot be explained by variation in productivity.

3. A model of polarization

We consider the following model of polarization stemmed from the classic Downsian model in political theory. Suppose there is a scale of opinions distributed on the interval [0, 1]. Suppose the group is divided into two equal subgroups A and B with opinions located on the polar positions 0 and 1, correspondingly. On the other hand, if all members of the group are located on the point $x \in [0,1]$, we can observe the absence of polarization in the second case, and the maximal polarization in the first case.

The rate of polarization Π also varies from 0 to 1; Π reaches its maximum when society is divided into two equal groups that belong to completely opposite social categories (views/cultures). Minimal value corresponds to belonging to the common social category (view/culture) or to categories that have the same value on the scale.

The polarization index proposed in [2] will be used to evaluate the difference in views of several parts of the society, which may lead to social tension among the ethnic, cultural or religious groups when the migrants with other views enter the country, the particular settlement of the country or the districts in the regions/cities etc.

We begin with the set $\Theta = \{1,...,P\}$ of opinions' positions on the scale [0, 1]. Next we consider the share of people residing in a particular settlement with the view $p \in \Theta$ as $v(p)$. Naturally,

$$\sum_{p\in\Theta} v(p) = 1.$$

We consider the following polarization index

$$\Pi = 2\sum_{p\in\Theta} v(p)\cdot|p-c|, \qquad (1)$$

where $c = \dfrac{\sum_{p\in\Theta} v(p)\cdot p}{\sum_{p\in\Theta} v(p)} = \sum_{p\in\Theta} v(p)\cdot p$, is the so-called "barycenter" (center of mass) of the system, similar to that in physics.

Denoting $v_i = v(p_i)$ for each group $i = \overline{1,P}$, with view position $p_i \in \Theta$, of size $v(p_i)$ the polarization index can be re-written in the following form

$$\Pi = 2\sum_{i=1}^{P} v_i \cdot |p_i - c|, \quad (2)$$

where $c = \dfrac{\sum_{j=1}^{P} v_j p_j}{\sum_{j=1}^{P} v_j}$.

So, each group $i = \overline{1,P}$ is characterized by two numbers, the position p_i of the views of the group on the segment [0, 1], and the size v_i of the group.

We denote the set of settlements as $\Omega = \{S_1,...,S_n\}$, S being a settlement from Ω, however, it should be noted that these might be districts or even quarters in big cities.

4. Examples

Let us illustrate the problem using the following example.

Example 1. Let there are three groups of people A, B, and C with the positions on the opinion scale and the population size as $p_A = 0$, $v_A = 1$ $p_B = 0.2$, $v_B = 1$, $p_C = 1$, $v_C = 98$. It is the initial distribution in the settlement. Using the formula (1), we obtain the following level of polarization Π = 0.07.

Hereafter, we will for simplicity consider that each group of people residing in the settlements and each group of migrants consist of three subgroups A, B, and C, and each subgroup has the following position on the opinions' scale: $p_A - 0$, $p_B - 0.2$, and $p_C - 1.0$. We will denote the settlement as consisting of three components. Consider the case $S(v_A, v_B, v_C)$, where v_A means the number of people with the opinion A, v_B means the number of people with the opinion B, etc. Let there are three groups of people in a settlement S(1,1,98).

Consider now the group of migrants with the following distribution of views G=G(0,60,0), which have been allocated in the settlement S. Then, we have a new distribution of views in the settlement S+G = (1,61,98). Using (1)

we obtain $\Pi = 0.76$. We can say that the polarization in the settlement has increased crucially.

Example 2. Now let us consider the case of 22 settlements with the following distribution of the population in the settlements among three groups with the positions A, B, and C. The initial polarization in the groups is presented in the last column of the Table 1. Note that while in the settlements S1, S2, S5 and some others, the polarization is below 0.1, in the settlements S6, S15, and S16 the level of polarization is above 0.5 which can be considered rather high.

Table 1. Initial polarization in the settlements

Settlements	A	B	C	Initial polarization
S1	1730	89	22	0.081
S2	1671	88	26	0.091
S3	614	83	85	0.408
S4	771	84	96	0.385
S5	1621	88	17	0.075
S6	650	83	155	0.566
S7	728	84	93	0.390
S8	1632	88	25	0.091
S9	1162	86	123	0.347
S10	1391	87	18	0.088
S11	1701	89	160	0.318
S12	1551	88	23	0.091
S13	1672	88	157	0.318
S14	1596	88	16	0.074
S15	929	85	197	0.542
S16	884	84	189	0.544
S17	1756	89	163	0.315
S18	1751	89	25	0.086
S19	888	84	104	0.371
S20	1358	87	19	0.092
S21	1634	88	16	0.073
S22	1682	88	22	0.083

Suppose now that there are 11 groups of migrants with the following distribution of population over positions, presented in Table 2. As before, G1(0,11,2) means that there are 0 people with the opinion A in the group, 11 people with the opinion B, and 2 people with the opinion C.

Table 2. Distributions of opinions in groups of migrants

	G1	**G2**	**G3**	**G4**	**G5**	**G6**	**G7**	**G8**	**G9**	**G10**	**G11**
A	0	0	0	0	2	0	0	0	0	0	2
B	11	4	1	37	52	35	25	5	34	37	41
C	2	9	12	20	38	70	72	72	75	106	113

Consider now two different allocations of migrants presented in Tables 3 and 4.

Table 3. Eventual polarization for 'bad' allocation of migrants

Bad allocation	**A**	**B**	**C**	**Polarization**	**Initial polarization**	**Difference**
S1	1730	89	22	0.081	0.081	0
S2	1671	88	26	0.091	0.091	0
S3+G3	614	84	97	0.442	0.408	0.034
S4+G4	771	121	116	0.426	0.385	0.041
S5	1621	88	17	0.075	0.075	0
S6+G8	650	88	227	0.702	0.566	0.137
S7+G6	728	119	163	0.533	0.390	0.143
S8	1632	88	25	0.091	0.091	0
S9+G5	1164	138	161	0.410	0.347	0.064
S10	1391	87	18	0.088	0.088	0
S11+G11	1703	130	273	0.459	0.318	0.141
S12	1551	88	23	0.091	0.091	0
S13+G10	1672	125	263	0.454	0.318	0.136
S14	1596	88	16	0.074	0.074	0
S15+G1	929	96	199	0.542	0.542	0
S16+G2	884	88	198	0.557	0.544	0.013
S17+G9	1756	123	238	0.412	0.315	0.097
S18	1751	89	25	0.086	0.086	0
S19+G7	888	109	176	0.511	0.371	0.140
S20	1358	87	19	0.092	0.092	0
S21	1634	88	16	0.073	0.073	0
S22	1682	88	22	0.083	0.083	0

The new allocations of the settlements are denoted as the sum of the populations of the settlements plus the population of the group with particular views distribution. For example, S3+G8 means that the group G8 is added to the settlement S3. In the following example one can see that some non-optimal allocation can lead to critical levels of polarization in some of the settlements.

The optimal allocation according to one of the models, considered in the next section, yields only moderate increase in polarization, as shown in Table 4.

In other words, a bad allocation of migrants leads to the situation where the number of settlements with polarization above 0.4 increased comparing to initial situation from 4 to 10. Moreover, we observe a settlement with the polarization above 0.7 which can be considered as extremely high. In the good allocation we have almost the same distribution of polarization among settlements as in the initial case.

Table 4. Eventual polarization for good allocation of migrants

Good allocation	**A**	**B**	**C**	**Polarization**	**Initial polarization**	**Difference**
S1+G10	1730	126	128	0.269	0.081	0.188
S2+G9	1671	122	101	0.234	0.091	0.142
S3	614	83	85	0.408	0.408	0
S4	771	84	96	0.385	0.385	0
S5+G5	1623	140	55	0.163	0.075	0.088
S6	650	83	155	0.566	0.566	0
S7	728	84	93	0.390	0.390	0
S8+G6	1632	123	95	0.228	0.091	0.137
S9	1162	86	123	0.347	0.347	0
S10+G2	1391	91	27	0.110	0.088	0.022
S11	1701	89	160	0.318	0.318	0
S12+G4	1551	125	43	0.143	0.091	0.052
S13	1672	88	157	0.318	0.318	0
S14+G3	1596	89	28	0.100	0.074	0.025
S15	929	85	197	0.542	0.542	0
S16	884	84	189	0.544	0.544	0
S17	1756	89	163	0.315	0.315	0
S18+G11	1753	130	138	0.282	0.086	0.195
S19	888	84	104	0.371	0.371	0
S20+G1	1358	98	21	0.101	0.092	0.009
S21+G8	1634	93	88	0.212	0.073	0.139
S22+G7	1682	113	94	0.220	0.083	0.137

Table 5. Changes of polarization for the two allocations

Settlements with polarization above	Initial situation	First (bad) allocation (Table 3)	Second (good) allocation (Table 4)
0.4	4	10	4
0.7	0	1	0

Consider Table 5. If some critical levels of polarization is set, the optimal allocation preserves the initial situation, if we count the number of settlements above the critical level, while some 'bad' allocation can worsen the situation substantially.

5. Models of migrants allocation over settlements

As it was understood from the above examples, to achieve an optimal allocation it is necessary to take into account the polarization in the society. To this end, we will present several optimization models of migrants' allocation over settlements and illustrate the results using the example above.

The Hungarian algorithm [16] was used to find an optimal assignment of the groups on the settlements. The algorithm solves the matching problem and initially was applied to the problem of assignment of workers to jobs in order to maximize some goal function. In other words, there is a different profit of assignment of a particular worker to do a particular job. The algorithm finds such an allocation that maximizes the total profit. For the models of the allocation of the groups of migrants over settlements, the Hungarian algorithm is used to find the optimal allocation that minimizes the sum of the polarization itself or the sum of increase of the polarization in settlements.

a) Minimization of the mean (over settlements) polarization rate

We define first the goal function as

$$\frac{1}{n}\sum_{j=1}^{n}\Pi'_j \to \min \tag{3}$$

Here Π'_j – polarization in settlement j after the migrants are placed in the settlement, n – number of settlements.

This optimization model gives exactly the allocation shown in Table 4 and have been discussed above. In Table 4 the polarization has been evaluated for

the case when the whole group is placed in a settlement. We can divide each group onto subgroups of migrants with the same views (3 subgroups in the example), and then allocate these subgroups to settlements.

The composition of groups are shown in the following tables.

Table 6. A denotation of subgroups

ID	G1(a)	G2(a)
Number of people	2	2

G1(b)	G2(b)	G3(b)	G4(b)	G5(b)	G6(b)	G7(b)	G8(b)	G9(b)	G10(b)	G11(b)
11	4	1	37	52	35	25	5	34	37	41

G1(c)	G2(c)	G3(c)	G4(c)	G5(c)	G6(c)	G7(c)	G8(c)	G9(c)	G10(c)	G11(c)
2	9	12	20	38	70	72	72	75	106	113

The distribution of polarization, if the groups are divided into subgroups, is shown in Table 7. In this Table we compare the polarization of the settlements with divided groups allocation and the settlements with the whole groups allocated with respect to the model a) (formula (3)).

Table 7. An allocation of subgroups in settlements

Settlements	Group	A	B	C	Polarization	Initial polarization	Difference
S1	G4(c)	1730	89	42	0.119	0.081	0.038
S2	G3(c)	1671	88	38	0.115	0.091	0.024
S3	G4(b)	614	120	85	0.399	0.408	–0.009
S4	G6(b)	771	119	96	0.38	0.385	–0.005
S5	G8(b)+G11(c)	1621	93	130	0.283	0.075	0.208
S6	G5(b)	650	135	155	0.536	0.566	–0.030
S7	G10(b)	728	121	93	0.385	0.390	–0.005
S8	G1(b)	1632	99	25	0.095	0.091	0.004
S9	G7(b)	1162	111	123	0.346	0.347	–0.001
S10	G1(a)	1393	87	18	0.088	0.088	0.000
S11	G8(c)	1701	89	232	0.416	0.318	0.098
S12	G3(b)	1551	89	23	0.092	0.091	0.001
S13	G7(c)	1672	88	229	0.417	0.318	0.099
S14	G2(b)	1596	92	16	0.076	0.074	0.002

Settlements	Group	A	B	C	Polarization	Initial polarization	Difference
S15	G6(c)	929	85	267	0.649	0.542	0.107
S16	G11(b)	884	125	189	0.527	0.544	–0.017
S17	G9(c)	1756	89	238	0.414	0.315	0.099
S18	G5(c)	1751	89	63	0.156	0.086	0.070
S19	G9(b)	888	118	104	0.368	0.371	–0.003
S20	G2(a)	1360	87	19	0.092	0.092	0.000
S21	G1(c)+G10(c)	1634	88	124	0.272	0.073	0.199
S22	G2(c)	1682	88	31	0.101	0.083	0.018

We can notice that not only the sum of polarization is decreased comparing with Table 4, but the polarization of the settlements is decreased in many of the settlements.

Note, that we use the Hungarian algorithm to put one subgroup to a settlement once at a time, so to allocate S5+ G8(b)+G11(c) as well as in similar cases two iterations were made.

b) Minimization of the growth of the mean polarization rate

Now, we evaluate the following model

$$\frac{1}{n}\sum_{j=1}^{n}\left(\Pi_j^{'}-\Pi_j\right)\to \min \tag{4}$$

Here Π_j and $\Pi_j^{'}$ – polarizations in settlement j before and after the migrants arrived in the settlement, thus, the mean increase of polarization is minimized, n – number of settlements.

The results are given in Table 8.

The distribution of polarization, if the groups are divided into subgroups according to the views, and the optimality condition is to minimize the increase of the polarization in the settlements, is presented in Table 9.

As the considered model deals with the increase of the polarization, having in mind that not the values of the polarization, but the difference in the situation is taken into account, it can be seen from tables 8 and 9, that the increase of polarization of settlements is less, when subgroups are allocated among settlements. Note that for some settlements there might be decrease in polarization since a subgroup with the position of views close to that of large subgroup in the settlement is added.

Table 8. An assignment of groups minimizing the total increase in polarization over settlements

Min. total increase allocation	A	B	C	Polarization	Initial polarization	Difference
S1+G3	1730	90	34	0.105	0.081	0.023
S2+G2	1671	92	35	0.110	0.091	0.019
S3+G1	614	94	87	0.411	0.408	0.003
S4	771	84	96	0.385	0.385	0
S5	1621	88	17	0.075	0.075	0
S6+G4	650	120	175	0.585	0.566	0.019
S7	728	84	93	0.390	0.390	0
S8	1632	88	25	0.091	0.091	0
S9+G6	1162	121	193	0.463	0.347	0.117
S10	1391	87	18	0.088	0.088	0
S11+G10	1701	126	266	0.452	0.318	0.134
S12	1551	88	23	0.091	0.091	0
S13+G7	1672	113	229	0.415	0.318	0.097
S14	1596	88	16	0.074	0.074	0
S15+G9	929	119	272	0.640	0.542	0.097
S16+G5	886	136	227	0.579	0.544	0.035
S17+G11	1758	130	276	0.453	0.315	0.139
S18+G8	1751	94	97	0.215	0.086	0.129
S19	888	84	104	0.371	0.371	0
S20	1358	87	19	0.092	0.092	0
S21	1634	88	16	0.073	0.073	0
S22	1682	88	22	0.083	0.083	0

Table 9. An allocation of subgroups with the goal function (4)

Settlements	Groups	A	B	C	Polarization	Initial polarization	Difference
S1	G4(c)	1730	89	42	0.119	0.081	0.038
S2	G3(c)	1671	88	38	0.115	0.091	0.024
S3	G10(b)	614	120	85	0.399	0.408	–0.009
S4	G4(b)	771	121	96	0.380	0.385	–0.005
S5	G3(b)	1621	89	17	0.076	0.075	0.000
S6	G5(b)	650	135	155	0.536	0.566	–0.030
S7	G6(b)	728	119	93	0.385	0.390	–0.005
S8	G1(b)	1632	99	25	0.095	0.091	0.004
S9	G7(b)	1162	111	123	0.346	0.347	0.000
S10	G2(a)	1393	87	18	0.088	0.088	0.000
S11	G7(c)+G11(c)	1701	89	345	0.542	0.318	0.223

Settlements	Groups	A	B	C	Polarization	Initial polarization	Difference
S12	G8(b)	1551	93	23	0.093	0.091	0.002
S13	G8(c)	1672	88	229	0.417	0.318	0.099
S14	G1(c)	1596	88	18	0.078	0.074	0.004
S15	G6(c)	929	85	267	0.649	0.542	0.107
S16	G11(b)	884	125	189	0.527	0.544	–0.016
S17	G9(c)+G10(c)	1756	89	344	0.530	0.315	0.215
S18	G5(c)	1751	89	63	0.156	0.086	0.070
S19	G9(b)	888	118	104	0.368	0.371	–0.003
S20	G1(a)	1360	87	19	0.092	0.092	0.000
S21	G2(b)	1634	92	16	0.074	0.073	0.001
S22	G2(c)	1682	88	31	0.101	0.083	0.018

c) An optimization of the number of settlements with the polarization rate above the predefined levels

In the next model we first minimize the number of settlements with polarization level more than 0.7, then minimize the number of settlements with polarization level more than 0.4. In order to use the Hungarian assignment algorithm, the actual polarization is replaced by the penalty function, i.e. we minimize the sum (over settlements) of the penalty function $R\left(\Pi_j\right)$ of the polarization rates Π_j of the settlements.

The optimization problem is defined as

$$\sum_j R\left(\Pi_j\right) \to \min. \tag{5}$$

The penalty function $R\left(\Pi\right)$ of the polarization rate Π is defined as follows

$$R = \begin{cases} 0, \text{if } \Pi < r_1, \\ 1, \text{if } \Pi \geq r_1 \text{ and } \Pi < r_2, \\ n+1, \text{if } \Pi \geq r_2. \end{cases}$$

Here Π is the polarization index, n is the number of settlements, r_1 and r_2 – critical levels, $r_1 < r_2$. For the example considered $r_1 = 0.4$, and $r_2 = 0.7$.

The intuition for this formula infers that the situation with any number of settlements with polarization rate less than critical level r_2 (equal to 0.7 in the

example) is better than the situation with at least one settlement with polarization rate exceeding this critical level. The same reason is used for r_1. In other words, if a particular settlement upon addition of a particular group yields the polarization not more than 0.4, it is coded by 0. If the polarization value lies between 0.4 and 0.7, then it is coded by 1, and if the polarization value is above 0.7, it is coded by the number of settlements plus 1 (i.e., 23 in the considered example). The assignment and the resulting polarization are presented in Table 9.

Table 9. An allocation for the critical levels 0.4 and 0.7

The allocation	A	B	C	Polarization	Initial polarization	Difference
S1+G8	1730	94	94	0.212	0.081	0.131
S2+G2	1671	92	35	0.110	0.091	0.019
S3	614	83	85	0.408	0.408	0
S4+G1	771	95	98	0.388	0.385	0.004
S5+G10	1621	125	123	0.275	0.075	0.199
S6	650	83	155	0.566	0.566	0
S7	728	84	93	0.390	0.390	0
S8+G7	1632	113	97	0.230	0.091	0.139
S9+G4	1162	123	143	0.382	0.347	0.035
S10+G9	1391	121	93	0.253	0.088	0.165
S11+G5	1703	141	198	0.370	0.318	0.051
S12+G11	1553	129	136	0.304	0.091	0.213
S13+G3	1672	89	169	0.335	0.318	0.018
S14+G6	1596	123	86	0.217	0.074	0.142
S15	929	85	197	0.542	0.542	0
S16	884	84	189	0.544	0.544	0
S17	1756	89	163	0.315	0.315	0
S18	1751	89	25	0.086	0.086	0
S19	888	84	104	0.371	0.371	0
S20	1358	87	19	0.092	0.092	0
S21	1634	88	16	0.073	0.073	0
S22	1682	88	22	0.083	0.083	0

As one can see, there are no settlements with polarization more than 0.7, and there are 4 settlements with polarization more than 0.4. This method yields the same situation as the assignment of groups to the settlements done by minimizing the sum of polarizations of the settlements (see Table 4). In both situations the number of settlements with the polarization above the critical

levels is the same, as in the initial situation (before the arrival of migrants). But, as can be seen from Tables 4 and 9, the allocations are different.

Table 10. An allocation of the subgroups for critical levels 0.4 and 0.7

Settlements	Groups	A	B	C	Polarization	Initial polarization	Difference
S1	G1(c)+G1(a)	1732	89	24	0.085	0.081	0.004
S2	G2(c)+G2(a)	1673	88	35	0.109	0.091	0.018
S3	G4(b)	614	120	85	0.399	0.408	–0.009
S4	G1(b)	771	95	96	0.383	0.385	–0.001
S5	G3(c)	1621	88	29	0.100	0.075	0.025
S6	G4(c)	650	83	175	0.608	0.566	0.042
S7	G2(b)	728	88	93	0.390	0.390	–0.001
S8	G5(c)	1632	88	63	0.166	0.091	0.074
S9	G3(b)	1162	87	123	0.347	0.347	0.000
S10	G6(c)	1391	87	88	0.239	0.088	0.151
S11	G5(b)	1701	141	160	0.319	0.318	0.001
S12	G7(c)	1551	88	95	0.232	0.091	0.141
S13	G6(b)	1672	123	157	0.319	0.318	0.001
S14	G8(c)	1596	88	88	0.215	0.074	0.140
S15	G9(c)	929	85	272	0.656	0.542	0.114
S16	G7(b)	884	109	189	0.534	0.544	–0.010
S17	G8(b)	1756	94	163	0.315	0.315	0.000
S18	G10(c)	1751	89	131	0.268	0.086	0.182
S19	G9(b)	888	118	104	0.368	0.371	–0.003
S20	G11(c)	1358	87	132	0.326	0.092	0.234
S21	G10(b)	1634	125	16	0.085	0.073	0.012
S22	G11(b)	1682	129	22	0.096	0.083	0.013

Yet another application of this model is used for the allocation not groups but subgroups as it was done above. The result is that the distribution of the polarization is even better than in the original situation – only 3 settlements with the polarization value more than 0.4 – is obtained (see Table 10).

Below we present some models, but we will not present examples of allocation.

d) Minimization of the maximal possible growth of the polarization rate

The next goal function minimizes the maximal possible growth of the polarization rate in the society, i.e.,

$$\max_j \left(\Pi_j^{'} - \Pi_j \right) \to \min,$$

where Π_j – original polarization rate of the settlement j, $\Pi_j^{'}$ – the polarization of the settlement j after the arrival of migrants.

e) Minimization of the average expenditures to conduct migration policy taking into account the growth of polarization rate,

$$\frac{1}{n} \sum_{j=1}^{n} c_j \cdot \left(\Pi_j^{'} - \Pi_j \right) \to \min,$$

where c_j are the expenditures to conduct the migration policy.

We would like to point out that c_j in general depend on polarization rate.

f) An optimization of the number of settlements with predefined polarization rate

The key migration policy problem may be posed in another way: to define the critical polarization rate of settlements (Π_0) and minimize the number of such settlements or their population.

$$a_i \cdot \left(n'_{\Pi_0} - n_{\Pi_0} \right) + \left(n'_{\Pi_1} - n_{\Pi_1} \right) \to \min,$$

where a_i – population of settlements with polarization rate Π_0, n_{Π_0} – number of settlements (or population size) before the resettlement of migrants with polarization rate exceeding Π_0, but less than Π_1, $n_{\Pi_0}^{'}$ – number of settlements (or population size) after the resettlement of migrants with polarization rate exceeding Π_0, but less than Π_1, $n_{\Pi_1}^{'}$ – number of settlements (or population size) before the resettlement of migrants with polarization rate more, than Π_1, $n_{\Pi_1}^{'}$ – number of settlements (or population size) after the resettlement of migrants with polarization rate more, than Π_1.

6. Conclusion – dynamic considerations

We propose five models of migrants' allocation over districts aiming to decrease or at least not increase the existing polarization in the districts.

We have considered one-dimensional model of polarization, however, the reasons of a polarization in the society might be various. Thus, we present in

the Appendix a model which takes into account multidimensional basis for polarization. This opens a wide way for sociologists to study the reasons for polarized society (dimensions of the polarization index).

Another direction of works is to consider the models with allocation of families of migrants taking into account as well the demand of certain labor skills, the level of unemployment and possible increasing of the level of polarization.

In this way we can also introduce several constraints, namely, the capacity of settlements to host certain number of migrants, the cost of their maintenance, naturally, again taking into account the level of polarization.

On the other hand, if we restrict the level of polarization in the settlements after allocation of migrants by some level $\overline{\Pi}$, it can be defined the maximum number of migrants which is possible to allocate in the settlements. Then, these constraints can define the total number of migrants to be allocated in a country under consideration.

Finally, the extension of the model presented above should include a dynamic component. Indeed, the simplest solution in the static model is to allocate migrants with the same position on the scale in some uninhabited place not increasing the polarization in the settlements. However, growing settlements and increasing ability of communication among settlements lead to the situation when nearest settlements should be considered as one unit. Then polarization in such unit might approach the level higher than any acceptable limits.

Appendix. Multi-dimensional polarization index

The multi-dimensional polarization indices were introduced in [3]. They were used to evaluate the polarization distribution in Finnish Parliament, in Russian Duma, and USA Congress [2, 3, 19].

For the number of dimensions being m, $\vec{p}_i = \left(p_1^i, ..., p_m^i\right)$ is the point for the group i in m-dimensional space. As before, v_i are the weights of the groups, it may be proportional to the number of people in the particular group i, the number of groups is denoted as P. The center of mass is the vector $\vec{c} = \left(c_1, ..., c_m\right)$ defined as

$$\vec{c} = \frac{\sum_{i=1}^{P} v_i \vec{p}_i}{\sum_{i=1}^{P} v_i}$$

The polarization index can be defined as follows

$$\Pi = \frac{2}{m} \sum_{i=1}^{P} v_i \cdot d\left(\vec{p}_i, \vec{c}\right),$$

where the Euclidian metrics is defined as $d\left(\vec{p}_i, \vec{c}\right) = \sqrt{\sum_{j=1}^{m} \left(p_j^i - c_j\right)^2}$.

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(*на английском языке*)

Зав. редакцией оперативного выпуска *А.В. Заиченко*
Технический редактор *Ю.Н. Петрина*

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