Bounded Skeptical Abduction

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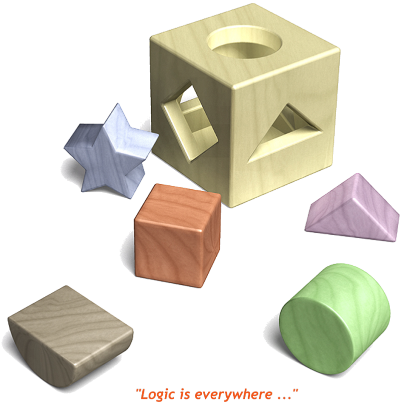
- Weak Completion Semantics
- The Suppression Task
- Skeptical Abduction
- Obligation and Factual Conditionals
- The Selection Task
- Syllogistic Reasoning
- The Need for Boundedness



Modus Ponens

- Byrne: Suppressing Valid Inferences with Conditionals. Cognition 31, 61-83: 1989

  Stenning, v. Lambalgen: Human Reasoning and Cognitive Science, MIT Press: 2008

- **If she has an essay to write, then she will go to the library**
  She has an essay to write

- **Program**

$e \leftarrow \top$	fact	definition of e
$\ell \leftarrow e \wedge \neg ab_1$	rule	definition of ℓ
$ab_1 \leftarrow \bot$	assumption	ab_1 is assumed to be false

- **Weakly completed program & least model**

$$e \leftrightarrow \top$$
$$\ell \leftrightarrow e \wedge \neg ab_1$$
$$ab_1 \leftrightarrow \bot$$

true	false	
e	ab_1	$\Phi \uparrow 1$
ℓ		$\Phi \uparrow 2$

- **Computing logical consequences with respect to the least model**
  - **She will go to the library**

- Łukasiewicz: O logice trójwartościowej. Ruch Filozoficzny 5, 169-171: 1920

  H., Kencana Ramli: Logic Programs under Three-Valued Łukasiewicz's Semantics LNCS 5649, 464-478: 2009

Alternative Arguments

- If she has an essay to write, then she will go to the library
  If she has a textbook to read, then she will go to the library
  She has an essay to write

- Program

e	$\leftarrow$	$\top$	fact	definition of e
ℓ	$\leftarrow$	$e \wedge \neg ab_1$	rule	definition of ℓ
ab_1	$\leftarrow$	$\bot$	assumption	ab_1 is assumed to be false
ℓ	$\leftarrow$	$t \wedge \neg ab_2$	rule	definition of ℓ
ab_2	$\leftarrow$	$\bot$	assumption	ab_2 is assumed to be false

- Weakly completed program & least model

$$
\begin{array}{rcl}
e & \leftrightarrow & \top \\
\ell & \leftrightarrow & (e \wedge \neg ab_1) \vee (t \wedge \neg ab_2) \\
ab_1 & \leftrightarrow & \bot \\
ab_2 & \leftrightarrow & \bot
\end{array}
$$

true	false	
e	ab_1	
	ab_2	$\Phi \uparrow 1$
ℓ		$\Phi \uparrow 2$

- Computing logical consequences with respect to the least model
  - She will go to the library

Additional Arguments

- If she has an essay to write, then she will go to the library
  If the library is open, then she will go to the library
  She has an essay to write

- **Programs**

$e \leftarrow \top$	fact	definition of e
$\ell \leftarrow e \wedge \neg ab_1$	rule	definition of ℓ
$ab_1 \leftarrow \bot$	assumption	ab_1 is assumed to be false
$\ell \leftarrow o \wedge \neg ab_3$	rule	definition of ℓ
$ab_3 \leftarrow \bot$	assumption	ab_3 is assumed to be false
$ab_1 \leftarrow \neg o$	rule	definition of ab_1
$ab_3 \leftarrow \neg e$	rule	definition of ab_3

- **Weakly completed program & least model**

$$e \leftrightarrow \top$$
$$\ell \leftrightarrow (e \wedge \neg ab_1) \vee (o \wedge \neg ab_3)$$
$$ab_1 \leftrightarrow \bot \vee \neg o$$
$$ab_3 \leftrightarrow \bot \vee \neg e$$

true	*false*	
e		$\Phi \uparrow 1$
	ab_3	$\Phi \uparrow 2$

- **Computing logical consequences with respect to the least model**
  - We can neither conclude that she will go nor that she will not go to the library

Affirmation of the Consequent

- **If she has an essay to write, then she will go to the library**
  She will go to the library
- **Program**

$$\begin{array}{rcl} \ell & \leftarrow & \top \\ \ell & \leftarrow & e \wedge \neg ab_1 \\ ab_1 & \leftarrow & \bot \end{array}$$

- **Weakly completed program & least model**

$$\begin{array}{rcl} \ell & \leftrightarrow & \top \vee (e \wedge \neg ab_1) \\ ab_1 & \leftrightarrow & \bot \end{array}$$

true	*false*
ℓ	ab_1

- **Computing logical consequences with respect to the least model**
  - **We cannot conclude that she has an essay to write**
  - Byrne 1989 **most humans conclude *e*!**

Abduction

- Hartshorne, Weiss: Collected Papers of C. S. Peirce. Harvard Univ. Press: 1932
- **Program & observation**

$$\begin{array}{rcl} \ell & \leftarrow & e \wedge \neg ab_1 \\ ab_1 & \leftarrow & \bot \end{array} \qquad \ell$$

- **Abducibles**

$$e \leftarrow \top \qquad e \leftarrow \bot$$

- **Weakly completed program plus explanation & least model**

$$\begin{array}{rcl} \ell & \leftrightarrow & e \wedge \neg ab_1 \\ ab_1 & \leftrightarrow & \bot \\ e & \leftrightarrow & \top \end{array}$$

true	false
e	ab_1
ℓ	

- **Computing logical consequences with respect to the least model**
  - **She has an essay to write**
- H., Philipp, Wernhard: An Abductive Model for Human Reasoning. In: Proc. Tenth Int. Symposium on Logical Formalizations of Commonsense Reasoning: 2011

Alternative Arguments and Affirmation of the Consequent

- **Program & observation**

$$\begin{array}{rcl} \ell & \leftarrow & e \wedge \neg ab_1 \\ ab_1 & \leftarrow & \bot \\ \ell & \leftarrow & t \wedge \neg ab_2 \\ ab_2 & \leftarrow & \bot \end{array} \qquad \ell$$

- **Abducibles**

$$e \leftarrow \top \qquad t \leftarrow \top \qquad e \leftarrow \bot \qquad t \leftarrow \bot$$

- **Weakly completed program plus explanations & least models**

$$\begin{array}{rcl} \ell & \leftrightarrow & (e \wedge \neg ab_1) \vee (t \wedge \neg ab_2) \\ ab_1 & \leftrightarrow & \bot \\ ab_2 & \leftrightarrow & \bot \\ e & \leftrightarrow & \top \quad \text{or} \quad t \leftrightarrow \top \end{array}$$

true	*false*
e	ab_1
	ab_2
ℓ	

true	*false*
t	ab_1
	ab_2
ℓ	

- **Computing skeptical consequences with respect to both models**
  - e **does not follow** — Byrne 1989 — **only 16% conclude** e

- Dietz, H., Ragni: A Computational Logic Approach to the Suppression Task Proc. COGSCI, 1500-1505: 2012

Obligation Conditional

- Dietz Saldanha, H., Lourêdo Rocha: Obligation versus Factual Conditionals under the Weak Completion Semantics. CEUR Workshop Proc. 1837, 55-64: 2017
- **If it rains then the roofs are wet**
  - **Its consequence is obligatory**
  - **We cannot easily imagine a case, where the antecedent is true and the consequence is not**
- **Necessary antecedent**
  - **The consequence cannot be true unless the antecedent is true**

Factual Conditional

- **If it rains then she takes her umbrella**
  - **We can easily imagine a situation,**
    where the antecedent is true and the consequence is not
  - **Its consequence is not obligatory**
- **Sufficient antecedent**
  - **The antecedent does not appear to be necessary**

Encoding Obligation and Factual Conditionals

- **Program**

$$
\begin{array}{rcl}
w & \leftarrow & r \wedge \neg ab_4 \\
ab_4 & \leftarrow & \bot \\
u & \leftarrow & r \wedge \neg ab_5 \\
ab_5 & \leftarrow & \bot
\end{array}
$$

- **Weakly completed program & least model**

$$
\begin{array}{rcl}
w & \leftrightarrow & r \wedge \neg ab_4 \\
ab_4 & \leftrightarrow & \bot \\
u & \leftrightarrow & r \wedge \neg ab_5 \\
ab_5 & \leftrightarrow & \bot
\end{array}
$$

true	false
	ab_4
	ab_5

- **Abducibles**

$r \leftarrow \top$ $\quad$ $r \leftarrow \bot$ $\quad$ $ab_5 \leftarrow \top$ $\quad$ $u \leftarrow \top$

The Evaluation of Indicative Conditionals

- **Indicative conditional**
  - ***If X then Y***
- **Background knowledge**
  - **Weakly completed program $wc\mathcal{P}$ with least model $\mathcal{M}$**
  - **Set of abducibles $\mathcal{A}$**
- **Evaluation**
  - **If $\mathcal{M}(X)$ is *true*, then the conditional is evaluated to $\mathcal{M}(Y)$**
  - **If $\mathcal{M}(X)$ is *false*, then the conditional is evaluated to *true***
  - **If $\mathcal{M}(X)$ is *unknown*, then the conditional is evaluated with respect to the skeptical consequences of $wc\mathcal{P}$ given $\mathcal{A}$ considering X as an observation**

If the roofs are not wet then it did not rain

- *If* $\neg w$ *then* $\neg r$
- Explaining $\neg w$ by $r \leftarrow \bot$ we obtain
  - $\{w \leftrightarrow r \wedge \neg ab_4,\ u \leftrightarrow r \wedge \neg ab_5,\ ab_4 \leftrightarrow \bot,\ ab_5 \leftrightarrow \bot,\ r \leftrightarrow \bot\}$
- It's least model is

true	*false*
	ab_4
	ab_5
	r
	w
	u

- The conditional is *true*

If she did not take her umbrella then it did not rain

- *If $\neg u$ then $\neg r$*
- The observation $\neg u$ can be explained by $r \leftarrow \bot$ and $ab_5 \leftarrow \top$, and we obtain
  - $\{w \leftrightarrow r \wedge \neg ab_4,\ u \leftrightarrow r \wedge \neg ab_5,\ ab_4 \leftrightarrow \bot,\ ab_5 \leftrightarrow \bot,\ r \leftrightarrow \bot\}$
  - $\{w \leftrightarrow r \wedge \neg ab_4,\ u \leftrightarrow r \wedge \neg ab_5,\ ab_4 \leftrightarrow \bot,\ ab_5 \leftrightarrow \bot \vee \top\}$
- Their least models are

true	false
	ab_4
	ab_5
	r
	w
	u

true	false
ab_5	ab_4
	u

- Reasoning skeptically, the conditional is *false*

If the roofs are wet then it rained

- *If w then r*
- Explaining w by $r \leftarrow \top$ we obtain
  - $\{w \leftrightarrow r \wedge \neg ab_4,\ u \leftrightarrow r \wedge \neg ab_5,\ ab_4 \leftrightarrow \bot,\ ab_5 \leftrightarrow \bot,\ r \leftrightarrow \top\}$
- Its least model is

true	*false*
r	ab_4
	ab_5
w	
u	

- The conditional is *true*

If she took her umbrella then it rained

- *If u then r*
- The observation u can be explained by $r \leftarrow \top$ or $u \leftarrow \top$, and we obtain
  - $\{w \leftrightarrow r \wedge \neg ab_4,\ u \leftrightarrow r \wedge \neg ab_5,\ ab_4 \leftrightarrow \bot,\ ab_5 \leftrightarrow \bot,\ r \leftrightarrow \top\}$
  - $\{w \leftrightarrow r \wedge \neg ab_4,\ u \leftrightarrow (r \wedge \neg ab_5) \vee \top,\ ab_4 \leftrightarrow \bot,\ ab_5 \leftrightarrow \bot\}$
- Their least models are

true	*false*
r	ab_4
	ab_5
w	
u	

true	*false*
u	ab_4
	ab_5

- Reasoning skeptically, the conditional is *false*

The Selection Task

- **If there is the letter d on one side of the card, then there is the number 3 on the other side**
  - **Factual conditional with necessary antecedent**
- **If a person is drinking beer, then the person must be over 19 years of age**
  - **Obligation conditional with sufficient antecedent**
- **Reasoning skeptically yields the adequate answers**
  - Dietz Saldanha, H., Lourêdo Rocha: Obligation versus Factual Conditionals under the Weak Completion Semantics. CEUR Workshop Proc. 1837, 55-64: 2017

The Abstract Case

- **Program**

$$\begin{array}{rcl} 3 & \leftarrow & d \wedge \neg ab_6 \\ ab_6 & \leftarrow & \bot \end{array}$$

- **Abducibles**

$$d \leftarrow \top \qquad d \leftarrow \bot \qquad ab_6 \leftarrow \top$$

- **Observations & least models**

Observation: d

true	false
d	ab_6
3	

turn
89%

Observation: a → $\neg d$

true	false
	d
	ab_6
	3

no turn
16%

Observation: 3

true	false
d	ab_6
3	

turn
62%

Observation: 7 → $\neg 3$

true	false
	d
	ab_6
	3

true	false
ab_6	
	3

no turn
25%

The Social Case

- **Program**

$$\begin{array}{rcl} o & \leftarrow & b \wedge \neg ab_7 \\ ab_7 & \leftarrow & \bot \end{array}$$

- **Abducibles**

$$b \leftarrow \top \qquad b \leftarrow \bot \qquad o \leftarrow \top$$

- **Observations & least models**

b

true	false
b	ab_7
o	

turn
95%

m $\neg b$

true	false
	b
	ab_7
	o

no turn
0,025%

o

true	false
o	ab_7

true	false
b	ab_7
o	

no turn
0,025%

y $\neg o$

true	false
	b
	ab_7
	o

turn
80%

Syllogistic Reasoning

- Oliviera da Costa, Dietz Saldanha, H., Ragni: A Computational Logic Approach to Human Syllogistic Reasoning. In: Proc. 39th Annual Conference of the Cognitive Science Society, 883-888: 2017

  Oliviera da Costa, Dietz Saldanha, H.: Monadic Reasoning using Weak Completion Semantics. In: CEUR Workshop Proc. 1837, 45-54: 2017

- **The Weak Completion Semantics achieves 89% when reasoning skeptically**
  - **It is better than 12 established cognitive theories**

The Complexity of Skeptical Abduction

- H., Philipp, Wernhard: An Abductive Model for Human Reasoning
  In: Logical Formalizations of Commonsense Reasoning
  Papers from the AAAI 2011 Spring Symposium
  AAAI Spring Symposium Series Technical Reports, AAAI Press, 135-138: 2011

  Dietz Saldanha, H., Philipp: Contextual Abduction and Its Complexity Issues
  In: Proc. 4th Int. Workshop on Defeasible and Ampliative Reasoning
  CEUR Workshop Proc. 1827, 58-70: 2017

- **Skeptical Reasoning is DP-complete**
  - **L is in the class DP**
    iff there are languages $L_1 \in$ NP and $L_2 \in$ coNP such that $L = L_1 \cap L_2$
  - **Deciding whether there exists an explanation is NP-complete**
  - **Deciding whether a formula follows from all explanations is coNP-complete**

Bounded Skeptical Abduction Hypotheses

- **Humans generate some, but usually not all possible explanations**
- **Humans reason skeptically with respect to them**

Which Explanations are Generated?

- Are short explanations preferred?
  - We have specified a connectionist system implementing the Weak Completion Semantics
  - Our system generates singleton sets first
- Are minimal explanations preferred?
  - The Weak Completion Semantics utilizes minimal explanations
- Are supersets of known explanations generated?
  - It is unnecessary to investgate explanations containing $A \leftarrow \top$ and $A \leftarrow \bot$
  - In the presented system, if $\mathcal{E}$ is an explanation, then so is each $\mathcal{E}' \supseteq \mathcal{E}$
  - However, this does not hold anymore in extensions of the presented system
- Are more explanations generated if more time is available?
- Is the generation of explanations biased and, if so, how is it biased?
- Does attention play a role?