On elliptical model for market network

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One way to analyze a stock market is to construct a market network. Nodes of the network corresponds to the stocks and weights of edges are given by some measure of similarity between fluctuations of stock returns. According to traditional point of view [1] stock returns are random variables. Therefore considered network is random variables network. To analyze the network it is necessary to model the distribution of these random variables.

Models of stock returns distribution attract a growing attention last decade. Popular multivariate model in financial analysis is the class of elliptically contoured distributions [2]. Consistency of real data with elliptically contoured model was studied in [3] where it was shown that the joint distribution of real market stock returns is not in accordance with hypothesis of elliptical distributions. This result is obtained by comparison of dependence measures of pairs of stock returns. However, as pointed by the authors, their methodology differs from usual hypothesis testing using statistical tools. In the case of large stock market the number of pairs of stocks is huge and it is necessary to take into account so called multiplicity phenomenon [4].

Let X_i be a random variable, corresponding to the return of stock i ($i = 1, \ldots, N$) and $X = (X_1, \ldots, X_N)$ be the random vector of stock returns. To study the consistency of real data with elliptical model for X we use one important property of elliptically contoured distributions, namely, the symmetry of density function of joint distribution with respect to the vector of means. To formulate the individual hypotheses we use the following pairwise sign symmetry property of multivariate elliptical distribution:

$$p_{1,1}^{i,j} = p_{-1,-1}^{i,j}; \quad p_{1,-1}^{i,j} = p_{-1,1}^{i,j}; \quad \forall i, j = 1, \ldots, N \tag{1}$$

where

$$p_{k,l}^{i,j} = P(k(X_i - E(X_i)) > 0, l(X_j - E(X_j)) > 0); k, l \in \{-1, 1\} \tag{2}$$

In the present paper we analyze sign symmetry property for stock returns distribution from multiple hypotheses testing theory point of view [4]. Our main goal is to detect pairs of stocks for which sign symmetry hypotheses are rejected and study associated rejection graph. Multiple statistical procedure for testing sign symmetry for stock returns distribution is proposed. Distribution

free uniformly most powerful tests of the Neyman structure are constructed for individual hypotheses testing. Associated stepwise multiple testing procedure is applied for the real market data. Numerical experiments shows that hypothesis of elliptical model is rejected. At the same time it is observed that the graph of rejected individual hypotheses has unexpected structure. Namely, this graph is sparse and has a few hubs of high degree. Removing this hubs leads to non-rejection of hypothesis of elliptical model.

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