On a State Feedback Anisotropy-based Control Problem for Linear Discrete-time Descriptor Systems

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***Abstract*—This paper deals with a suboptimal state feedback anisotropy-based control problem for linear time-invariant discrete-time descriptor systems. The goal is to design a state feedback for the system such that the closed-loop system is admissible, and its anisotropic norm (mean anisotropy level is set) is bounded by a given positive real value.**

***Keywords*—anisotropy-based control theory; analysis and synthesis problems; Gaussian random vectors; mean anisotropy; anisotropic norm; descriptor systems.**

I. INTRODUCTION

The theory of optimal stochastic anisotropy-based control for normal discrete-time systems was established in Russia since 1994 [1], [2]. This theory allows to design control, that minimizes specified norm of the closed-loop system (anisotropic norm). Since 2008 the theory of suboptimal stochastic anisotropy-based control, that provides boundedness of anisotropic norm for the referred above systems, was developed [3]. At present these mathematical theories find application in different control and filtration problems.

The created theory lies between the classical H_2-optimal and H_∞-optimal control theories (and suboptimal as well) in some sense. The basic concepts of these theories are anisotropy of the random signal and mean anisotropy of the input sequence. Anisotropy of the vector describes so-called "spectral color" of the signal as the distinction between its probability density function (p.d.f.) and the p.d.f. of the Gaussian white noise. Mean anisotropy of the sequence is the time averaging anisotropy.

This paper deals with generalization of suboptimal anisotropy-based control on the class of discrete-time descriptor systems. Descriptor systems, which have both dynamical and algebraic constraints, often appear in various engineering systems and control applications. The increased practical interest for a more general description reversed necessity of developing generalized methods of analysis and synthesis for descriptor systems.

The problem is to design a state feedback for a descriptor system such that the closed-loop system is admissible, and its anisotropic norm is less than a given positive real constant. A suboptimal state feedback anisotropy-based controller design problem for normal discrete-time systems is solved in [4]. The conditions of anisotropic norm boundedness for admissible descriptor systems, connected with existence of the solution of specified Riccati equation, are obtained in [5]. They are rewritten in terms of linear matrix inequalities (LMI) in [6]. Though LMI conditions for admissible descriptor systems have already been obtained while solving the analysis problem, they can't be directly extended on controller design.

A number of papers also addresses to H_∞-suboptimal state feedback control problem for descriptor systems. For example, in [7] under some rank assumptions, necessary and sufficient conditions are characterized by a symmetric solution of the generalized discrete-time algebraic Riccati inequality (GDARI) with one matrix variable; in [8] necessary and sufficient strict LMI conditions for H_∞-controller design are derived.

The paper is organized as follows. In the section II-A, anisotropy-based concepts are introduced. The section II-B is devoted to basic definitions and preliminaries of discrete-time descriptor systems theory. In the section III, the problem is formulated, the main results are given: under some rank assumptions sufficient conditions for the solution of the state feedback anisotropy-based control problem for descriptor systems are characterized by the solution of GDARI and a pair of inequalities, connected with the anisotropic norm boundedness. Numerical example is given in the section IV.

II. BACKGROUND

A. Mean anisotropy of the sequence and anisotropic norm of linear discrete-time systems

Here we provide some background material on anisotropy-based analysis of linear discrete-time systems. The concepts of mean anisotropy of the Gaussian random sequences and anisotropic norm of linear systems are introduced in [1], [9].

Let $W = \{w(k)\}_{k\geq 0}$ be a stationary sequence of square integrable vectors with values in $\mathbb{R}^m$, which is interpreted as a discrete-time random signal. Assembling the elements of W, associated with the interval $[0, N-1]$, into a random vector $W_{0:N-1} = \begin{bmatrix} w(0) \\ \dots \\ w(N-1) \end{bmatrix}$ we assume that $W_{0:N-1}$ is absolutely continuously distributed for every $N \geq 0$.

The anisotropy $\mathbf{A}(W_{0:N-1})$ is defined as the minimal value of the relative entropy [10] with respect to the Gaussian distributions in $\mathbb{R}^m$ with zero mean and scalar covariance matrices described by:

$$\mathbf{A}(W_{0:N-1}) = \frac{m}{2} \ln \left(\frac{2\pi e}{m} \mathbf{E}(|W_{0:N-1}|^2) \right) - h(W_{0:N-1})$$

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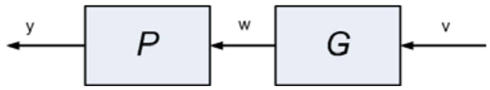

Fig. 1: Interpretation of the output signal $Y = PW$ and the input signal $W = GV$

where $h(W_{0:N-1}) = \mathbf{E} \ln f(W_{0:N-1}) = -\int_{\mathbb{R}^{mN}} f(w) \ln f(w) dw$ is the differential entropy, and $f : \mathbb{R}^{mN} \longrightarrow \mathbb{R}_+$ is the probability density function of mN-dimensional vector $W_{0:N-1}$. The mean anisotropy of the sequence W is defined by

$$\overline{\mathbf{A}}(W) = \lim_{N\to+\infty} \frac{\mathbf{A}(W_{0:N-1})}{N}. \quad (1)$$

Suppose W is generated from the Gaussian random white noise sequence V by a shaping filter G (see Fig.1).

The transfer function of the filter $G(z)$ is supposed to belong to the Hardy space $H_2^{m\times m}$ of matrix-valued functions, analytic in the disc $|z| < 1$ on the complex plane. The space is equipped with the H_2-norm, defined by

$$\|G\|_2 = \left(\frac{1}{2\pi} \int_{-\pi}^{\pi} \operatorname{Trace} S(\omega) d\omega \right)^{1/2}$$

where $S(\omega) = \widehat{G}(\omega)\widehat{G}(\omega)^* \, (-\pi \leq \omega \leq \pi)$ is a spectral density of W, $\widehat{G}(\omega) = \lim_{l\to 1} G(le^{i\omega})$ is the boundary value of the transfer function G.

The mean anisotropy (1) of the stationary Gaussian random sequence $W = GV$ may be computed in terms of the spectral density $S(\omega)$ and H_2-norm of the shaping filter G as

$$\overline{\mathbf{A}}(W) = -\frac{1}{4\pi} \int_{-\pi}^{\pi} \ln\det \frac{mS(\omega)}{\|G\|_2^2} d\omega.$$

It characterizes the divergence between the signal and the Gaussian white noise sequence. For more information, see [1], [11].

Let $Y = PW$ be an output of the linear system $P \in H_\infty^{p\times m}$ (see Fig.1), its transfer function $P(z)$ is analytic in the disc $|z| < 1$. $P(z)$ has a finite H_∞-norm.

Definition 1: The a-anisotropic norm of the system P for a given parameter $a \geq 0$ is defined by

$$|\!|\!| P |\!|\!|_a = \sup \left\{ \|PG\|_2 / \|G\|_2 : \; G \in \mathbf{G}_a \right\}. \quad (2)$$

The fraction on the right-hand side of (2) can also be interpreted as the ratio of the power norms of the output Y and the input W against the class of shaping filters

$$\mathbf{G}_a = \left\{ G \in H_2^{m\times m} : \; \overline{\mathbf{A}}(G) \leq a \right\}. \quad (3)$$

So the a-anisotropic norm $|\!|\!| P |\!|\!|_a$ describes a "stochastic gain" of the system P with respect to W.

Remark 1: The random sequence W is fully defined by its generating filter G, therefore, the notation $\overline{\mathbf{A}}(G)$ is used as equivalent to the notation $\overline{\mathbf{A}}(W)$.

B. Preliminary results on descriptor systems

Discrete-time descriptor systems are described by the following equations:

$$\begin{aligned} Ex(k+1) &= Ax(k) + Bf(k), \\ y(k) &= Cx(k) + Df(k) \end{aligned} \quad (4)$$

where $x(k) \in \mathbb{R}^n$ is the state, $f(k) \in \mathbb{R}^m$ and $y(k) \in \mathbb{R}^p$ are the input and output signals, respectively. A, B, C, D are constant real matrices of appropriate dimensions. The matrix $E \in \mathbb{R}^{n\times n}$ is singular, $\operatorname{rank}(E) = r \leq n$.

Definition 2: The system is assumed to be admissible if it is regular ($\exists \lambda \neq 0 : \det(\lambda E - A) \neq 0$), causal ($\deg\det(zE - A) = \operatorname{rank} E$), and stable ($\rho(E, A) = \max |\lambda|_{\lambda \in \{z | \det(zE-A)=0\}} < 1$). For more information, see [12], [13].

Consider a state feedback control in the following form:

$$f(k) = Fx(k) + v(k) \quad (5)$$

where $F \in \mathbb{R}^{m\times n}$ is a constant real matrix, $v(k)$ is a new control signal. The closed-loop system may be written in the form:

$$Ex(k+1) = (A + BF)x(k) + Bv(k). \quad (6)$$

Definition 3: The system (4) is called causal controllable if there exists a state feedback control in the form (5) such that the closed-loop system (6) is causal. For more information, see [12], [13].

Theorem 1: [12] The system (4) is causal controllable if the rank condition

$$\operatorname{rank} \left(\begin{bmatrix} E & 0 & 0 \\ A & E & B \end{bmatrix} \right) = \operatorname{rank}(E) + n$$

holds true.

Definition 4: The system (4) is called stabilizable if there exists a state feedback control in the form $f(k) = F_{st}x(k)$ such that the pair $(E, A + BF_{st})$ is stable.

The transfer function of the system (4) is given by

$$P(z) = C(zE - A)^{-1}B + D.$$

Consider the following system:

$$\begin{aligned} Ex(k+1) &= Ax(k) + Bw(k), \\ y(k) &= Cx(k) + Dw(k). \end{aligned} \quad (7)$$

Suppose the system (7) is admissible, and $W = \{w(k)\}_{k\geq 0}$ is a stationary Gaussian random sequence whose mean anisotropy does not exceed $a \geq 0$, i.e. W is generated from the m-dimensional Gaussian white noise sequence V (with zero mean and an identity covariance matrix) by an unknown shaping filter G, which belongs to the set (3).

The problem, stated and solved in [5], [6], was to find the conditions of anisotropic norm boundedness $|\!|\!| P |\!|\!|_a < \gamma$ for the given system P, parameters $a \geq 0$ and $\gamma > 0$. The following theorem gives the solution of this problem.

Theorem 2: [6] Let $w(k)$ be a stationary Gaussian random sequence, whose mean anisotropy does not exceed known $a \geq 0$. Suppose that the assumption

$$\operatorname{rank}(E) = \operatorname{rank}\left(\begin{bmatrix} E & B \end{bmatrix}\right)$$

holds. For the admissible system $P \in H_\infty^{p\times m}$ with a state space representation (7) a-anisotropic norm is bounded by a positive scalar $\gamma > 0$, i.e.

$$|||P|||_a \le \gamma$$

if there exists a scalar $q \in [0, \min(\gamma^{-2}, \|P\|_\infty))$ and symmetric matrix R satisfying

$$E^T RE \ge 0, \tag{8}$$

$$-(\det(I_m - B^T RB - qD^T D))^{1/m} < -(1-q\gamma^2)e^{2a/m}, \tag{9}$$

$$\begin{bmatrix} A^T RA - E^T RE & A^T RB \\ B^T RA & B^T RB - I_m \end{bmatrix} + \\ +q \begin{bmatrix} C^T \\ D^T \end{bmatrix} [C \quad D] < 0. \tag{10}$$

Corollary 1: Denote $\eta = q^{-1}$, $\xi = \gamma^2$ and $M = -(B^T\Phi B + D^T D - \eta I_m)$. Multiplying the inequalities (8) – (10) on η and taking into consideration ξ, the conditions of Theorem 2 for the admissible system (7) may be rewritten as

$$\eta - (e^{-2a}\det(M))^{1/m} \le \xi, \tag{11}$$

$$\begin{bmatrix} A^T\Phi A - E^T\Phi E + C^T C & A^T\Phi B + C^T D \\ B^T\Phi A + D^T C & -M \end{bmatrix} < 0, \tag{12}$$

$$E^T\Phi E \ge 0 \tag{13}$$

where $\Phi = \eta R$.

Taking into account that $M > 0$ [6] and applying Schur complement, we represent (12) as

$$A^T\Phi A - E^T\Phi E + C^T C - \\ -(A^T\Phi B + C^T D)(-M)^{-1}(B^T\Phi A + D^T C) < 0. \tag{14}$$

III. PROBLEM STATEMENT AND MAIN RESULTS

Consider a discrete-time descriptor system in the following form:

$$\begin{aligned} Ex(k+1) &= Ax(k) + B_1 w(k) + B_2 u(k), \\ z(k) &= Cx(k) + D_1 w(k) + D_2 u(k) \end{aligned} \tag{15}$$

where $w(k) \in \mathbb{R}^{m_1}$ and $z(k) \in \mathbb{R}^p$ are the input and output signals, respectively, $u(k) \in \mathbb{R}^{m_2}$ is the control vector. A, B_1, B_2, C, D_1, D_2 are constant real matrices of appropriate dimensions.

A state feedback anisotropy-based control problem may be formulated as follows: it is necessary to find a static state feedback in the form $u(k) = F_2 x(k)$ such that the closed-loop system is admissible and a-anisotropic norm of its transfer function P is limited by the given value γ for the set mean anisotropy level a. Denote $\xi = \gamma^2$.

The following theorem gives sufficient conditions for the solution of this problem.

Theorem 3: For the system (15) the following assumption holds:

$$\operatorname{rank}(E) = \operatorname{rank}([E \quad B_1]).$$

The state feedback anisotropy-based control problem is solvable if there exist $\Phi \in \mathbb{R}^{n\times n}$, $\Phi = \Phi^T$, $\Psi \in \mathbb{R}^{m_1\times m_1}$, $\Psi = \Psi^T > 0$ and a positive scalar η, satisfying the following conditions:

$$\begin{gathered} E^T\Phi E \ge 0, \\ B_1^T\Phi B_1 + D_1^T D_1 - \eta I_{m_1} < 0, \\ B_2^T\Phi B_2 + D_2^T D_2 > 0, \\ \Psi < \eta I_{m_1} - B_1^T\Phi B_1 - D_1^T D_1, \\ \eta - (e^{-2a}\det(\Psi))^{1/m_1} \le \xi, \\ A^T\Phi A - E^T\Phi E + C^T C - \\ -(A^T\Phi B + S)(B^T\Phi B + R)^{-1}(B^T\Phi A + S^T) < 0 \end{gathered}$$

where $B = [B_1 \quad B_2]$, $S = \begin{bmatrix} C^T D_1 & C^T D_2 \end{bmatrix}$,

$$R = \begin{bmatrix} D_1^T D_1 - \eta I_{m_1} & D_1^T D_2 \\ D_2^T D_1 & D_2^T D_2 \end{bmatrix}.$$

Moreover, $u(k) = F_2 x(k)$,

$$F_2 = -[0 \quad I_{m_2}](B^T\Phi B + R)^{-1}(B^T\Phi A + S^T).$$

Proof: Consider the system (15) with the state feedback $u(k) = F_2 x(k)$, so the closed-loop system is written in the following form:

$$\begin{aligned} Ex(k+1) &= (A + B_2F_2)x(k) + B_1 w(k), \\ z(k) &= (C + B_2F_2)x(k) + D_1 w(k). \end{aligned} \tag{16}$$

Denote $N = \eta I_{m_1} - B_1^T\Phi B_1 - D_1^T D_1 > 0$. For the system (16) the inequality (11) may be rewritten as

$$\eta - (e^{-2a}\det(N))^{1/m_1} \le \xi.$$

Consider a matrix $\Psi = \Psi^T > 0$, satisfying the inequality $\Psi < N$. So,the following condition

$$\eta - (e^{-2a}\det(\Psi))^{1/m_1} \le \xi$$

is true.

Denote

$$M = -(B^T\Phi B + R) = \\ = \begin{bmatrix} N & -B_1^T\Phi B_2 - D_1^T D_2 \\ -B_2^T\Phi B_1 - D_2^T D_1 & -B_2^T\Phi B_2 - D_2^T D_2 \end{bmatrix}.$$

M is invertible (see [14]), because $N > 0$ and $-B_2^T\Phi B_2 - D_2^T D_2 < 0$.

Let

$$F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = M^{-1}(B^T\Phi A + S^T),$$

which may be easily rewritten as

$$F_1^T N = (A + B_2F_2)^T\Phi B_1 + (C + D_2F_2)D_1 \tag{17}$$

and

$$F_1^T(B_1^T\Phi B_2 + D_1^T D_2) = \\ = -((A + B_2F_2)^T\Phi B_2 + (C + D_2F_2)^T D_2). \tag{18}$$

Using the equations (17) and (18), it is not difficult to show that [15]

$$
\begin{aligned}
&(A+B_2F_2)^{\mathrm{T}}\Phi(A+B_2F_2)-E^{\mathrm{T}}\Phi E+\\
&\quad +(C+D_2F_2)^{\mathrm{T}}(C+D_2F_2)+\\
&\quad +((A+B_2F_2)^{\mathrm{T}}\Phi B_1+(C+D_2F_2)^{\mathrm{T}}D_1)N^{-1}\times\\
&\quad \times(B_1^{\mathrm{T}}\Phi(A+B_2F_2)+D_1^{\mathrm{T}}(C+D_2F_2))=\\
&\quad =A^{\mathrm{T}}\Phi A-E^{\mathrm{T}}\Phi E+C^{\mathrm{T}}C-\\
&\quad -(A^{\mathrm{T}}\Phi B+S)(-M)^{-1}(B^{\mathrm{T}}\Phi A+S^{\mathrm{T}})<0. \quad (19)
\end{aligned}
$$

The inequality (19) coincides with (14), so, by Theorem 1 and Corollary 1, the closed-loop system (16) is admissible and a-anisotropic norm of its transfer function is limited by the given value γ for the set mean anisotropy level a. ■

Proposition 1: Denote $A_c = A + B_2F_2,\ C_c = C + D_2F_2$. Other denotations are from Theorem 3.

Direct application of the conditions (11) – (13) from Corollary 1 to the system (16) leads to the following inequalities

$$\eta-(e^{-2a}\det(N))^{1/m_1}\le\xi, \quad (20)$$

$$\begin{bmatrix}\Xi & A_c^{\mathrm{T}}\Phi B_1+C_c^{\mathrm{T}}D_1\\ B_1^{\mathrm{T}}\Phi A_c+D_1^{\mathrm{T}}C_c & -N\end{bmatrix}<0, \quad (21)$$

$$\Xi=A_c^{\mathrm{T}}\Phi A_c-E^{\mathrm{T}}\Phi E+C_c^{\mathrm{T}}C_c,$$

$$E^{\mathrm{T}}\Phi E\ge 0.$$

The inequality (20) is equivalent to

$$\eta-(e^{-2a}\det(\Psi))^{1/m_1}\le\xi$$

and

$$\begin{bmatrix}\Psi-\eta I_{m_1}+B_1^{\mathrm{T}}\Phi B_1 & D_1^{\mathrm{T}}\\ D_1 & -I_p^{-1}\end{bmatrix}<0.$$

Using Schur complement to (21), we get

$$\begin{bmatrix}A_c^{\mathrm{T}}\Phi A_c-E^{\mathrm{T}}\Phi E & * & *\\ B_1^{\mathrm{T}}\Phi A_c & B_1^{\mathrm{T}}\Phi B_1-\eta I_{m_1} & *\\ C_c & D_1 & -I_p^{-1}\end{bmatrix}<0 \quad (22)$$

or, equivalently,

$$\begin{bmatrix}\Upsilon & * & *\\ \Omega & B_1^{\mathrm{T}}\Phi B_1-\eta I_{m_1} & *\\ C+D_2F_2 & D_1 & -I_p^{-1}\end{bmatrix}<0 \quad (23)$$

where

$$
\begin{aligned}
\Upsilon=&A^{\mathrm{T}}\Phi A+F_2^{\mathrm{T}}B_2^{\mathrm{T}}\Phi A+A^{\mathrm{T}}\Phi B_2F_2+\\
&+F_2{}^{\mathrm{T}}B_2^{\mathrm{T}}\Phi B_2F_2-E^{\mathrm{T}}\Phi E,
\end{aligned}
$$

$$\Omega=B_1^{\mathrm{T}}\Phi A+B_1^{\mathrm{T}}\Phi B_2F_2.$$

The expression (23) is bilinear. So, the sufficient conditions for the solution of the state feedback anisotropy-based control problem may be written in the Bilinear Matrix Inequality (BMI) form.

IV. Numerical example

Consider the following system:

$$E=\begin{bmatrix}0.9 & 0\\ 0 & 0\end{bmatrix},\ A=\begin{bmatrix}0.85 & -0.3\\ 0.1 & 0.3\end{bmatrix},\ B_1=\begin{bmatrix}-0.02\\ 0\end{bmatrix},$$

$$B_2=\begin{bmatrix}-0.1\\ 0\end{bmatrix},\ C=\begin{bmatrix}0.35 & 0.09\end{bmatrix},\ D_1=0.035,\ D_2=0.1.$$

It is easy to see, that $\mathrm{rank}\,(E)=\mathrm{rank}\,([E \quad B_1])=1$. The system is causal, but unstable ($\rho(E,A)=1.0556$).

Now we find a state feedback control $u(k)=F_2x(k)$ for the given mean anisotropy level a and scalar value γ, using the technique from Theorem 3. The results of controller design for different a and γ are represented in Table I. The transfer function of the closed-loop system is denoted by $P_c(z)=C_c(zE-A_c)^{-1}B+D$.

As we can see, the obtained F_2 guarantees, that anisotropic norm of the closed-loop system is bounded by the given value: $|||P_c|||_a<\gamma$, and the system P_c is admissible.

V. Conclusion

In this paper, the suboptimal state feedback anisotropy-based control problem for linear time-invariant discrete-time descriptor systems is solved. The obtained conditions guarantee that the closed-loop system is admissible and its anisotropic norm is bounded by a given value.

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TABLE I: Conditions and results of anisotropy-based control design for different mean anisotropy levels

a	0.1	0.5	0.9
γ	0.050	0.055	0.060
Φ	$\begin{bmatrix} 1.0610 & 0.7604 \\ 0.7604 & 0.4337 \end{bmatrix}$	$\begin{bmatrix} 1.0769 & 0.7714 \\ 0.7714 & 0.4414 \end{bmatrix}$	$\begin{bmatrix} 1.0791 & 0.7730 \\ 0.7730 & 0.4425 \end{bmatrix}$
F_2	[2.5455, -0.8763]	[2.4077, -0.8772]	[2.3883, -0.8773]
$\rho(E, A_c)$	0.7403	0.7555	0.7577
$\|P_c\|_a$	0.0420	0.0423	0.0424

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