Syllabus

Differential Geometry 1

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Pre-requisites: Linear algebra ,Euclidean geometry, Calculus on Manifolds, Basic Topology
Course Type: optional

Abstract: The course will serve as an introductory guide to basic topics of Differential and Riemann geometry

Learning Objectives: Introducing the audience to the basic theory of Riemannian manifolds and geometry of the Levi-Chivita connections on them.

Learning Outcomes: After mastering the course, the student is expected to understand such fundamental notions as metric, curvature,, connection, geodesic and know how to operate with them

Course Plan:

1. **Differential and Riemann geometry of smooth hypersurfaces in the Euclidean space**:

Parallel transport. The Gauss Map .Shape operator.
The metric connection. Covariant derivatives. Parallel transport

Completeness and geodesics. The Exponential Map. The Hopf- Rinow theorem.

Geometry of a compact classical Lie group/.

Curvature. Geodesics.

2. **manifolds: Curvature and The Ricci tensor.**

Calculations with curvature tensor. The Gauss curvature.

The Ricci tensor.

Spaces of constant curvature...

3. **Variational theory of geodesics.**

First and second variation of arc length

Jacobi”s equation and conjugate points.

The Gauss lemma and polar coordinates.

4. **Connections,. Parallel transport and Covariant derivatives in vector bundle**

Reading List

Gallot,Hulin, Lafontaine,
Riemannian Geometry;
Milnor, Morse Theory.

Optional

Taubes, Differential Geometry. Bundles,
connections, metrics and curvature;
Chavel, Modern Riemann Geometry.

Grading System

Current control grade equals the percentage of the number of solved problems (including bonus problems) to the total number of problems given throughout the semester. The exam consists of a written 3 hour test, containing 6 problems. For a 100% result it suffices to solve at least 5 of 6 problems.

The total grade for the course is computed via the following formula:

Max(150, E+H)/15

where E equals the mark for the written exam and H is the percentage of number of solved problems to the total number of the problems.

Guidelines for Knowledge Assessment

A number of questions, which can be used for the examination:

Write down a projective transformation that maps the unit sphere of R^3 to the paraboloid $x_3={x_1}^2+={x_2}^2$

Calculate the curvature of a surface of revolution generating by rotating a plane curve $X(s),Y(s)$,where s is arc length

Show that a Hyperbolic manifold has flat sphere bundle

Methods of Instruction

The students are given home tasks, containing routine exercises, which assist in understanding theoretical material, and research problems, which require more effort to solve and motivate the students to study extra materials. The solutions are either submitted in written form to the lecturer and his assistants or can be sent via email. Some of the more difficult topics are made into talks, which then are given by students.