I. Course description

RS “Real and Complex analysis ”
prof. S. Lvovski.

1. Prerequisites include familiarity with the basic notions of point-set topology, linear algebra, and advanced calculus.
2. The course is electory.
3. This course will be devoted to the basic notions of functional analysis and their applications to problems of analysis. The main goal will be to study such classical results as Banach-Steinhaus, Hahn-Banach, and Banach-Alaoglu theorems and to get acquainted with their applications to analysis. The course will be based on the books ”Real and Complex Analysis” and “Functional Analysis” by W.Rudin.

II. The objectives and goals of the RS “Real and Complex Analysis” are as follows:
1. introducing the audience to main theorems of functional analysis for the case of Banach spaces;
2. introducing the audience to applications of these theorems in analysis.

III. After mastering the course, the student is expected to:
1. understand such fundamental notions as Lebesgue integral, Hilbert space, Banach space, Banach algebra;
2. know several key examples of non-trivial applications of these notions to analysis.

IV. Plan:
1. Lebesgue integral; construction of measure from positive linear functional.
2. L^p spaces and their completeness; Hilbert spaces.
3. Fourier series, their pointwise convergence. Fejer’s kernel.
4. Banach spaces.
5. Banach-Steinhaus theorem, applications to the divergence of Fourier series.
6. Hahn-Banach theorem. Open mapping and closed graph theorems, their applications.
7. Radon-Nikodym theorem as an application of Hilbert space techniques.
8. Banach-Alaoglu theorem and its applications to Banach algebras.

V. Reading lists:
1. Required
   1) W. Rudin, *Real and complex analysis,* McGraw Hill, 1987.
   2) W. Rudin, *Functional Analysis,* McGraw Hill, 1991.

VI. Current control grade equals the percentage of the number of solved problems (including bonus problems) to the total number of problems given throughout the semester. The exam consists of a written 4-hour test, containing 8 problems. For a 100% result it suffices to solve at least 6 of 8 problems.
The total grade for the course is computed via the following formula:

$$\mathrm{Max}(150, E+H)/15$$

where E equals the mark for the written exam and H is the percentage of number of solved problems to the total number of the problems.

VII. Guidelines for Knowledge Assessment:

1. Sample problems which will be used for knowledge assessment:

    1) Prove that there exists no continuous function $F:R\times[0;1]\to R$ with the following property: for any continuous function $\varphi:[0;1]\to R$ there exists a number t such that $\varphi(x)=F(t,x)$ for any x.
    2) Prove that if f is a continuous function on the unit circle such that all its negative Fourier coefficients vanish then f can be extended to a function that is continuous on the unit disk and holomorphic on its interior.

2. A number of questions which can be used for the examination:
    1) Prove that the dual to a bounded linear mapping between normed spaces ic continuous w.r.t. the weak* topologies.
    2) Prove that the set of functions continuous on the unit disk and holomorphic on its interior is a Banach algebra; describe its space of maximal ideals.
    3) Prove that if a Banach space is infinite-dimensional then its algebraic dimension is uncountable.
    4) True or false: for any function f from L^1 on the unit circle there exists a function g on the init circle such that the “positive” Fourier coefficients of g are equalk to those of f, while “negative” Fourier coefficients of g are zero?

VIII. The students are given home tasks, containing routine exercises, which assist in understanding theoretical material, and research problems, which require more effort to solve and motivate the students to study extra materials. The solutions are either submitted in written form to the lecturer and his assisstants or can be sent via email. Some of the more difficult topics are made into talks, which then are given by students.