SEMI-NUMERICAL METHOD FOR COMPUTATIONALLY EFFECTIVE ANALYSIS OF WORKING MEMORY MODELS

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Introduction

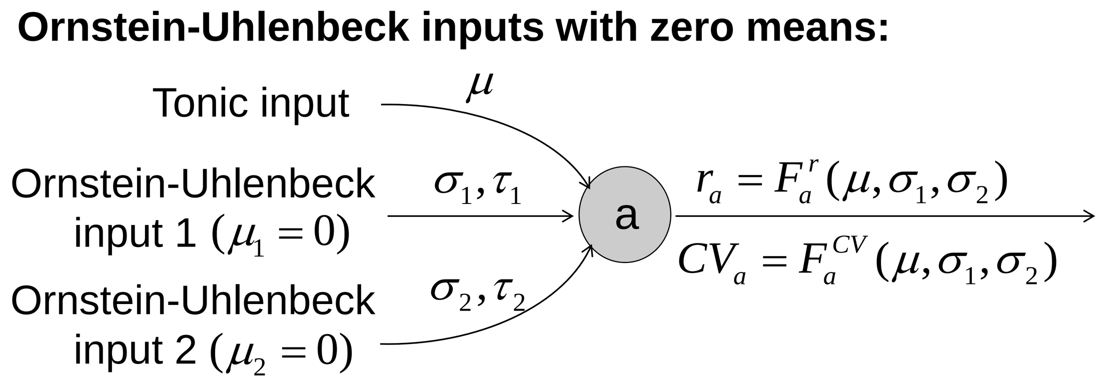
Working memory is the ability to temporarily retain information about sensory stimuli that are no longer directly perceived in such form that this information could be used to guide subsequent behavioral responses. Retention of working memory trace is associated with elevated spiking activity in specific subsets of neurons [1]. From the computational perspective, it could be implemented by a bistable neural network, one stable state of which corresponds to the background firing, and the other one – to the elevated spiking activity during retention of the memory trace [2]. A major problem of classical WM models is an excessive regularity of spike trains in the active state. Recently, a solution for this issue was proposed, based on a balanced network with short-term synaptic potentiation [3,4]. In such a model, mean input current to the neurons stays near zero in the active state, while variation of the current increases due to increased synaptic weights. The exact mathematical solution of this model is complicated because of the need to account for the effect of varying level of noise on the firing rates. In this work, we propose a computationally effective method for predicting the behavior of the model based on nullcline analysis and low-dimensional population models. The method is based on a pre-caclulation of neuronal gain functions under two Ornstein-Uhlenbeck current inputs (representing synaptic currents from excitatory and inhibitory subpopulations) in the nodes of 3-dimensional grid (coordinates: mean input, variances of both inputs) with subsequent numerical interpolation of the results. In turn, steady-state values of input means and variances can be found analytically as functions of instantaneous firing rates, which allows us to find self-consistent solutions numerically. Using the described method, we perform nullcline analysis of the WM model based on a balanced network with short-term synaptic potentiation, similarly to the model described in [4]. We demonstrate that the predicted stable states correspond to the ones obtained by spiking network simulation. We also lay out how to derive a low-dimensional model based on exponential convergence of firing rates, as well as means and variances of the synaptic currents, to the corresponding steady-states obtained using the method described above, and demonstrate that the behavior of this model of qualitatively similar to the behavior of the corresponding spiking network.

Methods and Results

LIF neuron parameters

Membrane conductance:	$g_m = 100\,\mu S/cm^2$
Membrane capacitance (E):	$C_{m,e} = 2\,\mu F/cm^2$
Membrane capacitance (I):	$C_{m,i} = 1\,\mu F/cm^2$
Resting potential:	$V_L = -70\,mV$
Spiking threshold:	$V_t = -50\,mV$
Reset voltage:	$V_r = -75\,mV$

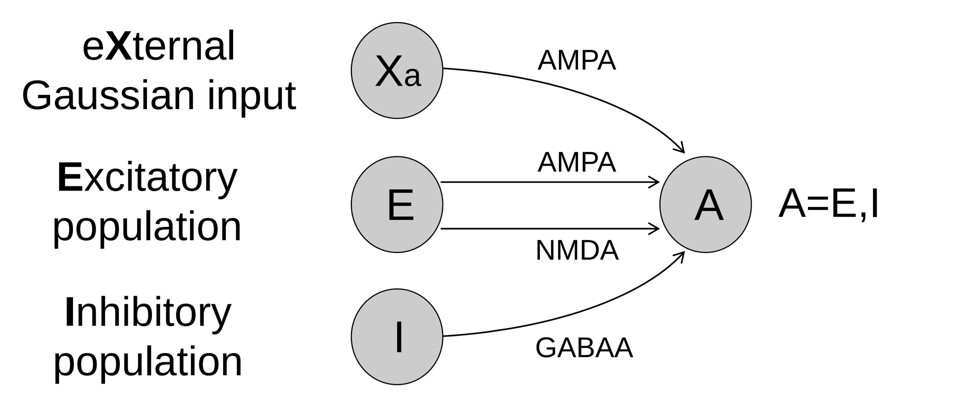
Consider a LIF neuron with tonic input and two Ornstein-Uhlenbeck inputs with zero means:



1. Fix neuron parameters and τ_1, τ_2 (usually known from experiment)
2. Choose rectangular grid: $\{(\mu^i, \sigma_1^j, \sigma_2^k)\}$
3. For each combination *(i,j,k)*, calculate gain function $r_a = F_a^r(\mu^i, \sigma_1^j, \sigma_2^k)$ and $CV_a = F_a^{CV}(\mu^i, \sigma_1^j, \sigma_2^k)$
4. Now r_a, CV_a can be calculated for arbitrary $(\mu, \sigma_1, \sigma_2)$ using numerical interpolation

*This scheme allows to **quickly explore connectivity effects and perform low-dimensional simulations**, because steps 1-3 (computationally expensive) do not depend on synaptic weights or instantaneous firing rates of populations*

Network parameters



E-E synapses have STP (Tsodyks-Markram model)

Synaptic weights:

$\tilde{J}_{ee}^{AMPA} = 1.5\ \mu A/cm^2$

$\tilde{J}_{ee}^{NMDA} = 0.25\ \mu A/cm^2$

$J_{ei}^{AMPA} = 0.26\ \mu A/cm^2$

$J_{ei}^{NMDA} = 0.004\ \mu A/cm^2$

$J_{ie} = -1.15\ \mu A/cm^2$

$J_{ii} = -0.42\ \mu A/cm^2$

Background inputs:

$\mu_{xe} = 2.0\,\mu A/cm^2$

$\mu_{xi} = 0.6\,\mu A/cm^2$

$\sigma_{xe} = 2.0\,\mu A/cm^2$

$\sigma_{xi} = 2.0\,\mu A/cm^2$

Numbers of inputs:

$K_{ee} = K_{ei} = 400$

$K_{ie} = K_{ii} = 100$

Synaptic time constants:

$\tau_e^{AMPA} = 2\,ms$

$\tau_e^{NMDA} = 50\,ms$

$\tau_i = 5\,ms$

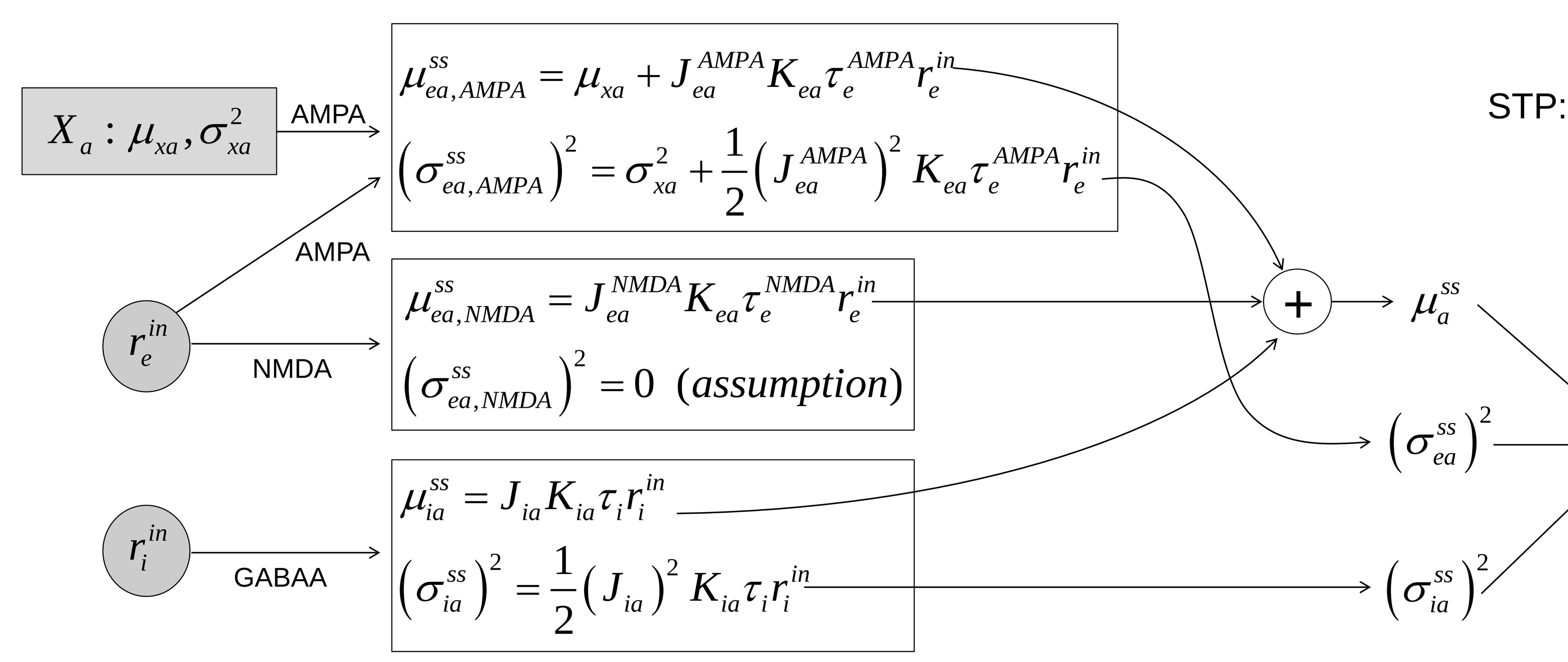
Short-term plasticity:

$U = 0.03$

$\tau_F = 450\,ms$

$\tau_F = 200\,ms$

Calculating steady-state

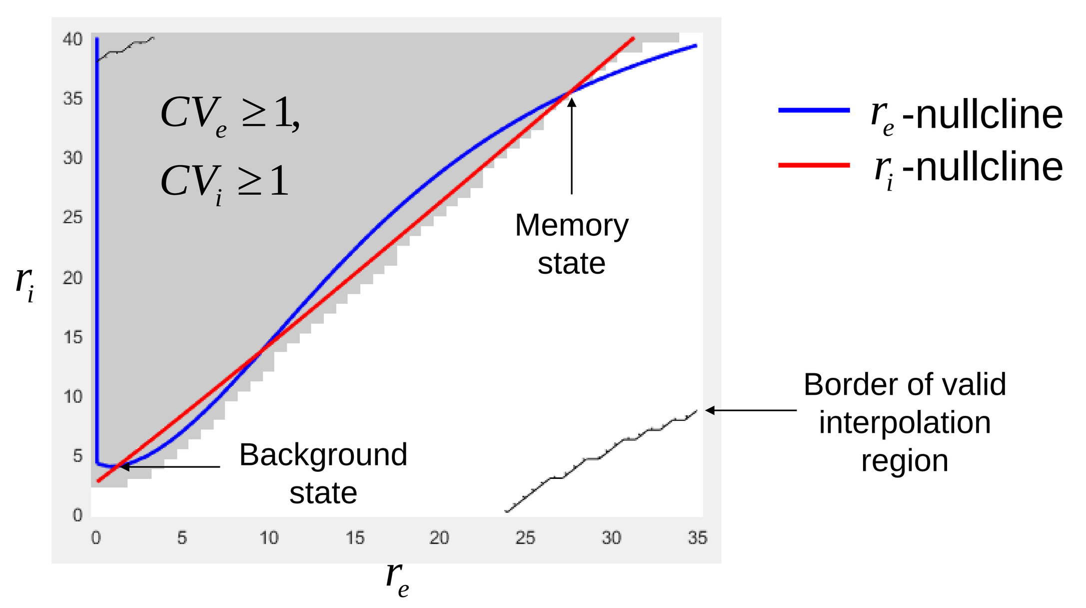


STP:

$$\begin{cases} J_{ea}^{AMPA} = \tilde{J}_{ea}^{AMPA} x_{ss} u_{ss} \\ J_{ea}^{NMDA} = \tilde{J}_{ea}^{NMDA} x_{ss} u_{ss} \\ u_{ss} = U/(1-\alpha_F(1-U)) \\ x_{ss} = (1-\alpha_D)/(1-\alpha_D(1-u_{ss})) \\ \alpha_F = \exp(-1/(r_e^{in}\tau_F)) \\ \alpha_D = \exp(-1/(r_e^{in}\tau_D)) \end{cases}$$

Nullcline analysis

1. For each combination (r_e^{in}, r_i^{in}) find r_e^{ss}, r_i^{ss} and CV_e^{ss}, CV_i^{ss}
2. Find nullclines from conditions: $r_e^{ss} = r_e^{in}$, $r_i^{ss} = r_i^{in}$



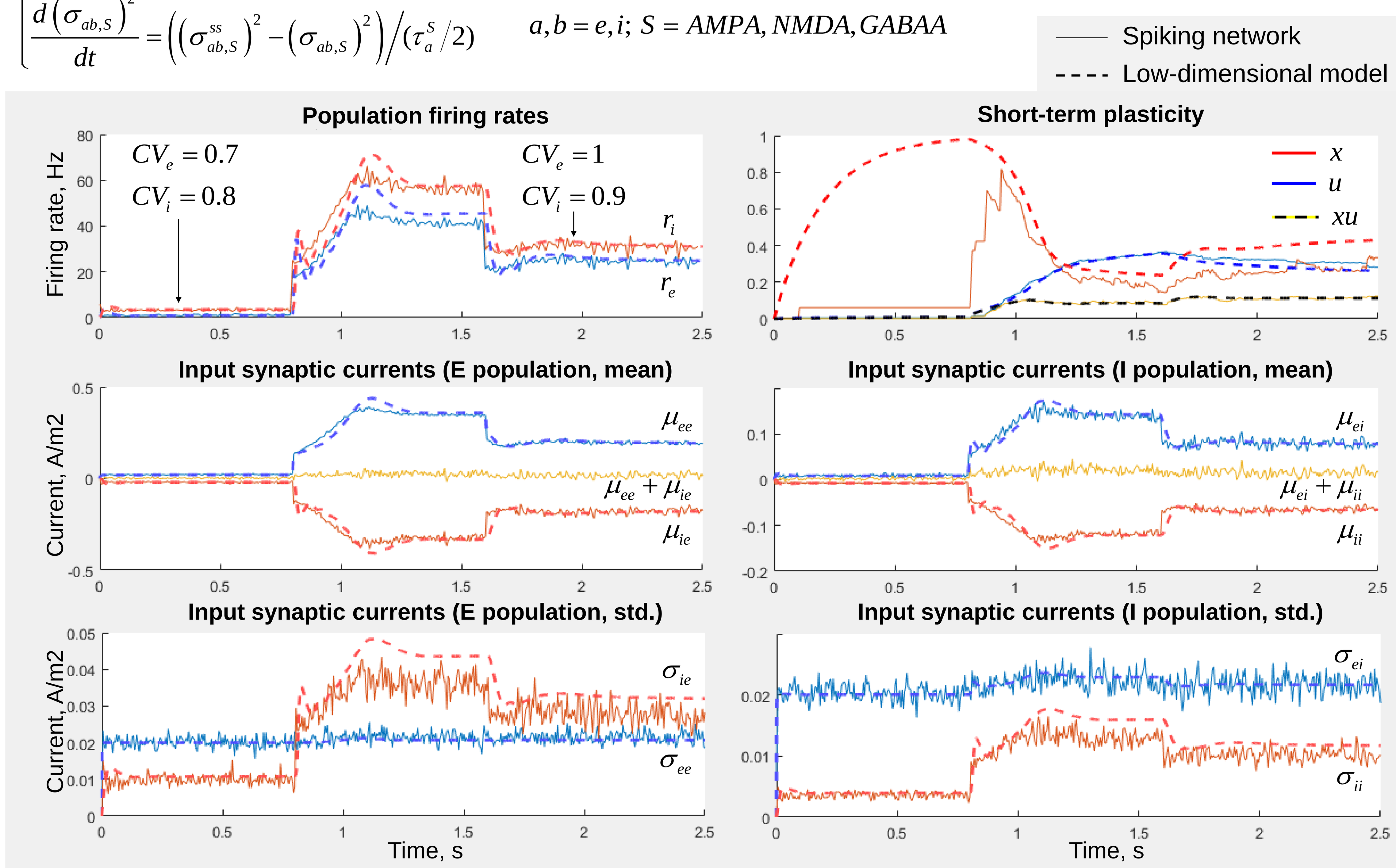
Low-dimensional dynamics

$$\begin{cases} \frac{dr_a}{dt} = (r_a^{ss} - r_a)/\tau_{pop,a} \\ \frac{d\mu_{ab,S}}{dt} = (\mu_{ab,S}^{ss} - \mu_{ab,S})/\tau_a^S \\ \frac{d(\sigma_{ab,S})^2}{dt} = \left((\sigma_{ab,S}^{ss})^2 - (\sigma_{ab,S})^2\right)/(\tau_a^S/2) \end{cases}$$

$$\begin{cases} \frac{du}{dt} = -\frac{u}{\tau_F} + U(1-u)r_e \\ \frac{dx}{dt} = -\frac{(1-x)}{\tau_D} - uxr_e \end{cases}$$

$a, b = e, i;\ S = AMPA, NMDA, GABAA$

We use large population time constants: $\tau_{pop,e} = \tau_{pop,i} = 50\,ms$ (Otherwise, low-dimensional model produces oscillations)



Conclusion

1. Our semi-numerical scheme allows to quickly perform nullcline analysis and find steady states, as well as the corresponding CV values
2. The scheme also allows to build a low-dimensional model that takes into account dynamics of (1) population firing rates, (2) means and variances of synaptic currents, (3) synaptic weights (STP).
3. The predictions of the nullcline analysis and the results of the low-dimensional model simulation are in the good agreement with the results of spiking network simulation
4. The low-dimensional model is much more predisposed to oscillations than the spiking model, so we had to use unrealistically high time constants for convergence of population firing rates to steady-states

References

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